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Solitary Wave solutions to the (3+1)-dimensional Jimbo–Miwa equation

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Abstract The homogeneous balance method can be used to construct exact traveling wave solutions of nonlinear partial differential equations. In this paper, this method is used to construct new soliton solutions of the (3+1) Jimbo–Miwa equation.

Keywords. Homogeneous balance method; $(3+1)$ Jimbo-Miwa equation; Solitary wave solutions.
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1. Introduction

The study of exact solutions of nonlinear evolution equations plays an important role in soliton theory and explicit formulas of NPDEs. Also, explicit formulas may provide physical information and help us to understand the mechanism of related physical models. A large number of such equations have been studied in these contexts, and numerous analytic and computational effective techniques have been proposed to investigate these types of equations.
Phenomena in physics and other fields are often described by nonlinear evolution equations. Nonlinear partial differential equations (NPDEs) are widely used to describe complex phenomena in various fields of sciences, especially in physics. Some of these nonlinear wave solutions are the cnoidal waves, solitons, solitary waves, shock waves, compactons, stumpons, covatons, cuspons, peakons propeller solitons and several many others. These solutions are all indeed very useful in various areas of Applied Mathematics and Theoretical Physics.Therefore solving nonlinear problems plays an important role in nonlinear sciences.
Some of these methods that have been recently developed are exponential function method [4,5], Fan's F-expansion method [6,7], the tanh–sech method [8–10], extended tanh method [11–13], sine–cosine method [14–16], homogeneous balance method [17,18], first integral method [19-21] and so on [1-3, 23-29].
In this paper we consider the (3+1) Jimbo–Miwa equation [22],

$$u_{xxx} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0. \tag{1.1}$$

This work is organized as follows. In the next section we give brief description of the Homogeneous balance method. In the Sections 3 we construct soliton solutions for the (3+1) Jimbo–Miwa equation. In the last section we summarize our results.

2. Algorithm of the Homogeneous balance method

For a given partial differential equation

$$G(u, u_x, u_t, u_{xx}, u_{tt}, ...) = 0, \tag{2.1}$$

Our method mainly consists of four steps:
Step 1: We seek complex solutions of Eq.(2.1) as the following form:

$$u = u(\xi), \qquad \xi = ik(x - ct), \tag{2.2}$$

Where k and c are real constants. Under the transformation (2.2), Eq.(2.1) becomes an ordinary differential equation

$$N(u, iku', -ikcu', -k^2u'', ...) = 0, \tag{2.3}$$

Where$u' = \frac{du}{d\xi}$.
Step 2: We assume that the solution of Eq.(2.3) is of the form

$$u(\xi) = \sum_{i=0}^{n} a_i \phi^i(\xi), \tag{2.4}$$

Where $a_i (i = 1, 2, .., n)$ are real constants to be determined later and ϕ satisfy the Riccati equation

$$\phi' = a\phi^2 + b\phi + c. \tag{2.5}$$

Eq.(2.5) admits the following solutions:
Case1: Let $\phi = \sum_{i=0}^{n} b_i \tanh^i \xi$,Balancing $\phi'$with ϕ^2 in Eq.(2.5) gives $m = 1$so

$$\phi = b_0 + b_1 \tanh \xi, \tag{2.6}$$

Substituting Eq. (2.6)into Eq.(2.5), we obtain the following solution of Eq.(2.5)

$$\phi = -\frac{1}{2a}\left(b + 2\tanh \xi\right), \qquad ac = \frac{b^2}{4} - 1. \tag{2.7}$$

Case2: when $a = 1, b = 0$,the Riccati Eq.(2.5) has the following solutions

$$\begin{array}{l} \phi = -\sqrt{-c}\tanh\left(\sqrt{-c}\xi\right), \qquad c < 0, \\ \phi = -\frac{1}{\xi}, \qquad c < 0 \\ \phi = \sqrt{c}\tan\left(\sqrt{c}\xi\right), \qquad c > 0. \end{array} \tag{2.8}$$

