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New analytical soliton type solutions for double layers structure model of extended KdV equation

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Abstract In this present study, the double layers structure model of extended Korteweg-de Vries (KdV) equation will be obtained with the help of the reductive perturbation method, which admits a double layer structure in current plasma model. Then by using of new analytical method we obtain the new exact solitary wave solutions of this equation. Double layer is a structure in plasma and consists of two parallel layers with opposite electrical charge. The sheets of charge cause a strong electric field and a correspondingly sharp change in electrical potential across the double layer. As a result, they are expected to be an important process in many different types of space plasmas on earth and on many astrophysical objects. These nonlinear structures can occur naturally in a variety of space plasmas environment. They are described by the Korteweg-de Vries (KdV) equation with additional term of cubic nonlinearity in different homogeneous plasma systems. The performance of this method is reliable, simple and gives many new exact solutions. The (G'/G)-expansion method has more advantages: It is direct and concise.

Keywords. Double layers, Extended Korteweg-de Vries, Analytical method.

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1. Introduction

A solitary wave is a wave which propagates without any temporal evolution in shape or size when viewed in the reference frame moving with the group velocity of the wave. The envelope of the wave has one global peak and decays far away from the peak. Solitary waves arise in many contexts, including the elevation of the surface of water and the intensity of light in optical fibers. A soliton is a nonlinear solitary wave with the additional property that the wave retains its permanent structure, even after interacting with another soliton. For example, two solitons propagating in opposite directions effectively pass through each other without breaking. Solitons form a special class of solutions of model equations, including the Korteweg de-Vries (KdV) and

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the Nonlinear Schrodinger (NLS) equations. These model equations are approximations, which hold under a restrictive set of conditions. The soliton solutions obtained from the model equations provide important insight into the dynamics of solitary waves. Exact solutions of NPDEs play an important role in the proper understanding of qualitative features of many phenomena and processes in the mentioned areas of natural science.

These equations are mathematical models of complex physical phenomena that arise in engineering, applied mathematics, chemistry, biology, mechanics, physics, etc. Thus, it is very important to reveal exact solutions of nonlinear partial differential equations. Two-temperature plasmas are one of very interesting subjects in plasma physic because they can open new research area for scientists. The theoretical and experimental results have shown that the characteristics of solitary waves strongly modified by the presence of minority population of cold electrons [2, 9, 13], Many authors [1, 3, 4, 14] have investigated the propagation of linear and nonlinear waves and their stability properties in Maxwellian plasmas. However, observations show that astrophysical and space plasmas have particle distribution function that are quasi-Maxwellian up to the mean thermal velocities, and possess non-Maxwellian superthermal tails at the high velocities or energies. Such superthermal plasmas may be found naturally in the magnetosphere of earth, Saturn, Mercury, Uranus, and in the solar wind [12] and can be modeled by the socalled Lorentzian (or kappa) distribution [10]. The Lorentzian distribution function in three-dimensional is given as [6, 8, 10, 11]

$$f_k\left(\nu\right) = \frac{n_0}{\left(\pi\kappa\theta^2\right)^{\frac{3}{2}}} \frac{\Gamma\left(\kappa+1\right)}{\Gamma\left(\kappa-1/2\right)} \left(1+\frac{\nu^2}{\kappa\theta^2}\right)^{-(\kappa+1)}, \tag{1.1}$$

where n_0 is the species equilibrium number density, $\theta = \left[1-3/\left(2\kappa_B T/m\right)\right]^{\frac{1}{2}}$ is the characteristic velocity, T is the kinetic temperature and m is the species mass. Here z obviously denotes the square velocity norm of the velocity ν, $\Gamma\left(x\right)$ is the usual gamma function and κ is the spectral index that measuring deviation from Maxwellian equilibrium. We shall note that the effective thermal speed θ is only defined for $\kappa > 3/2$, and thus, when considering physical quantities derived from Eq. (1.1), such as the density, we shall use $\kappa > 3/2$. For large values of κ(in limit $\kappa \to \infty$) kappa distributions reduces to Maxwellian distribution.

