Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. 8, No. 3, 2020, pp. 585-596
DOI:10.22034/cmde.2020.31374.1471

Complex wave surfaces to the extended shallow water wave model with (2+1)-dimensional

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Abstract In this paper, we apply an analytical method, namely, the sine-Gordon expansion method and extract some complex optical soliton solutions to the (2+1)-dimensional extended shallow water wave model, which describes the evolution of shallow water wave propagation. We obtain some complex mixed-dark and bright soliton solutions to this nonlinear model. Considering some suitable values of parameters, we plot the various dimensional simulations of every results found in this manuscript. We observe that our result may be useful in detecting some complex wave behaviors.

Keywords. Extended shallow water wave model, Sine-Gordon expansion method, Complex mixed-dark and bright solitons.

2010 Mathematics Subject Classification. 35Axx, 35Dxx.

1. Introduction

Plasma is a mixture of ions, free electrons, and also neutral atoms or molecules [26]. The major difference between plasma and other matter is that plasma contain charged particles, furthermore, an important characteristic of plasmas is their ability to sustain a great variety of wave phenomena [3, 26]. Charged particles produce wave propagation in various powers. Such a kind propagation symbolize different prototype of matter, and, give important clues about charged particles from cell wave propagations to water wave propagations. In this sense, many nonlinear evolution equations (NLEEs) for various complex physical problems arising in the fields of nonlinear science, such as fluid mechanic, plasma physics, along with sets of waves such as magnetohydrodynamic waves. Moreover, in the last several decades, both various approaches, power tools and their modifications such as exponential function method, the modified simple equation method, the Kudryashov method, sumudu transform method, $(G^{'}/G)$-expansion method, Hirota bilinear method and many more techniques [1, 2, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 54] have been

Received: 6 January 2019 ; Accepted: 15 June 2019.
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presented to the literature. In this direct, one of such models is the extended shallow water wave model (ESWWM) with (2+1)-dimensional defined as [19, 40, 46, 54]

$$u_{yt} + 3u_{xxxy} - 3u_{xx}u_y - 3u_x u_{xy} + ku_{xy} = 0, \tag{1.1}$$

where k is constant. Equation (1.1) is a frequently utilized model for exploring dynamics of solitons and nonlinear waves to describe the (2+1)-dimensional interaction of a Riemann wave propagating along the y-axis with a long wave propagating along the x-axis in fluid dynamics, plasma physics and weakly dispersive media [40].
This paper is constructed in the following sections. We present the general properties of the powerful sine-Gordon expansion method (SGEM) [5, 6, 7], in the second section. We find some combined complex dark-bright optical soliton solutions to the Eq. (1.1) by utilizing SGEM in the third section. We present a conclusion in the last section of the paper.

2. General Properties of SGEM

In this section, we discuss the general facts of SGEM. Consider the following sine-Gordon equation [51, 52, 53]:

$$u_{xx} - u_{tt} = m^2 sin(u), \tag{2.1}$$

where $u = u(x,\ t)$ and m is a real constant.

Applying the wave transformation $u = u(x,\ t) = U(\xi)$, $\xi = \mu(x - ct)$ to Eq. (2.1), yields the following nonlinear ordinary differential equation (NODE):

$$U^{''} = \frac{m^2}{\mu^2(1-c^2)} sin(U), \tag{2.2}$$

where $U = U(\xi)$, ξ is the amplitude of the travelling wave and c is the velocity of the travelling wave. Reconsidering Eq. (2.2), we can write its full simplification as:

$$\left[\left(\frac{U}{2}\right)^{'}\right]^2 = \frac{m^2}{\mu^2(1-c^2)} sin^2\left(\frac{U}{2}\right) + K, \tag{2.3}$$

where K is the integration constant.