Case3: We suppose that the Riccati Eq.(2.5) have the following solutions of the form:

$$\phi = A_0 + \sum_{i=1}^{n} \sinh^{i-1}\left(A_i \sinh \omega + B_i \cosh \omega\right), \tag{2.9}$$

Where $\frac{d\omega}{d\xi} = \sinh\omega$ or$\frac{d\omega}{d\xi} = \cosh\omega$. It is easy to find that $m = 1$by Balancing ϕ'withϕ^2. So we choose

$$\phi = A_0 + A_1 \sinh\omega + B_1 \cosh\omega, \tag{2.10}$$

Where $\frac{d\omega}{d\xi} = \sinh\omega$, we substitute (2.10) and $\frac{d\omega}{d\xi} = \sinh\omega$, into (2.5) and set the coefficients of $\sinh^i\omega, \cosh^i\omega\,(i = 0, 1, 2)$ to zero. We obtain a set of algebraic equations and solving these equations we have the following solutions

$$A_0 = -\frac{b}{2a}, A_1 = 0, B_1 = \frac{1}{2a}, \tag{2.11}$$

where $c = \frac{b^2-4}{4a}$ and

$$A_0 = -\frac{b}{2a}, A_1 = \pm\sqrt{\frac{1}{2a}}, B_1, = \frac{1}{2a} \tag{2.12}$$

where $c = \frac{b^2-1}{4a}$. To $\frac{d\omega}{d\xi} = \sinh\omega$ we have

$$\sinh\omega = -\csc h\xi, \cosh\omega = -\coth\xi. \tag{2.13}$$

From (2.11)-(2.13), we obtain

$$\phi = -\frac{b + 2\coth\xi}{2a}, \tag{2.14}$$

where $c = \frac{b^2-4}{4a}$ and

$$\phi = -\frac{b \pm \csc h\xi + \coth\xi}{2a}, \tag{2.15}$$

where $c = \frac{b^2-1}{4a}$.

Step3. Substituting (2.6-2.15) into (2.3) along with (2.5), then the left hand side of Eq.(2.3) is converted into a polynomial in$F(\xi)$; equating each coefficient of the polynomial to zero yields a set of algebraic equations.

Step4. Solving the algebraic equations obtained in step 3, and substituting the results into (2.4), then we obtain the exact traveling wave solutions for Eq. (2.1).

Remark 1: If $c = 0$, then the Riccati Eq. (2.5) reduces to the Bernoulli equation

$$\phi' = a\phi^2 + b\phi, \tag{2.16}$$

The solution of the Bernoulli Eq. (2.16) can be written in the following form [23]:

$$\phi = b\left[\frac{\cosh\left[b\left(\xi+\xi_0\right)\right] + \sinh\left[b\left(\xi+\xi_0\right)\right]}{1 - a\cosh\left[b\left(\xi+\xi_0\right)\right] - a\sinh\left[b\left(\xi+\xi_0\right)\right]}\right], \tag{2.17}$$

where ξ_0 is integration constant.

Remark 2: If $b = 0$, then the Riccati Eq. (2.5) reduces to the Riccati equation

$$\phi' = a\phi^2 + c,$$

which the equation above is the special case of the Riccati Eq. (2.5).

Remark 3: Also, the Riccati Eq.(2.5) admits the following exact solution [23]:

$$\phi = -\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{\theta}{2}\xi\right) + \frac{\sec h\left(\frac{\theta}{2}\xi\right)}{C\cosh\left(\frac{\theta}{2}\xi\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta}{2}\xi\right)}, \tag{2.18}$$

where $\theta^2 = b^2 - 4ac$ and C is a constant of integration.