The objective of this article is the constructing exact solitary wave solutions for double layers structure model of extended Korteweg-de Vries (KdV) equation by using of the new extension of the (G'/G)-expansion method [8]. We assume the solution of NLEEs is of the form $u\left(\xi\right) = \sum\limits_{i=0}^{n} \alpha_i \left(m+F\left(\xi\right)\right)^i + \sum\limits_{i=1}^{n} \beta_i \left(m+F\left(\xi\right)\right)^{-i}$ where $F\left(\xi\right) = G'/G$, and $G = G\left(\xi\right)$ satisfies the ordinary differential equation $G''\left(\xi\right)+\lambda G'\left(\xi\right)+\mu G\left(\xi\right) = 0$, where k and l are arbitrary constants. From our observation we found that if we set $m = 0$ and leave out the portion $\sum\limits_{i=1}^{n} \beta_i \left(m+F\left(\xi\right)\right)^{-i}$ in our solution, then our solutions coincides with the solutions introduced by Wang et al. [16] and improved by other researchers [5, 7, 15, 16].

On comparing between the (G'/G)-expansion method and the other methods such as the modified tanhcoth method, we conclude that the (G'/G)- expansion method is more powerful, effective and convenient. The performance of this method is reliable, simple and gives many new exact solutions. The (G'/G)-expansion method has more advantages: It is direct and concise. It is also a standard and computerizable method which allows us to solve complicated nonlinear evolution equations in mathematical physics. We have noted that the (G'/G)-expansion method changes the given difficult problems into simple problems which can be solved easily.
The remainder of this paper is organized as follows. In Section 2, basic equations for derivation the extended KdV equation in current plasma model is discussed. In Section 3, basic structure of analytical method and application to the extended Korteweg-de Vries (KdV) equation is expressed. Finally, some conclusions are given in Section 4.

2. Basic equations for derivation the extended KdV equation in current plasma model

In this section, we consider the plasma consisting of cold ions and two distinct group of electrons, cold electrons (n_c, T_c) and hot electrons (n_h, T_h), where the Lorentzian (kappa) distribution assumed for electrons in the form:

$$n_c = \left(1 - \frac{\varphi}{\kappa_c - \frac{3}{2}}\right)^{-\kappa_c + \frac{1}{2}}, \tag{2.1}$$

$$n_h = \left(1 - \frac{\sigma\varphi}{\kappa_h - \frac{3}{2}}\right)^{-\kappa_h + \frac{1}{2}}. \tag{2.2}$$

The nonlinear dynamics of the cold ions, is described as follow normalized equations

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{2.3}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \varphi}{\partial x}, \tag{2.4}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = (1-f)n_c + f n_h - n. \tag{2.5}$$

The quantities n, n_c and n_h are the ion, cold and hot electrons densities normalized by the unperturbed density $n_{j0}\,(j = i, c, h)$, u is the ion fluid velocity normalized by ion-acoustic speed $C_s = (k_B T_c/m_i)^{1/2}$, where k_B and T_c are the Boltzmann's constant and cold electron temperature, respectively. The space coordinate x and time t are normalized by the ion Debye length $\lambda_D = (k_B T_c/4\pi e^2 n_{i0})^{1/2}$ and ion plasma period $\omega_{pi} = (4\pi e^2 n_{i0}/m_i)^{1/2}$, respectively. The quantity φ is the normalized by $k_B T_c/e$, where e is the electron charge. In equilibrium state we have $n_{i0} = n_{c0} + n_{h0}$, or $n_{c0}/n_{i0} = 1 - f$. Here $\sigma = T_c/T_h$, $f = n_{h0}/n_{i0}$ and the real parameters κ_c and κ_h are the spectral index that measuring deviation from Maxwellian equilibrium for cold and hot electrons, respectively.

Now for solving the nonlinear equations (2.3)–(2.5), we apply the reductive perturbation technique to find the evolution equation, extended KdV equation, in present plasma model. The perturbed quantities can be expanded in the power series of ε as follows

$$\begin{bmatrix} n \\ u \\ \varphi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_1 \\ u_1 \\ \varphi_1 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_2 \\ u_2 \\ \varphi_2 \end{bmatrix} + \varepsilon^3 \begin{bmatrix} n_3 \\ u_3 \\ \varphi_3 \end{bmatrix} + ... \tag{2.6}$$

To study the double layers and obtaining extended KdV equation the independent variables are stretched as

$$\xi = \varepsilon(x - \lambda t), \tau = \varepsilon^3 t, \tag{2.7}$$

where ε is a small parameter and λ is the phase velocity of solitons.
Substituting (2.6) and (2.7) into the system of (2.1) - (2.5) and collecting the terms of various powers of ε, we obtain the following expressions to the lowest order of ε:

$$n_1 = \frac{1}{\lambda^2}\varphi_1, u_1 = \frac{1}{\lambda}\varphi_1. \tag{2.8}$$