Substituting $K = 0$, $w(\xi) = \frac{U}{2}$ and $a^2 = \frac{m^2}{\mu^2(1-c^2)}$ in Eq. (2.3), gives:

$$w^{'} = asin(w), \tag{2.4}$$

Putting $a = 1$ in Eq. (2.4), we have:

$$w^{'} = sin(w). \tag{2.5}$$

Equation (2.5) is variables separable equation, we obtain the following two significant equations from solving it;

$$sin(w) = sin(w(\xi)) = \left.\frac{2pe^{\xi}}{p^2e^{2\xi}+1}\right|_{p=1} = sech(\xi), \tag{2.6}$$

$$cos(w) = cos(w(\xi)) = \frac{p^2e^{2\xi}-1}{p^2e^{2\xi}+1}\Big|_{p=1} = tanh(\xi), \tag{2.7}$$

where p is the integral constant.

For the solution of the following nonlinear partial differential equation;

$$P(u,\ u_x,\ u_t,\ u_{xx},\ u_{tt},\ u_{xt},\ u_{xxx},\ldots) = 0, \tag{2.8}$$

we consider,

$$U(\xi) = \sum_{i=1}^{n} tanh^{i-1}(\xi)\big[B_i sech(\xi) + A_i tanh(\xi)\big] + A_0. \tag{2.9}$$

Equation(2.9) can be rewritten according to Eqs. (2.6) and (2.7) as follows:

$$U(w) = \sum_{i=1}^{n} cos^{i-1}(w)\big[B_i sin(w) + A_i cos(w)\big] + A_0. \tag{2.10}$$

We determine the value n under the terms of NODE by balance principle. Letting the coefficients of $sin^i(w)cos^j(w)$ to be all zero, yields a system of equations. Solving this system by using various computational programs gives the values of $A_i,\ B_i,\ \mu$ and c. Finally, substituting the values of $A_i,\ B_i,\ \mu$ and c in Eq. (2.9), we obtain the new travelling wave solutions to Eq. (2.8).

3. Application of SGEM

In this section, the application of SGEM to the ESWWM is presented. Firstly, we start by transforming Eq.(1.1) into a nonlinear ordinary differential equation by using the following travelling wave transformation:

$$u(x,y,t) = mx + ny - ct. \tag{3.1}$$

Substituting Eq. (3.1) into Eq. (1.1), the following nonlinear ordinary differential equation is obtained:

$$3m^3U^{(4)} - 6m^2U'U'' + (km-c)U'' = 0. \tag{3.2}$$

Integrating once of Eq. (3.2) and getting to zero of integral constant gives the following nonlinear model

$$3m^3U''' - 3m^2(U')^2 + (km-c)U' = 0. \tag{3.3}$$

For simplicity, when we reconsider as $V = U'$, then, we can rewrite Eq.(3.3) as

$$3m^3V'' - 3m^2V^2 + (km-c)V = 0. \tag{3.4}$$

With the help of the Balance principle, between V^2 and $V^{''}$ in Eq(3.4), yields $n = 2$. Using $n = 2$ into Eq. (2.9) produces

$$V(\xi) = B_1 sech(\xi) + A_1 tanh(\xi) + tanh(\xi)(B_2 sech(\xi) + A_2 tanh(\xi)) + A_0. \quad (3.5)$$

and into Eq. (2.10) gives

$$V(w) = B_1 sin(w) + A_1 cos(w) + B_2 cos(w) sin(w) + A_2 cos(w)^2 + A_0. \quad (3.6)$$

Differentiating Eq. (3.6) twice and getting integrate constants to zero, yields

$$V^{''} = B_1 cos^2(w) sin(w) - B_1 sin^3(w) - 2A_1 sin^2(w) cos(w) + B_2 sin(w) cos(w)^3$$

$$-5B_2 sin(w)^3 cos(w) - 4A_2 sin(w)^2 cos(w)^2 + 2A_2 sin(w)^4. \quad (3.7)$$

Inserting Eqs. (3.6) and (3.7) into Eq. (3.4), gives the an algebraic equation in trigonometric function including various form of $sin^i(w)cos^j(w)$. Getting the coefficients of trigonometric terms in the same power to zero, give a system. Solving this system with aid of symbolic software to obtain the values of the coefficients involved, we find the following coefficients in each case obtained from the set of algebraic equation systems, and it gives the travelling wave solutions to Eq.(1.1).

