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Numerical Solution of Semiconductor Medium Under Photothermal Theory by Finite-Difference Method

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Abstract

This paper examines the numerical solution within a semiconducting medium, employing the Finite Difference method. The research focuses on solving the fundamental equations for a 2-D, semi-infinite semiconducting medium with thermally insulated, stress-free surfaces to determine wave solutions. The study formulates both explicit and implicit finite difference schemes to address this problem. Numerical methods based on finite difference principles are employed to obtain solutions for these schemes. Subsequently, a comparative analysis of these solutions reveals unexpected similarities as a noteworthy outcome.

Keywords. Tridiagonal matrix algorithm (TDMA), Thermoelastic coupling, Carrier density, Semiconducting.
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1. Introduction

The physical characteristics of elastic materials are significantly influenced by temperature variations. In various research scenarios, the variable nature of thermal conductivity in elastic materials is often overlooked. However, the significance of thermal conductivity becomes particularly pronounced when it varies, especially in relation to temperature variation. The interplay between variable thermal conductivity and mass diffusion is referred to as thermo-diffusion. Thermo-diffusion occurs when particles transition from regions of higher concentration to lower concentration as a result of temperature change.

The investigation of thermal conductivity due to mass diffusion has significant practical implications in various fields of modern engineering, especially in the aerospace, electronics, and integrated circuit industries. Semiconductors, such as silicon, are well-suited for exploring this process when subjected to incident light or laser beams. In these circumstances, excited electrons appear on the material's surface, leading to the creation of free carriers or charge carriers, including plasma waves.

In the field of materials science, the temperature change in the medium is of paramount importance and finds numerous applications in various natural processes. Recent studies have revealed that the dependence of semiconductor thermal conductivity on temperature has a significant impact on physical properties, particularly deformation and thermo-mechanical behavior. For instance, the thermal conductivity of materials like silicon nitride can decrease by half within the temperature range of 1.0 C to 400.0 C. Generally, semiconductor materials exhibit high sensitivity to temperature fluctuations.

Plasma density plays a pivotal role in governing the diffusion of silicon based on the intensity of incident light. Regrettably, many researchers have tended to overlook the interaction between plasma waves and thermal-elastic waves during the deformation process in semiconductor materials [11, 23, 25]. However, recent studies have harnessed the power of photoacoustic spectroscopy, a technique involving the illumination of a semiconductor with a laser beam,

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to delve into photothermal phenomena [15, 26, 30]. This innovative method enables the determination of temperature values, carrier intensity, and the measurement of thermal diffusion in semiconductors [17, 31–33].The intricate interplay between elastic-thermal-plasma waves unfolds as thermal waves propagate, giving rise to elastic oscillations. Meanwhile, plasma waves, generated by photo-excited free carriers, directly induce periodic elastic deformation [2, 13].

Numerous investigations in the past have delved into generalized thermoelasticity [1, 3, 8, 9, 12, 16, 18, 34–36]. In response, several authors have introduced the concept of variable thermal conductivity to probe generalized thermoelasticity across various domains. The deformation of a semiconducting medium has been extensively studied in the recent past [4–6, 19–22, 27, 28].Numerical methods have been employed by the researchers to solve various types of equations. A few methods applied in the previous decade are listed [10, 24].

In this article, a 2-D problem is addressed, focusing on the numerical solution in a semiconducting medium under the framework of photothermal theory, using the theory of Finite Differences. The governing equations for a two-dimensional semi-infinite semiconducting medium featuring a thermally insulated, stress-free surface have been resolved to identify surface wave solutions. These solutions have been obtained through the utilization of the Finite Difference method. The Finite Difference method is employed to explore this scenario, resulting in discrete equations for both explicit and implicit schemes. Explicit schemes are solved by deriving recurrence relations involving displacement, temperature, and moisture. Implicit schemes are efficiently solved using the Tridiagonal Matrix Algorithm (TDMA) along the valid directions. The problem is numerically solved using MATLAB software for a specific material, and the results are presented graphically. This article aims to shed light on the influence of time and space on semiconducting media under the framework of photothermal theory problems. A similar method was used to analyze the thermal-mechanical behavior of different media [7, 14, 29].