Now, using the first order relations given in Eq. (2.8) into the lowest order of the Poisson equation we have

$$\lambda = \sqrt{\frac{1}{C_1(1-f) + D_1 f\sigma}}, \tag{2.9}$$

$$C_1 = \frac{\kappa_c - \frac{1}{2}}{\kappa_c - \frac{3}{2}}, \ D_1 = \frac{\kappa_h - \frac{1}{2}}{\kappa_h - \frac{3}{2}}. \tag{2.10}$$

The next order of ε for the density and velocity of ions lead to

$$n_2 = \frac{3}{2\lambda^4}\varphi_1^2 + \frac{1}{\lambda^2}\varphi_2, \tag{2.11}$$

$$u_2 = \frac{1}{2\lambda^3}\varphi_1^2 + \frac{1}{\lambda}\varphi_2. \tag{2.12}$$

Also, from the Poisson equation for the next higher order ε have

$$\left((1-f)C_1 + fD_1\sigma - \frac{1}{\lambda^2}\right)\varphi_2 = \left[\frac{3}{2\lambda^4} - \left((1-f)C_2 + fD_2\sigma^2\right)\right](\varphi_1)^2, \tag{2.13}$$

where

$$C_2 = \frac{(\kappa_c - \frac{1}{2})(\kappa_c + \frac{1}{2})}{2(\kappa_c - \frac{3}{2})^2}, \ D_2 = \frac{(\kappa_h - \frac{1}{2})(\kappa_h + \frac{1}{2})}{2(\kappa_h - \frac{3}{2})^2}. \tag{2.14}$$

According to Eq. (2.9), the left-hand side of Eq. (2.13) is zero. Since on the right-hand side $\varphi_1 \neq 0$, then term in the square bracket on the right-hand side of Eq. (2.13) should be at least of the first order of ε showing that Eq. (2.13) is now of the third order of ε. Therefore, the term in the square bracket in Eq. (2.13) should be added in the third order terms of Poisson equation, which gives a favorable condition for

DL, i.e., shock like structures instead of solitons (Watanabe 1984). Therefore, from the next higher order of ε we have

$$\frac{\partial n_1}{\partial \tau} - \lambda \frac{\partial n_3}{\partial \xi} + \frac{\partial (n_2 u_1)}{\partial \xi} + \frac{\partial (n_1 u_2)}{\partial \xi} + \frac{\partial u_3}{\partial \xi} = 0, \tag{2.15}$$

$$\frac{\partial u_1}{\partial \tau} + u_2 \frac{\partial u_1}{\partial \xi} + u_1 \frac{\partial u_2}{\partial \xi} - \lambda \frac{\partial u_3}{\partial \xi} + \frac{\partial \varphi_3}{\partial \xi} = 0, \tag{2.16}$$

$$\begin{aligned} &\tfrac{\partial^2 \varphi_1}{\partial \xi^2} + \left[\tfrac{3}{2\lambda^4} - \left((1-f)C_2 + f\sigma^2 D_2\right)\right](\varphi_1)^2 = \\ &\tfrac{1}{\lambda^2}\varphi_3 + 2\left((1-f)C_2 + f\sigma^2 D_2\right)(\varphi_1\varphi_2) + \\ &\left(C_3(1-f) + f\sigma^3 D_3\right)\varphi_1^3 - n_3. \end{aligned} \tag{2.17}$$

By solving Eqs. (2.15)– (2.17) along with the first and second order solutions, we obtain the following extended KdV equation expressed in terms of first-order perturbed potential φ_1, i.e.,

$$\frac{\partial \varphi_1}{\partial \tau} + P \frac{\partial \varphi_1^2}{\partial \xi} + Q \frac{\partial \varphi_1^3}{\partial \xi} + R \frac{\partial^3 \varphi_1}{\partial \xi^3} = 0, \tag{2.18}$$

and

$$\begin{aligned} P &= \frac{\lambda^3}{2}\left[\frac{3}{2\lambda^4} - \left(C_2(1-f) + D_2 f\sigma^2\right)\right], \\ Q &= \frac{\lambda^3}{2}\left(\frac{5}{2\lambda^6} - C_3(1-f) - f\sigma^3 D_3\right), \\ R &= \frac{\lambda^3}{2}, \\ C_3 &= \frac{(\kappa_c - \frac{1}{2})(\kappa_c + \frac{1}{2})(\kappa_c + \frac{3}{2})}{6(\kappa_c - \frac{3}{2})^3}, \ D_3 = \frac{(\kappa_h - \frac{1}{2})(\kappa_h + \frac{1}{2})(\kappa_h + \frac{3}{2})}{6(\kappa_h - \frac{3}{2})^3}. \end{aligned} \tag{2.19}$$