Case 1:

$$A_0 = \frac{-2iB_2}{3}, A_1 = 0, A_2 = iB_2, B_1 = 0, m = \frac{iB_2}{3}, c = \frac{1}{9} iB_2(3k + B_2^2)$$

substituting these coefficients into Eq. (3.5), we have

$$u_1(x, y, t) = -sech[ny + \frac{1}{3} iB_2 x - \frac{1}{9} iB_2(3k + B_2^2)t]B_2 + \frac{1}{3} iB_2[ny + \frac{1}{3} iB_2 x$$

$$-\frac{1}{9} iB_2(3k + B_2^2)t] - iB_2 tanh[ny + \frac{1}{3} iB_2 x - \frac{1}{9} iB_2(3k + B_2^2)t]. \quad (3.8)$$

FIGURE 1. The 3D surfaces of Eq. (3.8) under the values $B_2 = 1.5$, $n = 2$, $k = 3$, $y = 0.03$, $-12 < x < 12$, $-12 < t < 12$.

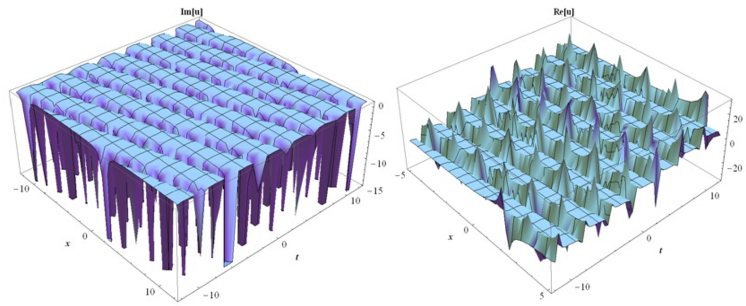


FIGURE 2. The contour surfaces of Eq. (3.8) under the values $B_2 = 1.5$, $n = 2$, $k = 3$, $y = 0.03$, $-180 < x < 180$, $-180 < t < 180$.

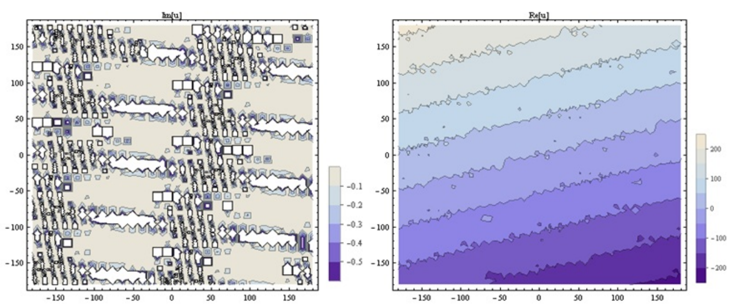
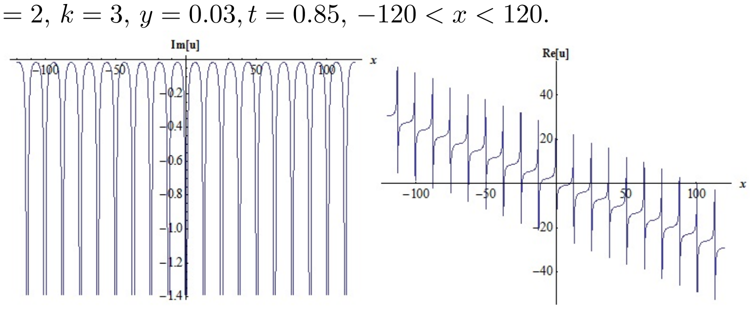


FIGURE 3. The 2D surfaces of Eq. (3.8) under the values $B_2 = 1.5$, $n = 2$, $k = 3$, $y = 0.03, t = 0.85$, $-120 < x < 120$.