2. Basic Equations

In a semiconducting medium characterized by isotropic and homogeneous properties, Song et al. [28] have provided equations that describe the coupled transport of plasma, thermal, and elastic effects.

$$\mu\nabla^2\vec{u}(\vec{r},t) + (\lambda+\mu)\nabla(\nabla\cdot\vec{u}(\vec{r},t)) - \gamma\nabla T(\vec{r},t) - \delta_d\nabla N(\vec{r},t) = \rho\frac{\partial^2\vec{u}(\vec{r},t)}{\partial t^2}, \tag{2.1}$$

$$A_e\nabla^2 N(\vec{r},t) - \frac{1}{\tau}N(\vec{r},t) + \kappa T(\vec{r},t) - \frac{\partial N(\vec{r},t)}{\partial t} = 0, \tag{2.2}$$

$$\tau_c\nabla^2 T(\vec{r},t) - \frac{S_e}{\tau}N(\vec{r},t) + \gamma T_0\nabla\cdot\frac{\partial\vec{u}(\vec{r},t)}{\partial t} - \rho C_e\frac{\partial T(\vec{r},t)}{\partial t} = 0. \tag{2.3}$$

The third and fourth terms on the left-hand side L.H.S. of Equation (2.1) account for the presence of source terms and the impact of thermal, plasma, and elastic waves. Meanwhile, in Equation (2.3), the second term on the L.H.S. represents the influence of heat generation due to carrier volume and surface de-excitation within the sample, and the third term describes the heat generated by stress waves.

A rectangular Cartesian coordinate system, denoted as OXYZ with the $z-axis$ oriented vertically downward, is adopted for reference. Within this framework, the displacement vector u is defined as $u = (u, v, 0)$, where $u = u(x, y, t)$ and $v = v(x, y, t)$.

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y} + \mu\frac{\partial^2 u}{\partial y^2} - \gamma\frac{\partial T}{\partial x} - \delta_d\frac{\partial N}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2.4}$$

$$\mu\frac{\partial^2 v}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y} + (\lambda+2\mu)\frac{\partial^2 v}{\partial y^2} - \gamma\frac{\partial T}{\partial y} - \delta_d\frac{\partial N}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{2.5}$$

$$A_e\nabla^2 N - \frac{1}{\tau}N + \kappa T - \frac{\partial N}{\partial t} = 0, \tag{2.6}$$

$$t_c\nabla^2 T - \frac{S_e}{\tau}N + \gamma T_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \rho C_e\frac{\partial T}{\partial t} = 0. \tag{2.7}$$

In the given equations, we encounter various physical parameters, including Lam's constants (λ and μ), the stress tensor (σ_{ij}), density (ρ), carrier density (N), the energy gap of a semiconductor (S_e), specific heat at constant strain (C_e), the difference in deformation potential between conduction and valence bands (δ_d), carrier diffusion coefficient (A_e), coefficient of thermal conductivity (t_c), coefficient of linear thermal expansion (α_t), and the rate of change of equilibrium carrier concentration with respect to time,

$$\kappa = \left(\frac{\partial N_0}{\partial t}\right)\left(\frac{T}{\tau}\right),$$

where N_0 represents the equilibrium carrier concentration at temperature T, and τ signifies the photo-generated carrier lifetime. Additionally, we have the thermodynamic temperature (T) and the parameter γ, which is calculated as

$$\gamma = (3\lambda + 2\mu)\alpha_t,$$

and δ_d is expressed as

$$\delta_d = (3\lambda + 2\mu)\delta_n.$$

To simplify numerical computations, we introduce the following dimensionless quantities into the equations.

$$x' = \frac{x}{c_T t^*}, \quad y' = \frac{y}{c_T t^*}, \quad u' = \frac{u}{c_T t^*}, \quad v' = \frac{v}{c_T t^*}, \quad t' = \frac{t}{t^*},$$

$$T' = \frac{\gamma T}{\lambda + 2\mu}, \quad N' = \frac{\delta_n N}{\lambda + 2\mu}, \quad c_T^2 = \frac{\lambda + 2\mu}{\rho}, \quad t^* = \frac{t_c}{\rho C_e c_T^2}. \tag{2.8}$$