3. Basic structure of analytical method

Suppose the general nonlinear partial differential equation,

$$P\left(u, u_\tau, u_\xi, u_{\tau\tau}, u_{\xi\xi}, ...\right) = 0, \tag{3.1}$$

where $u = u(\xi, \tau)$ is an unknown function,P is a polynomial in $u(\xi, \tau)$ and its partial derivatives in which the highest order partial derivatives and the nonlinear terms are involved. The main steps of new extension of (G'/G)-expansion method combined with the algebra expansion are as follows:

Step 1. The traveling wave variable ansatz,

$$\chi = \xi \pm \omega\tau, \qquad u(\xi, \tau) = u(\chi), \tag{3.2}$$

where $\omega \in \Re - \{0\}$ is the speed of the traveling wave, permits us to transform the Eq. (3.1) into the following ODE

$$Q\left(u, u', u'', ...\right) = 0, \tag{3.3}$$

where the superscripts stand for the ordinary derivatives with respect to ξ.

Step 2. Suppose the traveling wave solution of Eq. (3.3) can be expressed by a polynomial in $F(\xi)$ as follows

$$u(\chi) = \sum_{i=0}^{n} \alpha_i (m + F(\chi))^i + \sum_{i=1}^{n} \beta_i (m + F(\chi))^{-i}, \tag{3.4}$$

where $F(\xi) = G'/G, \alpha_n$ and β_n are not zero simultaneously. Also $G = G(\xi)$ satisfies the ordinary differential equation,

$$G''(\chi) + \lambda G'(\chi) + \mu G(\chi) = 0, \tag{3.5}$$

where λ and μ are arbitrary constants to be determined later. The solutions of Eq. (3.5) can be written as follows

Hyperbolic function solutions:
When $\Omega = \lambda^2 - 4\mu > 0$,

$$F_1 = \frac{\sqrt{\Omega}}{2} \coth\left(A + \frac{\sqrt{\Omega}}{2}\chi\right) - \frac{\lambda}{2}, \tag{3.6}$$

$$F_2 = \frac{\sqrt{\Omega}}{2} \tanh\left(A + \frac{\sqrt{\Omega}}{2}\chi\right) - \frac{\lambda}{2}. \tag{3.7}$$

Trigonometric function solutions:
When $\Omega = \lambda^2 - 4\mu < 0$,

$$F_3 = \frac{\sqrt{\Omega}}{2} \cot\left(A + \frac{\sqrt{\Omega}}{2}\chi\right) - \frac{\lambda}{2}, \tag{3.8}$$

$$F_4 = \frac{\sqrt{\Omega}}{2} \tan\left(A - \frac{\sqrt{\Omega}}{2}\chi\right) - \frac{\lambda}{2}. \tag{3.9}$$

Rationalfunction solutions:
When $\Omega = \lambda^2 - 4\mu = 0$,

$$F_5 = \frac{B}{A + B\chi} - \frac{\lambda}{2}. \tag{3.10}$$

Step 3. The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (3.1) or Eq. (3.3). Moreover precisely, we define the degree of $u(\chi)$ as $D(u(\chi)) = n$ which gives rise to the degree of other expression as follows

$$D\left(\frac{d^z u}{d\chi^z}\right) = n + z, \qquad D\left(u^r \left(\frac{d^z u}{d\chi^z}\right)^s\right) = nr + s(n + z). \tag{3.11}$$

Therefore we can find the value of n in Eq. (2.3), using Eq. (3.11).
Step 4. Substituting Eq. (3.4) along with Eq. (3.5) into Eq. (3.3) together with the

value of n obtained in step 3, we obtain polynomials in F^i and $F^{-i}\,(i = 1, 2, 3, ...)$, then setting each coefficient of the resulted polynomial to zero, yields a system of algebraic equations for α_n, β_n and ω.