3. Methodology to the (3+1) Jimbo–Miwa equation

Using the wave transformation

$$u(x,y,z,t) = U(\xi), \xi = kx + ly + mz + \omega t \tag{3.1}$$

Eq. (1.1) is carried out to the following ODE:

$$k^3U''' + 6k^2lU'U'' + (2l\omega - 3km)U'' = 0. \tag{3.2}$$

After integrating with respect to ξ they obtained the nonlinear ordinary differential equation in the form

$$k^3lU''' + 3k^2l(U')^2 + (2l\omega - 3km)U' = C.$$

Here C is the constant of integration. Denoting $U' = V(\xi)$ in Eq. above we have the following equation

$$k^3lV'' + 3k^2lV^2 + (2l\omega - 3km)V = C. \tag{3.3}$$

For the solutions of Eq. (3.3), with the aid of Homogeneous balance method we make the following ansatz

$$V(\xi) = \sum_{i=0}^{n} a_i\phi^i(\xi), \tag{3.4}$$

where $a_i$are all real constants to be determined, n is a positive integer which can be determined by balancing the highest order derivative term with the highest order nonlinear term, then gives$n = 2$. Therefore, we may choose

$$V(\xi) = a_2\phi^2 + a_1\phi + a_0. \tag{3.5}$$

Substituting (3.5) along with (2.5) in Eq.(3.3) and then setting the coefficients of $\phi^j(j = 0,1,2,3,4,5)$to zero in the resultant expression, we obtain a set of algebraic equations and solving these equations with the aid of Maple we have

$$\begin{aligned} a_0 &= \frac{1}{6}\frac{-2lw + 3km \pm \sqrt{4l^2w^2 - 12lwkm + 9k^2m^2 + 48k^6la^2c^2 - 24k^6lab^2c}}{k^2l}, \\ a_1 &= 2kab, \\ a_2 &= -2ka^2. \end{aligned} \tag{3.6}$$

By substituting (3.6) in (3.5) along with (3.7) and (3.1) we have solution of the Eq. (1.1) as follows

$$V(\xi)=-\frac{1}{2}ka^2\left(b+2\tanh\xi\right)^2-kb\left(b+2\tanh\xi\right)+\\ \frac{-2lw+3km\pm\sqrt{4l^2w^2-12lwkm+9k^2m^2+48k^6la^2c^2-24k^6lab^2c}}{6k^2l}.$$

So

$$U=\int\left(-\frac{1}{2}ka^2\left(b+2\tanh\xi\right)^2-kb\left(b+2\tanh\xi\right)\right)d\xi+\\ \left(\frac{-2lw+3km\pm\sqrt{4l^2w^2-12lwkm+9k^2m^2+48k^6la^2c^2-24k^6lab^2c}}{6k^2l}\right)\xi.$$

The solution of the (3+1) Jimbo–Miwa equation (1.1) when $c<0$ will obtain by substituting (3.5),(3.6) along with (2.8) and (3.1) as follows

$$V(\xi)=2ka^2c\tanh^2\left(\sqrt{-c}\xi\right)-2kab\left(\sqrt{-c}\tanh\left(\sqrt{-c}\xi\right)\right)+\\ \frac{1}{6}\frac{-2lw+3km\pm\sqrt{4l^2w^2-12lwkm+9k^2m^2+48k^6la^2c^2-24k^6lab^2c}}{k^2l},$$

So

$$U=\int\left(2ka^2c\tanh^2\left(\sqrt{-c}\xi\right)-2kab\left(\sqrt{-c}\tanh\left(\sqrt{-c}\xi\right)\right)\right)d\xi+\\ \frac{1}{6}\frac{-2lw+3km\pm\sqrt{4l^2w^2-12lwkm+9k^2m^2+48k^6la^2c^2-24k^6lab^2c}}{k^2l}\xi,$$

Soliton solution of (1.1) when $c=0$ is

$$V(\xi)=-\frac{2ka^2}{\xi^2}-\frac{2kab}{\xi}+\\ \frac{1}{6}\frac{-2lw+3km\pm\sqrt{4l^2w^2-12lwkm+9k^2m^2+48k^6la^2c^2-24k^6lab^2c}}{k^2l},$$

and

$$U=\frac{2ka^2}{\xi}-2kab\ln\xi+\\ \frac{1}{6}\frac{-2lw+3km\pm\sqrt{4l^2w^2-12lwkm+9k^2m^2+48k^6la^2c^2-24k^6lab^2c}}{k^2l}\xi,$$