Upon applying the dimensionless variables mentioned above to Equations (2.4) through (2.7) and removing the primes, we obtain the following set of dimensionless equations:

$$\frac{\partial^2 u}{\partial x^2} + A_{11}\frac{\partial^2 v}{\partial x \partial y} + A_{12}\frac{\partial^2 u}{\partial y^2} - \frac{\partial T}{\partial x} - \frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{2.9}$$

$$A_{12}\frac{\partial^2 v}{\partial x^2} + A_{11}\frac{\partial^2 u}{\partial x \partial y} + A_{12}\frac{\partial^2 v}{\partial y^2} - \frac{\partial T}{\partial y} - \frac{\partial N}{\partial y} = \frac{\partial^2 v}{\partial t^2}, \tag{2.10}$$

$$B_{01}\nabla^2 N - B_{11}N + B_{12}T - B_{13}\frac{\partial N}{\partial t} = 0, \tag{2.11}$$

$$C_{01}\nabla^2 T - C_{11}N + C_{12}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - C_{13}\frac{\partial T}{\partial t} = 0, \tag{2.12}$$

$$A_{11} = \frac{\lambda + \mu}{\lambda + 2\mu}, \qquad A_{12} = \frac{\mu}{\lambda + 2\mu},$$

$$B_{01} = \frac{D_e}{c_T^2 \tau}, \qquad B_{11} = \frac{1}{\tau}, \qquad B_{12} = \frac{\kappa^* \delta_d}{\gamma}, \qquad B_{13} = \frac{1}{t},$$

$$C_{01} = \frac{t_c}{\gamma {t^*}^2 c_T^2}, \qquad C_{11} = \frac{S_e}{\gamma t^* c_T^2 \delta_d}, \qquad C_{12} = \frac{\gamma T_0}{(\lambda + 2\mu)t^*}, \qquad C_{13} = \frac{C_e}{\gamma t^*}. \tag{2.13}$$

3. Finite Difference Equations

We are examining a rectangular area defined by the boundaries $x = 0$, $y = 0$, $x = l_1$, and $y = l_2$, within which a wave is propagating in the xy plane. The displacement vector within this two-dimensional region of a semiconducting medium is denoted as $\vec{u} = (u, v, 0)$, where $u = u(x, y, t)$ and $v = v(x, y, t)$.

Our analysis takes into account the equations governing motion, coupled with the equations describing heat conduction and carrier density, as well as the constitutive relations (2.9)-(2.12) in two dimensions. These equations are transformed according to the central finite difference approximation method, and we consider a region where time $t \geq 0$, x ranges from 0 to l_1, and y ranges from 0 to l_2. The internal points of region are defined as $x = ih_1$, $y = jh_2$, and $t = k\tau$, where $i = 0, 1, 2, \ldots, n_1$, $j = 0, 1, 2, \ldots, n_2$, and $k = 0, 1, 2, \ldots$. Subsequently, we apply the finite difference approximation to the respective derivatives.

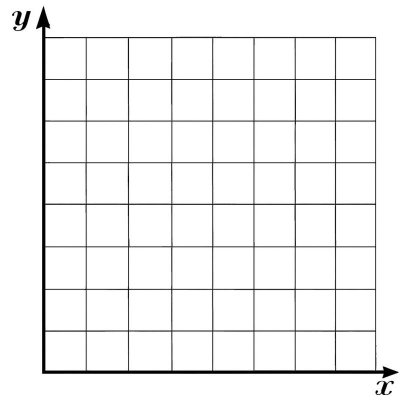

FIGURE 1. Problem set up in 2-D semiconductor region.

$$
\begin{aligned}
&\frac{u^k_{i+1,j} - 2u^k_{i,j} + u^k_{i-1,j}}{h_1^2} + A_{11}\frac{v^k_{i+1,j+1} - v^k_{i+1,j-1} - v^k_{i-1,j+1} + v^k_{i-1,j-1}}{4h_1h_2} \\
&+ A_{12}\frac{u^k_{i,j+1} - 2u^k_{i,j} + u^k_{i,j-1}}{h_2^2} - \frac{T^k_{i+1,j} - T^k_{i-1,j}}{2h_1} - \frac{N^k_{i+1,j} - N^k_{i-1,j}}{2h_1} = \frac{u^{k+1}_{i,j} - 2u^k_{i,j} + u^{k-1}_{i,j}}{\tau^2},
\end{aligned} \tag{3.1}
$$