Step 5. Suppose the values of the constants α_n, β_n and ω can be determined by solving the system of algebraic equations obtained in step 4. Since the general solutions of Eq. (3.5) are known, substituting α_n, β_n and ω into Eq. (3.4), we obtain some exact traveling wave solutions of the nonlinear evolution Eq. (3.1).
Now from above, we will exert the analytical method to obtain new and more general exact solutions and then the solitary wave solutions of the extended KdV equation (2.18).
For this aim we by combining the variables ξ and τ into one variable $\chi = \xi - V\tau($ V is the wave velocity) and to integrating with respect to χ, Eq. (2.18) is transformed as

$$-V\varphi_1 + P\varphi_1^2 + Q\varphi_1^3 + R\varphi_1^{''} + s = 0, \tag{3.12}$$

where $\varphi_1^{'} = d\varphi_1/d\chi$.
Thus using (3.12) and considering the homogeneous balance between φ_1^3 and $d^2\varphi_1/d\chi^2$ in Eq. (3.12) we obtain that $n = 1$. This, indubitably, allows us to assume that the solution is in form

$$\varphi_1 = \alpha_0 + \alpha_1\,(m + F) + \beta_1\,(m + F)^{-1}\,. \tag{3.13}$$

Now substituting Eq. (3.13) along with Eq. (2.4) into Eq. (3.12), we get a polynomial in $F\,(\chi)$. Equating the coefficient of same power of
$F^i\,(\chi)\,(i = 0, \pm1, \pm2, ...)$, we attain the system of algebraic equations and by solving these obtained system of equations for α_0, α_1, β_1 and m and by solving obtained system we get the following values:

Set 1.

$$\begin{aligned}
&\alpha_1 = \sqrt{\tfrac{-2R}{Q}},\\
&\beta_1 = -\tfrac{2}{3}\tfrac{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q} + \sqrt{\tfrac{-2R}{Q}}\lambda - \tfrac{2}{3}\sqrt{\tfrac{-2R}{Q}}m,\\
&\alpha_0 = \tfrac{1}{3}\tfrac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q},
\end{aligned}$$

Set 2.

$$\begin{aligned}
&\alpha_1 = 0,\\
&\beta_1 = \mp\tfrac{R\mu(m^2-\lambda m+\mu)}{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}},\\
&\alpha_0 = \tfrac{1}{3}\tfrac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q},
\end{aligned}$$

Set 3.

$$\alpha_1 = \mp \frac{R(-3\lambda+2m)}{\sqrt{P^2+3Qv-3QR\lambda^2-6QR\mu+6QRm\lambda}},$$
$$\beta_1 = 0,$$
$$\alpha_0 = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q},$$

Where

$$P = \frac{\lambda^3}{2}[\frac{3}{2\lambda^4} - \left(C_2(1-f) + D_2 f\sigma^2\right)],$$

$$Q = \frac{\lambda^3}{2}\left(\frac{5}{2\lambda^6} - C_3(1-f) - f\sigma^3 D_3\right),$$

$$R = \frac{\lambda^3}{2},$$

$$C_2 = \frac{(\kappa_c - \frac{1}{2})(\kappa_c + \frac{1}{2})}{2(\kappa_c - \frac{3}{2})^2}, D_2 = \frac{(\kappa_h - \frac{1}{2})(\kappa_h + \frac{1}{2})}{2(\kappa_h - \frac{3}{2})^2},$$

$$C_3 = \frac{(\kappa_c - \frac{1}{2})(\kappa_c + \frac{1}{2})(\kappa_c + \frac{3}{2})}{6(\kappa_c - \frac{3}{2})^3}, D_3 = \frac{(\kappa_h - \frac{1}{2})(\kappa_h + \frac{1}{2})(\kappa_h + \frac{3}{2})}{6(\kappa_h - \frac{3}{2})^3}.$$

3.1. **Hyperbolic function solutions.** When $\Omega = \lambda^2 - 4\mu > 0$, we get the following solutions;