when $c>0$

$$V(\xi)=-2ka^2c\tan^2\left(\sqrt{c}\xi\right)+2kab\left(\sqrt{c}\tan\left(\sqrt{c}\xi\right)\right)+\\ \frac{1}{6}\frac{-2lw+3km\pm\sqrt{4l^2w^2-12lwkm+9k^2m^2+48k^6la^2c^2-24k^6lab^2c}}{k^2l},$$

and

$$U=\int\left(-2ka^2c\tan^2\left(\sqrt{c}\xi\right)+2kab\left(\sqrt{c}\tan\left(\sqrt{c}\xi\right)\right)\right)d\xi+\\ \frac{1}{6}\frac{-2lw+3km\pm\sqrt{4l^2w^2-12lwkm+9k^2m^2+48k^6la^2c^2-24k^6lab^2c}}{k^2l}\xi.$$

From (3.6) in (3.5) along with (2.14) and (3.1) we have solution of the Eq.(1.1) as follows

$$V(\xi) = -\frac{1}{2}k\left(b + 2\coth\xi\right)^2 - kb\left(b + 2\coth\xi\right) + \frac{1}{6}\frac{-2lw + 3km \pm \sqrt{4l^2w^2 - 12lwkm + 9k^2m^2 + 48k^6la^2c^2 - 24k^6lab^2c}}{k^2l},$$

so

$$U = \int\left(-\frac{1}{2}k\left(b + 2\coth\xi\right)^2 - kb\left(b + 2\coth\xi\right)\right)d\xi + \frac{1}{6}\frac{-2lw + 3km \pm \sqrt{4l^2w^2 - 12lwkm + 9k^2m^2 + 48k^6la^2c^2 - 24k^6lab^2c}}{k^2l}\xi,$$

where $c = \frac{b^2 - 4}{4a}$ and

$$V(\xi) = -\frac{1}{2}k\left(b \pm \csc h\xi + \coth\xi\right)^2 - kb\left(b \pm \csc h\xi + \coth\xi\right) + \frac{1}{6}\frac{-2lw + 3km \pm \sqrt{4l^2w^2 - 12lwkm + 9k^2m^2 + 48k^6la^2c^2 - 24k^6lab^2c}}{k^2l}$$

and

$$U = \int\left(-\frac{1}{2}k\left(b \pm \csc h\xi + \coth\xi\right)^2 - kb\left(b \pm \csc h\xi + \coth\xi\right)\right)d\xi + \frac{1}{6}\frac{-2lw + 3km \pm \sqrt{4l^2w^2 - 12lwkm + 9k^2m^2 + 48k^6la^2c^2 - 24k^6lab^2c}}{k^2l}\xi,$$

where $c = \frac{b^2 - 1}{4a}$.

In this section we will obtain the solution of Eq. (1.1) from (3.5), (3.6) along with (2.17) and (2.19)

$$V(\xi) = -2kb^2a^2\left[\frac{\cosh\left[b\left(\xi + \xi_0\right)\right] + \sinh\left[b\left(\xi + \xi_0\right)\right]}{1 - a\cosh\left[b\left(\xi + \xi_0\right)\right] - a\sinh\left[b\left(\xi + \xi_0\right)\right]}\right]^2 + 2kab^2\left[\frac{\cosh\left[b\left(\xi + \xi_0\right)\right] + \sinh\left[b\left(\xi + \xi_0\right)\right]}{1 - a\cosh\left[b\left(\xi + \xi_0\right)\right] - a\sinh\left[b\left(\xi + \xi_0\right)\right]}\right] + \frac{1}{6}\frac{-2lw + 3km \pm \sqrt{4l^2w^2 - 12lwkm + 9k^2m^2 + 48k^6la^2c^2 - 24k^6lab^2c}}{k^2l},$$