$$
\begin{aligned}
&A_{12}\frac{v^k_{i+1,j} - 2v^k_{i,j} + v^k_{i-1,j}}{h_1^2} + A_{11}\frac{u^k_{i+1,j+1} - u^k_{i+1,j-1} - u^k_{i-1,j+1} + u^k_{i-1,j-1}}{4h_1h_2} + A_{12}\frac{v^k_{i,j+1} - 2v^k_{i,j} + v^k_{i,j-1}}{h_2^2} \\
&- \frac{T^k_{i,j+1} - T^k_{i,j-1}}{2h_2} - \frac{N^k_{i,j+1} - N^k_{i,j-1}}{2h_2} = \frac{v^{k+1}_{i,j} - 2v^k_{i,j} + v^{k-1}_{i,j}}{\tau^2},
\end{aligned} \tag{3.2}
$$

$$
B_{01}\frac{N^k_{i+1,j} - 2N^k_{i,j} + N^k_{i-1,j}}{h_1^2} + B_{01}\frac{N^k_{i,j+1} - 2N^k_{i,j} + N^k_{i,j-1}}{h_2^2} - B_{11}N^k_{i,j} + B_{12}T^k_{i,j} - B_{13}\frac{N^{k+1}_{i,j} - N^{k-1}_{i,j}}{\tau} = 0, \tag{3.3}
$$

$$
\begin{aligned}
&C_{01}\left(\frac{T^k_{i+1,j} - 2T^k_{i,j} + T^k_{i-1,j}}{h_1^2} + \frac{T^k_{i,j+1} - 2T^k_{i,j} + T^k_{i,j-1}}{h_2^2}\right) + C_{12}\frac{u^{k+1}_{i+1,j} - u^{k+1}_{i-1,j} - u^{k-1}_{i+1,j} + u^{k-1}_{i-1,j}}{2h_1\tau} \\
&+ C_{12}\frac{v^{k+1}_{i,j+1} - v^{k+1}_{i,j-1} - v^{k-1}_{i,j+1} + v^{k-1}_{i,j-1}}{2h_2\tau} - C_{13}\frac{T^{k+1}_{i,j} - T^k_{i,j}}{\tau} - C_{11}N^k_{i,j} = 0.
\end{aligned} \tag{3.4}
$$

Appropriate initial condition:

$$
\begin{aligned}
&u(x,y,t)\big|_{t=t_0} = \phi_1, \quad v(x,y,t)\big|_{t=t_0} = \phi_2, \quad N(x,y,t)\big|_{t=t_0} = N_0, \quad T(x,y,t)\big|_{t=t_0} = T_0, \\
&\left.\frac{\partial u}{\partial t}\right|_{t=t_0} = \psi_1, \quad \left.\frac{\partial v}{\partial t}\right|_{t=t_0} = \psi_2, \quad \left.\frac{\partial T}{\partial t}\right|_{t=t_0} = 0, \quad \left.\frac{\partial N}{\partial t}\right|_{t=t_0} = 0,
\end{aligned} \tag{3.5}
$$

and boundary condition:

$$
\begin{aligned}
&u(x,y,t)\big|_{x=0} = u_0, \quad u(x,y,t)\big|_{x=l_1} = \overline{u_0}, \quad u(x,y,t)\big|_{y=0} = u_0^*, \quad u(x,y,t)\big|_{y=l_2} = \overline{u_0^*}, \\
&v(x,y,t)\big|_{x=0} = v_0, \quad v(x,y,t)\big|_{x=l_1} = \overline{v_0}, \quad v(x,y,t)\big|_{y=0} = v_0^*, \quad v(x,y,t)\big|_{y=l_2} = \overline{v_0^*}, \\
&T(x,y,t)\big|_{x=0} = T_0, \quad T(x,y,t)\big|_{x=l_1} = \overline{T_0}, \quad T(x,y,t)\big|_{y=0} = T_0^*, \quad T(x,y,t)\big|_{y=l_2} = \overline{T_0^*}, \\
&N(x,y,t)\big|_{x=0} = N_0, \quad N(x,y,t)\big|_{x=l_1} = \overline{N_0}, \quad N(x,y,t)\big|_{y=0} = N_0^*, \quad N(x,y,t)\big|_{y=l_2} = \overline{N_0^*}.
\end{aligned} \tag{3.6}
$$