Family 1. By using set 1 and Eqs. (26-27) along with (3.13) we have solutions of Eq. (2.18) as follow

$$\varphi_{1-1} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q} +$$
$$\sqrt{\frac{-2R}{Q}}\left(m + \frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(\xi - V\tau)\right) - \frac{\lambda}{2}\right) +$$
$$\left(-\frac{2}{3}\frac{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q} + \sqrt{\frac{-2R}{Q}}\lambda - \frac{2}{3}\sqrt{\frac{-2R}{Q}}m\right) \times$$
$$\left(m + \frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(\xi - V\tau)\right) - \frac{\lambda}{2}\right)^{-1},$$

and

$$\varphi_{1-2} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q} +$$
$$\sqrt{\frac{-2R}{Q}}\left(m + \frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(\xi - V\tau)\right) - \frac{\lambda}{2}\right) +$$
$$\left(-\frac{2}{3}\frac{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q} + \sqrt{\frac{-2R}{Q}}\lambda - \frac{2}{3}\sqrt{\frac{-2R}{Q}}m\right) \times$$
$$\left(m + \frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(\xi - V\tau)\right) - \frac{\lambda}{2}\right)^{-1}.$$

Family 2. By using set 2 and Eqs. (26-27) along with (3.13) we have solutions of Eq. (2.18) as follow

$$\varphi_{1-3}=\frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+ \left(\mp\frac{R\mu(m^2-\lambda m+\mu)}{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}\right)\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right)^{-1},$$

and

$$\varphi_{1-4}=\frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+ \left(\mp\frac{R\mu(m^2-\lambda m+\mu)}{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}\right)\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right)^{-1}.$$

Family 3. By using set 3 and Eqs. (26-27) along with (3.13) we have solutions of Eq. (2.18) as follow

$$\varphi_{1-5}=\frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}\mp \frac{R(-3\lambda+2m)}{\sqrt{P^2+3Qv-3QR\lambda^2-6QR\mu+6QRm\lambda}}\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right),$$

and

$$\varphi_{1-6}=\frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}\mp \frac{R(-3\lambda+2m)}{\sqrt{P^2+3Qv-3QR\lambda^2-6QR\mu+6QRm\lambda}}\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right).$$

3.2. **Trigonometric function solutions.** When $\Omega=\lambda^2-4\mu<0$;
Family 4. By using set 1 and Eqs. (28-29) along with (3.13) we have solutions of Eq. (2.18) as follow

$$\varphi_{1-7}=\frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+ \sqrt{\frac{-2R}{Q}}\left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right)+ \left(-\frac{2}{3}\frac{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+\sqrt{\frac{-2R}{Q}}\lambda-\frac{2}{3}\sqrt{\frac{-2R}{Q}}m\right)\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right)^{-1},$$

and

$$\varphi_{1-8} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+ \sqrt{\frac{-2R}{Q}}\left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\tan\left(A-\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right)+ \left(-\frac{2}{3}\frac{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+\sqrt{\frac{-2R}{Q}}\lambda-\frac{2}{3}\sqrt{\frac{-2R}{Q}}m\right)\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\tan\left(A-\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right)^{-1}.$$

Family 5. By using set 2 and Eqs. (28-29) along with (3.13) we have solutions of Eq. (2.18) as follow

$$\varphi_{1-9} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+ \left(\mp\frac{R\mu(m^2-\lambda m+\mu)}{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}\right)\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right)^{-1},$$

and

$$\varphi_{1-10} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+ \left(\mp\frac{R\mu(m^2-\lambda m+\mu)}{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}\right)\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\tan\left(A-\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right)^{-1}.$$

Family 6. By using set 3 and Eqs. (28-29) along with (3.13) we have solutions of Eq. (2.18) as follow

$$\varphi_{1-11} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}\mp \frac{R(-3\lambda+2m)}{\sqrt{P^2+3Qv-3QR\lambda^2-6QR\mu+6QRm\lambda}}\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right),$$

and

$$\varphi_{1-12} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}\mp \frac{R(-3\lambda+2m)}{\sqrt{P^2+3Qv-3QR\lambda^2-6QR\mu+6QRm\lambda}}\times \left(m+\frac{\sqrt{\lambda^2-4\mu}}{2}\tan\left(A-\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi-V\tau)\right)-\frac{\lambda}{2}\right).$$

3.3. **Rational function solutions.** When $\Omega = \lambda^2 - 4\mu = 0$;