and

$$U = \int\left(-2kb^2a^2\left[\frac{\cosh\left[b\left(\xi + \xi_0\right)\right] + \sinh\left[b\left(\xi + \xi_0\right)\right]}{1 - a\cosh\left[b\left(\xi + \xi_0\right)\right] - a\sinh\left[b\left(\xi + \xi_0\right)\right]}\right]^2\right)d\xi + \int\left(2kab^2\left[\frac{\cosh\left[b\left(\xi + \xi_0\right)\right] + \sinh\left[b\left(\xi + \xi_0\right)\right]}{1 - a\cosh\left[b\left(\xi + \xi_0\right)\right] - a\sinh\left[b\left(\xi + \xi_0\right)\right]}\right]\right)d\xi + \frac{1}{6}\frac{-2lw + 3km \pm \sqrt{4l^2w^2 - 12lwkm + 9k^2m^2 + 48k^6la^2c^2 - 24k^6lab^2c}}{k^2l}\xi.$$

From (3.5),(3.6) along with (2.18) and (3.1), we set

$$V(\xi) = -2ka^2 \left(-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{\theta}{2}\xi\right) + \frac{\sec h\left(\frac{\theta}{2}\xi\right)}{C\cosh\left(\frac{\theta}{2}\xi\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta}{2}\xi\right)} \right)^2 +$$
$$2kab\left(-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{\theta}{2}\xi\right) + \frac{\sec h\left(\frac{\theta}{2}\xi\right)}{C\cosh\left(\frac{\theta}{2}\xi\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta}{2}\xi\right)} \right) +$$
$$\frac{1}{6}\frac{-2lw + 3km \pm \sqrt{4l^2w^2 - 12lwkm + 9k^2m^2 + 48k^6la^2c^2 - 24k^6lab^2c}}{k^2 l}$$

So

$$U = \int \left(-2ka^2 \left(-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{\theta}{2}\xi\right) + \frac{\sec h\left(\frac{\theta}{2}\xi\right)}{C\cosh\left(\frac{\theta}{2}\xi\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta}{2}\xi\right)} \right)^2 \right) d\xi +$$
$$\int \left(2kab\left(-\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{\theta}{2}\xi\right) + \frac{\sec h\left(\frac{\theta}{2}\xi\right)}{C\cosh\left(\frac{\theta}{2}\xi\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta}{2}\xi\right)} \right) \right) d\xi +$$
$$\frac{1}{6}\frac{-2lw + 3km \pm \sqrt{4l^2w^2 - 12lwkm + 9k^2m^2 + 48k^6la^2c^2 - 24k^6lab^2c}}{k^2 l}\xi,$$

Where $\theta^2 = b^2 - 4ac$ and C is a constant of integration.

4. Conclusion

In this paper, the homogeneous balance method has been applied to obtain the new exact solutions of the (3+1) Jimbo-Miwa equation. This method has the advantages of being direct and concise. The method proposed in this paper can also be extended to solve some nonlinear evolution equations in mathematical physics.

References

[1] A. Biswas, Optical Solitons with Time-Dependent Dispersion, Nonlinearity and Attenuation in a Kerr-Law Media, Int. J. Theor. Phys., 48 (2009), 256-260.

[2] A. Biswas, 1-Soliton solution of the $K(m, n)$ equation with generalized evolution. Phys. Lett. A., 372(25) (2008), 4601-460.

[3] A. Biswas, 1-Soliton solution of the K(m,n) equation with generalized evolution and time-dependent damping and dispersion. Comput. Math. Appl., 59(8)(2010), 2538-2542.

[4] J.H. He, X.H. Wu, Exp-function method for nonlinear wave equations. Chaos, Solitons Fractals, 30(2006), 700-708.

[5] J.H. He, M.A. Abdou, New periodic solutions for nonlinear evolution equations using Exp-function method. Chaos, Solitons Fractals, 34(5) (2007), 1421-1429.

[6] M.A. Abdou, The extended F-expansion method and its application for a class of nonlinear evolution equations. Chaos, Solitons Fractals, 31 (2007), 95-104.