Case 2: When

$$A_1 = 0, A_2 = \frac{-3A_0}{2}, B_1 = 0, B_2 = \frac{-3iA_0}{2}, m = \frac{-A_0}{2}, c = \frac{1}{8}A_0(-4k + 3A_0^2),$$

we have

$$u_2(x,y,t)=\frac{3}{2}iA_0sech[ny-\frac{A_0}{2}x-\frac{1}{8}A_0(-4k+3A_0^2)t]-\frac{1}{2}A_0[ny-\frac{A_0}{2}x$$

$$-\frac{1}{8}A_0(-4k+3A_0^2)t]+\frac{3}{2}A_0tanh[ny-\frac{A_0}{2}x-\frac{1}{8}A_0(-4k+3A_0^2)t]. \tag{3.9}$$

FIGURE 4. The 3D surfaces of Eq. (3.9) under the values $A_0 = 2$, $n = -1$, $k = 3$, $y = 0.4$, $-12 < x < 12$,$-12 < t < 12$.

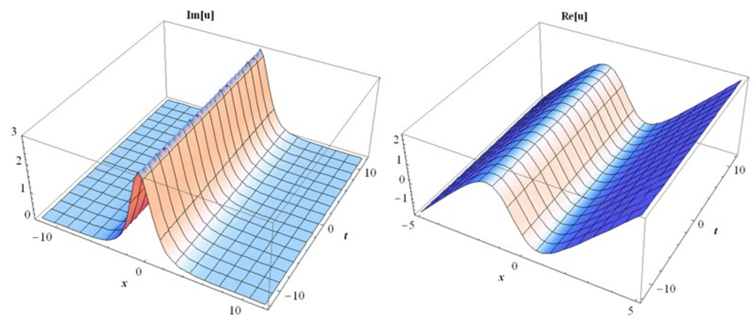


FIGURE 5. The contour surfaces of Eq. (3.9) under the values $A_0 = 2$, $n = -1$, $k = 3$, $y = 0.4$, $-10 < x < 10$,$-10 < t < 10$.

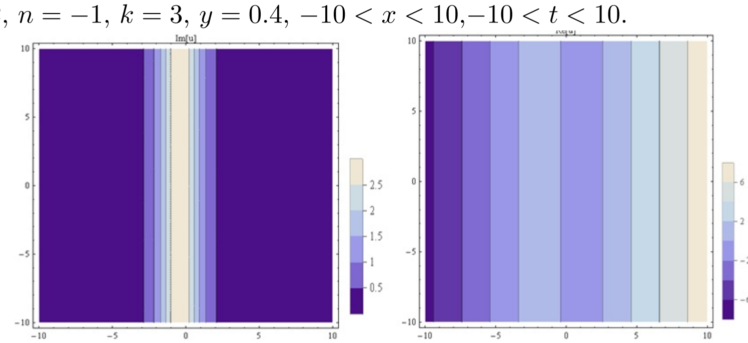
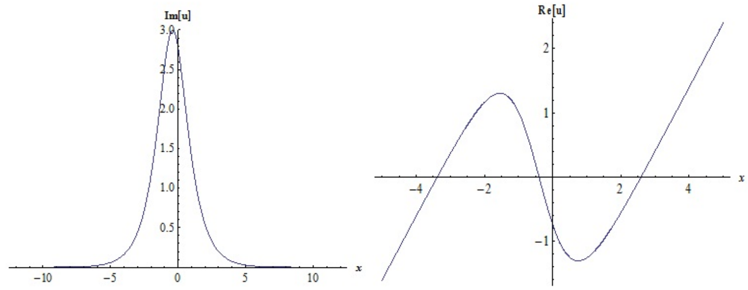


FIGURE 6. The 2D surfaces of Eq. (3.9) under the values $A_0 = 2$, $n = -1$, $k = 3$, $y = 0.4, t = 0.85$, $-12 < x < 12$.