3.1. **Explicit Scheme.** Upon a closer examination of Equations (3.1)–(3.4), we can derive the values of $u(x,y,t)$, $v(x,y,t)$, $T(x,y,t)$, and $N(x,y,t)$ at the subsequent time layer, t^{k+1}. More precisely, by applying the initial conditions specified for $k=0$, we can determine the values of $u(x,y,t)$ and $v(x,y,t)$ for two levels, $k=0,1$.

$$\begin{aligned} u_{i,j}^{k+1} &= \tau^2\Bigg(\frac{u_{i+1,j}^k - 2u_{i,j}^k + u_{i-1,j}^k}{h_1^2} + A_{11}\frac{v_{i+1,j-1}^k - v_{i-1,j-1}^k - v_{i+1,j+1}^k + v_{i-1,j+1}^k}{4h_1h_2} \\ &\quad + A_{12}\frac{u_{i,j+1}^k - 2u_{i,j}^k + u_{i,j-1}^k}{h_2^2} - \frac{T_{i+1,j}^k - T_{i-1,j}^k}{2h_1} - \frac{N_{i+1,j}^k - N_{i-1,j}^k}{2h_1}\Bigg) + 2u_{i,j}^k - u_{i,j}^{k-1}, \end{aligned} \tag{3.7}$$

$$\begin{aligned} v_{i,j}^{k+1} &= \tau^2\Bigg(\frac{v_{i+1,j}^k - 2v_{i,j}^k + v_{i-1,j}^k}{h_1^2} + A_{11}\frac{u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k}{4h_1h_2} \\ &\quad + A_{12}\frac{v_{i,j+1}^k - 2v_{i,j}^k + v_{i,j-1}^k}{h_2^2} - \frac{T_{i+1,j}^k - T_{i-1,j}^k}{2h_1} - \frac{N_{i+1,j}^k - N_{i-1,j}^k}{2h_1}\Bigg) + 2v_{i,j}^k - v_{i,j}^{k-1}, \end{aligned} \tag{3.8}$$

$$N_{i,j}^{k+1} = \frac{\tau}{B_{13}}\left(B_{01}\left(\frac{N_{i+1,j}^k - 2N_{i,j}^k + N_{i-1,j}^k}{h_1^2} + \frac{N_{i,j+1}^k - 2N_{i,j}^k + N_{i,j-1}^k}{h_2^2}\right) - B_{11}N_{i,j}^k + B_{12}T_{i,j}^k\right) - N_{i,j}^k, \tag{3.9}$$

$$\begin{aligned} T_{i,j}^{k+1} &= \frac{\tau}{C_{13}}\Bigg(C_{01}\left(\frac{T_{i+1,j}^k - 2T_{i,j}^k + T_{i-1,j}^k}{h_1^2} + \frac{T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k}{h_2^2}\right) - C_{11}N_{i,j}^k \\ &\quad + C_{12}\left(\frac{u_{i+1,j}^{k+1} - u_{i-1,j}^{k+1} - u_{i+1,j}^{k-1} + u_{i-1,j}^{k-1}}{2h_1\tau} + \frac{v_{i,j+1}^{k+1} - v_{i,j-1}^{k+1} - v_{i,j+1}^{k-1} + v_{i,j-1}^{k-1}}{2h_2\tau}\right)\Bigg) - T_{i,j}^k. \end{aligned} \tag{3.10}$$

As we know initially

$$u(x_i,y_j) = \phi_1, \quad v(x_i,y_j) = \phi_2, \quad T(x_i,y_j) = T_0, \quad N(x_i,y_j) = N_0$$

$$\left.\frac{\partial u}{\partial t}\right|_{t=t_0} = \psi_1(x_i,y_j), \quad \Rightarrow \frac{u_{i,j}^1 - u_{i,j}^{-1}}{2\tau} = \psi_1(x_i,y_j), \quad \Rightarrow u_{i,j}^1 = u_{i,j}^{-1} + 2\tau\psi_1(x_i,y_j), \quad \Rightarrow u_{i,j}^{-1} = u_{i,j}^1 - 2\tau\psi_1(x_i,y_j).$$