[7] J.L. Zhang, M.L. Wang, Y.M.Wang, Z.D. Fang, The improved Fexpansion method and its applications. Phys Lett A ,350 (2006), 103-109.
[8] W. Malfliet, Solitary wave solutions of nonlinear wave equations. Am J Phys., 60 (1992), 650-654.
[9] W. Malfliet, W. Hereman, The tanh method. I: Exact solutions of nonlinear evolution and wave equations. Phys Scr., 54 (1996), 563-568.
[10] A.M. Wazwaz, The tanh method for travelling wave solutions of nonlinear equations. Appl Math Comput, 154(3) (2004), 713-723.
[11] S.A. El-Wakil, M.A. Abdou, New exact travelling wave solutions using modified extended tanh-function method. Chaos, Solitons Fractals, 31(4) (2007), 840-852.
[12] E. Fan, Extended tanh-function method and its applications to nonlinear equations. Phys Lett A, 277 (2000), 212-218.
[13] A.M. Wazwaz, The extended tanh method for new soliton solutions for many forms of the fifth-order KdV equations. Appl Math Comput., 184(2) (2007), 1002-1014.
[14] A.M. Wazwaz, The tanh method and the sine–cosine method for solving the KP-MEW equation. Int J Comput Math., 82(2) (2005), 235-246.
[15] A.M. Wazwaz, A sine-cosine method for handling nonlinear wave equations. Math Comput Model, 40 (2004), 499-508.
[16] A. Bekir, New solitons and periodic wave solutions for some nonlinear physical models by using the sine–cosine method. Phys Scr., 77(4) (2008), 501-504.
[17] E. Fan, H. Zhang, A note on the homogeneous balance method. Phys Lett A., 246 (1998), 403-406.
[18] M.L Wang, Exact solutions for a compound KdV-Burgers equation. Phys Lett A., 213 (1996), 279-287.
[19] Z.S. Feng, The first integral method to study the Burgers-KdV equation. J Phys A: Math Gen., 35 (2002), 343.
[20] M. Eslami, B. Fathi Vajargah, M. Mirzazadeh, A. Biswas, Application of first integral method to fractional partial differential equations, Indian Journal of Physics 88(2), (2014), 177-184.
[21] M. Mirzazadeh, M. Eslami, Exact solutions of the Kudryashov-Sinelshchikov equation and nonlinear telegraph equation via the first integral method, Nonlinear Analysis Modelling and Control, 4(17) (2012), 481-488.
[22] L. Zitian, D. Zhenge, Abundant new exact solutions for the (3+1)-dimensional Jimbo–Miwa equation, J. Math. Anal. Appl. 361 (2010), 587-590.
[23] A. H. Bhrawy, M. A. Abdelkawy, Anjan Biswas,Topological Solitons and Cnoidal waves to a few nonlinear wave equations in theoretical physics, Indian Journal of Physics. 87(11) (2013), 1125-1131.
[24] L. Girgis, D. Milovic, S. Konar, A. Yildirim, H. Jafari and A. Biswas, Optical Gaussons in birefringent fibers and DWDM system with Inter-Modal Romanian Reports in Physics, 64(3) (2012), 663-671.
[25] C. Masood Khalique, Anjan Biswas, A lie symmetry approach tononlinear Schrödinger's equation with non-kerrlaw nonlinearity, Communications in Nonlinear Science and Numerical Simulation, 14(12) (2009), 4033-4040.
[26] M. Eslami, M. Mirzazadeh, Anjan Biswas,Soliton solutions of the resonant nonlinear Schrödinger's equation in optical fibers with time-dependent coefficients by simplest equation approach, Journal of Modern Optics, 60(19) (2013), 1627-1636.
[27] A. Biswas, Solitary eave solution for the generalized KdV equation with Time-Dependent and dispersion,Communications in Nonlinear Science and Numerical Simulation, 14(9-10) (2009), 3503-3506.
[28] M. Eslami, M. Mirzazadeh, Topological 1-soliton solution of nonlinear Schrödinger equation with dual-power law nonlinearity in nonlinear optical fibers, The European Physical Journal Plus, 128(11) (2013),1-7.
[29] A. Nazarzadeh, M. Eslami, M. Mirzazadeh, Exact solutions of some nonlinear partial differential equations using functional variable method, Pramana, 81(2) (2013), 225-236.