Case 3: If

$$A_0 = \frac{2iB_2}{3}, A_1 = 0, A_2 = -iB_2, B_1 = 0, m = \frac{-iB_2}{3}, c = \frac{-1}{9} iB_2(3k + B_2^2),$$

we obtain

$$u_3(x, y, t) = -sech[ny - \frac{1}{3} iB_2 x + \frac{1}{9} iB_2(3k + B_2^2)t]B_2 - \frac{1}{3} iB_2[ny - \frac{1}{3} iB_2 x$$

$$+\frac{1}{9} iB_2(3k + B_2^2)t] + iB_2 tanh[ny - \frac{1}{3} iB_2 x + \frac{1}{9} iB_2(3k + B_2^2)t]. \quad (3.10)$$

FIGURE 7. The 3D surfaces of Eq. (3.10) under the values $B_2 = 2$, $n = -1$, $k = 3$, $y = -0.2$, $-6 < x < 6$,$-6 < t < 6$.

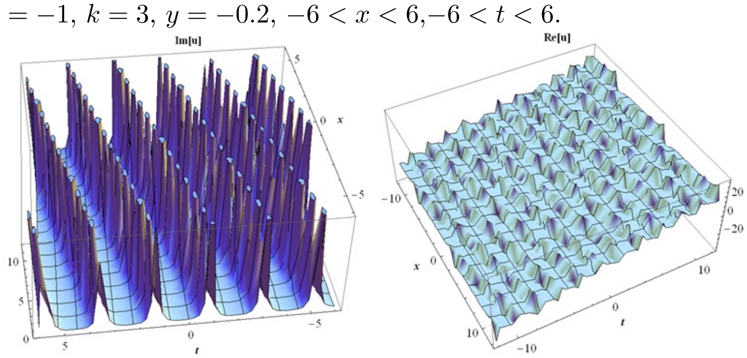


FIGURE 8. The contour surfaces of Eq. (3.10) under the values $B_2 = 2$, $n = -1$, $k = 3$, $y = -0.2$, $-180 < x < 180$,$-180 < t < 180$.

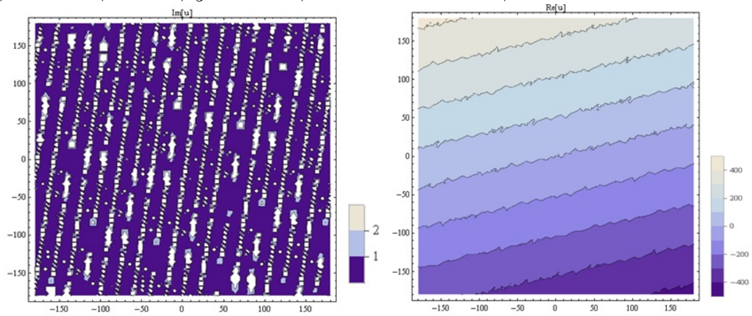
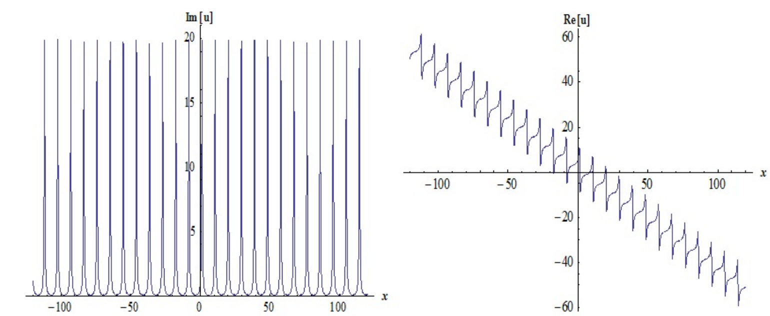


FIGURE 9. The 2D surfaces of Eq. (3.10) under the values $B_2 = 2$, $n = -1$, $k = 3$, $y = -0.2, t = 0.85$, $-120 < x < 120$.