Family 7. By using set 1 and Eq. (3.10) along with (3.13) we have solutions of Eq. (2.18) as follow

$$\varphi_{1-13} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+ \sqrt{\frac{-2R}{Q}}\left(m+\frac{B}{A+B(\xi-V\tau)}-\frac{\lambda}{2}\right)+ \left(-\frac{2}{3}\frac{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+\sqrt{\frac{-2R}{Q}}\lambda-\frac{2}{3}\sqrt{\frac{-2R}{Q}}m\right)\times \left(m+\frac{B}{A+B(\xi-V\tau)}-\frac{\lambda}{2}\right)^{-1}.$$

Family 8. By using set 2 and Eq. (3.10) along with (3.13) we have solutions of Eq. (2.18) as follow

$$\varphi_{1-14} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}+ \mp\frac{R\mu(m^2-\lambda m+\mu)}{\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}\times\left(m+\frac{B}{A+B(\xi-V\tau)}-\frac{\lambda}{2}\right)^{-1}.$$

Family 9. By using set 3 and Eq. (3.10) along with (3.13) we have solutions of Eq. (2.18) as follow

$$\varphi_{1-15} = \frac{1}{3}\frac{-P\pm\sqrt{P^2+3Qv-3QR\mu\lambda-3QR\lambda m^2+3QR\lambda^2 m}}{Q}\mp \frac{R(-3\lambda+2m)}{\sqrt{P^2+3Qv-3QR\lambda^2-6QR\mu+6QRm\lambda}}\left(m+\frac{B}{A+B(\xi-V\tau)}-\frac{\lambda}{2}\right).$$

In all above solutions we have,

$$P=\frac{\lambda^3}{2}[\frac{3}{2\lambda^4}-\left(C_2(1-f)+D_2 f\sigma^2\right)],$$

$$Q=\frac{\lambda^3}{2}\left(\frac{5}{2\lambda^6}-C_3(1-f)-f\sigma^3 D_3\right),$$

$$R=\frac{\lambda^3}{2},$$

$$C_2=\frac{(\kappa_c-\frac{1}{2})(\kappa_c+\frac{1}{2})}{2(\kappa_c-\frac{3}{2})^2}, D_2=\frac{(\kappa_h-\frac{1}{2})(\kappa_h+\frac{1}{2})}{2(\kappa_h-\frac{3}{2})^2},$$

$$C_3=\frac{(\kappa_c-\frac{1}{2})(\kappa_c+\frac{1}{2})(\kappa_c+\frac{3}{2})}{6(\kappa_c-\frac{3}{2})^3}, D_3=\frac{(\kappa_h-\frac{1}{2})(\kappa_h+\frac{1}{2})(\kappa_h+\frac{3}{2})}{6(\kappa_h-\frac{3}{2})^3}.$$

Next for more reviews we consider e graphical behavior of hyperbolic, trigonometric and rational functions solutions of Eq. (2.18) in one case.

FIGURE 1. Graphical behavior of hyperbolic solutions of Eq. (2.18).

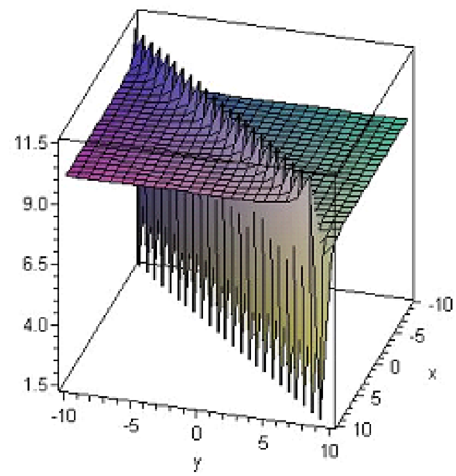
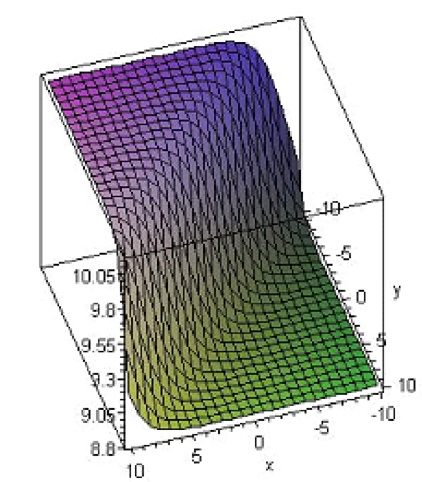


FIGURE 2. Graphical behavior of hyperbolic solutions of Eq. (2.18).