Also

$$\left.\frac{\partial v}{\partial t}\right|_{t=t_0} = \psi_2(x_i,y_j), \quad \Rightarrow \frac{v_{i,j}^1 - v_{i,j}^{-1}}{2\tau} = \psi_2(x_i,y_j), \quad \Rightarrow v_{i,j}^1 = v_{i,j}^{-1} + 2\tau\psi_2(x_i,y_j), \quad \Rightarrow v_{i,j}^{-1} = v_{i,j}^1 - 2\tau\psi_2(x_i,y_j),$$

$$\left.\frac{\partial T}{\partial t}\right|_{t=t_0} = 0, \Rightarrow \frac{T_{i,j}^1 - T_{i,j}^{-1}}{\tau} = 0, \quad \Rightarrow T_{i,j}^1 = T_{i,j}^{-1}, \qquad \left.\frac{\partial N}{\partial t}\right|_{t=t_0} = 0, \Rightarrow \frac{N_{i,j}^1 - N_{i,j}^{-1}}{\tau} = 0, \quad \Rightarrow N_{i,j}^1 = N_{i,j}^{-1}, \tag{3.11}$$

using above boundary and initial values in Equation (3.7)–(3.10) we can get relation to calculate the values of $u(x,y,t), v(x,y,t), T(x,y,t)$and $N(x,y,t)$, for $k=1$

$$\begin{aligned} u_{i,j}^1 &= \tau^2\Bigg(\frac{u_{i+1,j}^0 - 2u_{i,j}^0 + u_{i-1,j}^0}{h_1^2} + A_{11}\frac{v_{i+1,j-1}^0 - v_{i-1,j-1}^0 - v_{i+1,j+1}^0 + v_{i-1,j+1}^0}{4h_1h_2} \\ &\quad + A_{12}\frac{u_{i,j+1}^0 - 2u_{i,j}^0 + u_{i,j-1}^0}{h_2^2} - \frac{T_{i+1,j}^0 - T_{i-1,j}^0}{2h_1} - \frac{N_{i+1,j}^0 - N_{i-1,j}^0}{2h_1}\Bigg) + 2u_{i,j}^0 - u_{i,j}^{-1}, \\ u_{i,j}^1 &= \frac{\tau^2}{2}\Bigg(\frac{u_{i+1,j}^0 - 2u_{i,j}^0 + u_{i-1,j}^0}{h_1^2} + A_{11}\frac{v_{i+1,j-1}^0 - v_{i-1,j-1}^0 - v_{i+1,j+1}^0 + v_{i-1,j+1}^0}{4h_1h_2} \\ &\quad + A_{12}\frac{u_{i,j+1}^0 - 2u_{i,j}^0 + u_{i,j-1}^0}{h_2^2} - \frac{T_{i+1,j}^0 - T_{i-1,j}^0}{2h_1} - \frac{N_{i+1,j}^0 - N_{i-1,j}^0}{2h_1}\Bigg) + u_{i,j}^0, \end{aligned} \tag{3.12}$$

$$v^1_{i,j} = \tau^2\Bigg(A_{12}\frac{v^0_{i,j+1} - 2v^0_{i,j} + v^0_{i,j-1}}{h_1^2} + A_{11}\frac{u^0_{i+1,j} - u^0_{i-1,j} - e^0_{i+1,j} + e^0_{i-1,j}}{4h_1h_2} + \frac{N^0_{i,j+1} - N^0_{i,j-1} - T^0_{i,j+1} + T^0_{i,j-1}}{2h_2}\Bigg) + 2v^0_{i,j} - v^{-1}_{i,j},$$

$$v^1_{i,j} = \tau^2\Bigg(A_{12}\frac{v^0_{i,j+1} - 2v^0_{i,j} + v^0_{i,j-1}}{h_1^2} + A_{11}\frac{u^0_{i+1,j} - u^0_{i-1,j} - e^0_{i+1,j} + e^0_{i-1,j}}{4h_1h_2} + \frac{N^0_{i,j+1} - N^0_{i,j-1} - T^0_{i,j+1} + T^0_{i,j-1}}{2h_2}\Bigg) + 2v^0_{i,j} - v^{-1}_{i,j}, \tag{3.13}$$