Case-4: Considered into Eq. (3.5) as

$$A_0 = -3m, A_1 = 0, A_2 = 3m, B_1 = 0, B_2 = -3im, c = m(k + 3m^2),$$

we find the singular soliton solution

$$u_4(x, y, t) = 3imsech[m(k+3m^2)t-mx-ny]+3mtanh[m(k+3m^2)t-mx-ny]. \tag{3.11}$$

4. Conclusion

In this manuscript, we have successfully applied the powerful SGEM to extract the complex solutions to the extended shallow water wave model with (2+1)-dimensional. We have found firstly new complex mixed-type optical soliton solutions to this equation. 3D surfaces can be seen in Figures (1), (4), (7) and (10), and contour graphs may also be viewed in Figures (2), (5) and (8) and 2D surfaces can be observed in the Figures (3), (6), (9) and (11) in this study, under the choice of suitable parameters.

FIGURE 10. The 3D surfaces of Eq.(3.11) under the values $m = 1$, $n = 2$, $k = 3$, $y = 0.2$, $-6 < x < 6$, $-6 < t < 6$.

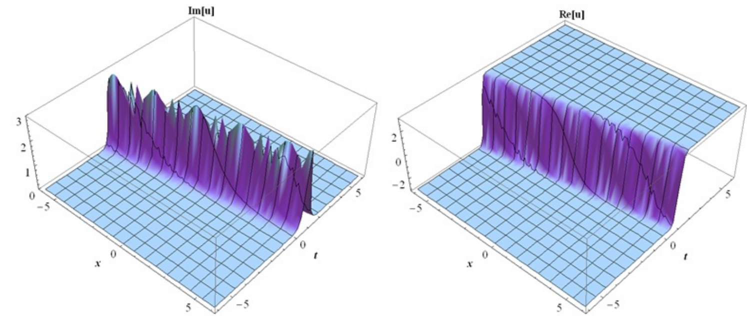
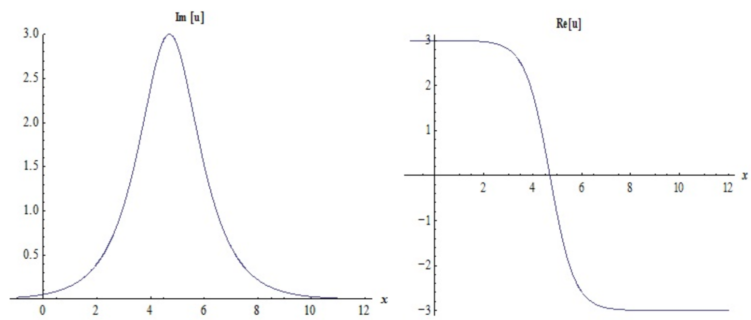


FIGURE 11. The 2D surfaces of Eq. (3.11) under the values $m = 1$, $n = 2$, $k = 3$, $y = 0.2, t = 0.85$, $-1 < x < 12$.



Moreover, we have compared our results with some of the existing results in the literature. Periodic and lump wave structures to this equation have been obtained with the aid of Hirota bilinear form in [40]. We observed that our results are entirely newly constructed in the sense of complex structures. Furthermore, the obtained results, in this study, have some important physical meaning which is related to the studied models, for example, it has been presented that the hyperbolic tangent arises in the calculation of magnetic moment and rapidity of special relativity, the hyperbolic secant arises in the profile of a laminar jet in [48]. It can be therefore seen that our results may also be useful in explaining the physical behavior of the studied models and many other nonlinear models arising in the field of nonlinear science. What is more and more interestingly, it has been observed that soliton waves of the solutions found in this paper remain unchanged. To the best of our knowledge, the application of SGEM has not applied to the model previously. should be numbered with roman numerals in the order of appearance. Every table must have a caption, which should be typed above the table.

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