4. Conclusion

In this Letter, the new extended analytical (G'/G)-expansion method has been successfully applied to find the exact solitary wave solutions for double layers structure model of extended Korteweg-de Vries (KdV) equation. The obtained results show that the method is very powerful and convenient mathematical tool for nonlinear evolution

FIGURE 3. Graphical behavior of trigonometric solutions of Eq. (2.18).

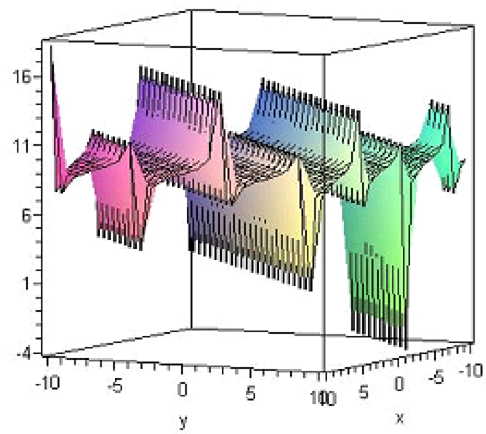
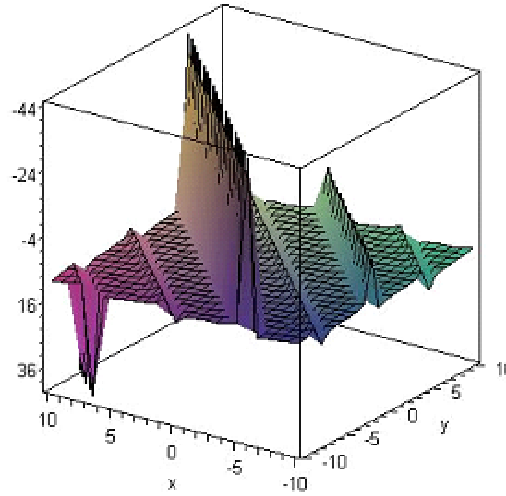
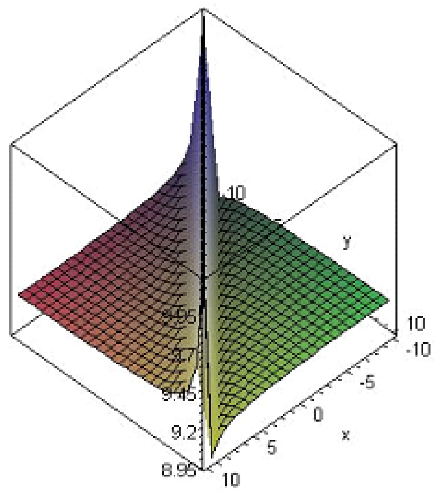


FIGURE 4. Graphical behavior of trigonometric solutions of Eq. (2.18).



equations in science and engineering. Now we briefly summarize the method in the following. Firstly, the main points of the method are that assuming the solution of the ODE reduced by using the travelling wave variable as well as integrating can be expressed by an mth degree polynomial in (G'/G), where $G = G(\xi)$ is the general

FIGURE 5. Graphical behavior of rational solutions of Eq. (2.18).



solutions of a second order LODE, the positive integer m is determined by the homogeneous balance between the highest order derivatives and nonlinear terms appearing in the reduced ODE, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations resulted from the process of using the method. Secondly, it is important to solve the algebraic equations resulted. Generally speaking, if the order of the reduced ODE is equal to or less than 3 (in the circumstances, the number of the equations included in the set of algebraic equations is generally equal to or less than that of the unknowns), with the aid of MATHEMATICA or MAPLE it is mostly possible to find out a useful solution of the algebraic equations resulted. Otherwise, it is generally unable to guarantee the existence of a solution of the algebraic equations resulted, this is because the number of the equations included in the set of algebraic equations is generally great than that of the unknowns. In spite of this, the (G'/G)-expansion is still a useful method for finding travelling wave solutions of nonlinear evolution equations, the reason is as follows. On comparing between the (G'/G)-expansion method and the other methods such as the modified tanhcoth method, we conclude that the (G'/G)- expansion method is more powerful, effective and convenient. The performance of this method is reliable, simple and gives many new exact solutions. The (G'/G)-expansion method has more advantages: It is direct and concise. It is also a standard and computerizable method which allows us to solve complicated nonlinear evolution equations in mathematical physics. We have noted that the (G'/G)-expansion method changes the given difficult problems into simple problems which can be solved easily.

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