$$N^1_{ij} = \frac{\tau}{B_{13}}\Bigg(\frac{B_{01}\left(N^0_{i-1,j} - 2N^0_{i,j} + N^0_{i+1,j}\right)}{h_1^2} + \frac{B_{01}\left(N^0_{i,j-1} - 2N^0_{i,j} + N^0_{i,j+1}\right)}{h_2^2} - B_{11}T^{x^0}_{ij} + B_{12}T^{y^0}_{ij}\Bigg) + N^0_{ij}, \tag{3.14}$$

$$T^1_{ij} = \frac{\tau}{C_{13}}\Bigg[C_{01}\left(\frac{T^0_{i+1,j} - 2T^0_{i,j} + T^0_{i-1,j}}{h_1^2} + \frac{T^0_{i,j+1} - 2T^0_{i,j} + T^0_{i,j-1}}{h_2^2}\right) - C_{11}N^{x^0}_{ij} + C_{12}N^{y^0}_{ij} + C_{12}\left(\frac{u^1_{i+1,j} - u^1_{i-1,j} - u^0_{i+1,j} + u^0_{i-1,j}}{2h_1\tau}\right) + C_{12}\left(\frac{v^1_{i,j+1} - v^1_{i,j-1} - v^0_{i,j+1} + v^0_{i,j-1}}{2h_2\tau}\right)\Bigg] + T^0_{ij}. \tag{3.15}$$

3.2. **Implicit Scheme.** Now, to deduce an implicit method for solving coupled equations numerically, we need to rewrite equations (3.1)–(3.4) in the following form by replacing k with $k+1$ in the term which second derivative with respect to x:

$$\frac{1}{h_1^2}u^{k+1}_{i+1,j} - \left(\frac{2}{h_1^2} + \frac{1}{\tau^2}\right)u^{k+1}_{i,j} + \frac{1}{h_2^2}u^{k+1}_{i,j-1} = A_{11}\frac{v^k_{i+1,j+1} - v^k_{i-1,j+1} - v^k_{i+1,j-1} + v^k_{i-1,j-1}}{4h_1h_2} - A_{12}\frac{u^k_{i,j+1} - 2u^k_{i,j} + u^k_{i,j-1}}{h_2^2} + \frac{T^k_{i+1,j} - T^k_{i-1,j}}{2h_1} + \frac{N^k_{i+1,j} - N^k_{i-1,j}}{2h_1} - \frac{2}{\tau^2}u^k_{i,j} + \frac{1}{\tau^2}u^{k-1}_{i,j},$$

$$a_1u^{k+1}_{i,j} + bu^k_{i,j} + cu^{k-1}_{i,j} = f_{i,j}, \tag{3.16}$$

where

$$a_i = \frac{1}{h_1^2}, \quad b_i = -\left(\frac{2}{h_1^2} + \frac{\rho}{\tau^2}\right), \quad c_i = \frac{1}{h_1^2},$$

and

$$f_{i,j} = A_{11}\frac{v^k_{i+1,j+1} - v^k_{i-1,j+1} - v^k_{i+1,j-1} + v^k_{i-1,j-1}}{4h_1h_2} - A_{12}\frac{u^k_{i,j+1} - 2u^k_{i,j} + u^k_{i,j-1}}{h_2^2} + \frac{T^k_{i+1,j} - T^k_{i-1,j}}{2h_1} + \frac{N^k_{i+1,j} - N^k_{i-1,j}}{2h_1} - \frac{2}{\tau^2}u^k_{i,j} + \frac{1}{\tau^2}u^{k-1}_{i,j},$$

$$\frac{A_{12}}{h_1^2}v^{k-1}_{i-1,j} - \left(\frac{2A_{12}}{h_1^2} + \frac{1}{\tau^2}\right)v^{k-1}_{i,j} + \frac{A_{12}}{h_1^2}v^{k-1}_{i+1,j} = -A_{11}\frac{u^{k-1}_{i,j+1} - u^{k-1}_{i,j-1} - u^{k-2}_{i,j+1} + u^{k-2}_{i,j-1}}{4h_1\tau} - \frac{v^{k-1}_{i,j+1} - 2v^{k-1}_{i,j} + v^{k-1}_{i,j-1}}{h_2^2} + \frac{T^{k-1}_{i,j+1} - T^{k-1}_{i,j-1}}{2h_2} + \frac{N^{k-1}_{i,j+1} - N^{k-1}_{i,j-1}}{2h_2} - \frac{2}{\tau^2}v^{k-1}_{i,j} + \frac{1}{\tau^2}v^k_{i,j}, \tag{3.17}$$

$$a_i = \frac{A_{12}}{h_1^2}, \qquad b_i = -\left(\frac{2A_{12}}{h_1^2} + \frac{1}{\tau^2}\right), \qquad c_i = \frac{A_{12}}{h_1^2},$$

$$f_{ij} = \frac{1}{\tau^2}\left(v_{ij}^k - 2v_{ij}^{k-1}\right) - A_{11}\frac{u_{i,j+1}^{k-1} - u_{i,j-1}^{k-1} - u_{i,j+1}^{k-2} + u_{i,j-1}^{k-2}}{4h_1\tau} - \frac{v_{i,j+1}^{k-1} - 2v_{i,j}^{k-1} + v_{i,j-1}^{k-1}}{h_2^2}$$
$$+ \frac{N_{i,j+1}^{k-1} - N_{i,j-1}^{k-1}}{2h_2} + \frac{T_{i,j+1}^{k-1} - T_{i,j-1}^{k-1}}{2h_2},$$

$$\frac{B_{01}}{h_1^2}N_{i-1,j}^{k+1} - \left(\frac{2B_{01}}{h_1^2} + \frac{B_{13}}{\tau}\right)N_{i,j}^{k+1} + \frac{B_{01}}{h_1^2}N_{i+1,j}^{k+1} = -B_{01}\frac{N_{i,j+1}^k - 2N_{i,j}^k + N_{i,j-1}^k}{h_2^2} + B_{11}T_{ij}^{x^k}$$
$$+ B_{12}T_{ij}^{y^k} - \frac{B_{13}}{\tau}N_{ij}^k, \tag{3.18}$$

$$a_i = \frac{B_{01}}{h_1^2}, \qquad b_i = -\left(\frac{2B_{01}}{h_1^2} + \frac{B_{13}}{\tau}\right), \qquad c_i = \frac{B_{01}}{h_1^2},$$

$$f_{ij} = B_{11}T_{ij}^{x^k} + B_{12}T_{ij}^{y^k} - B_{01}\frac{N_{i,j+1}^k - 2N_{i,j}^k + N_{i,j-1}^k}{h_2^2} - \frac{B_{13}}{\tau}N_{ij}^k,$$

$$\frac{C_{01}}{h_1^2}T_{i-1,j}^{k+1} - \left(\frac{2C_{01}}{h_1^2} + \frac{C_{13}}{\tau}\right)T_{i,j}^{k+1} + \frac{C_{01}}{h_1^2}T_{i+1,j}^{k+1} = -C_{01}\frac{T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k}{h_2^2} + C_{11}N_{ij}^{x^k} - C_{12}N_{ij}^{y^k}$$
$$- C_{12}\left(\frac{v_{i,j+1}^k - v_{i,j-1}^k - v_{i,j+1}^{k-1} + v_{i,j-1}^{k-1}}{2h_2\tau}\right) + \frac{C_{13}}{\tau}T_{ij}^k, \tag{3.19}$$

$$a_i = \frac{C_{01}}{h_1^2}, \qquad b_i = -\left(\frac{2C_{01}}{h_1^2} + \frac{C_{13}}{\tau}\right), \qquad c_i = \frac{C_{01}}{h_1^2},$$

$$f_{ij} = \frac{C_{01}}{h_1^2}\left(T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k\right) + C_{11}N_{ij}^{x^k} - C_{12}N_{ij}^{y^k} - C_{12}\left(\frac{v_{i,j+1}^k - v_{i,j-1}^k - v_{i,j+1}^{k-1} + v_{i,j-1}^{k-1}}{2h_2\tau}\right) - \frac{C_{13}}{\tau}T_{ij}^k.$$

4. Numerical Verification

To validate the theoretical results obtained in the previous sections, a numerical simulation is conducted to solve the problem outlined by Equations (2.1)-(2.4). This simulation involves using recurrence formulas and the Tridiagonal Matrix Algorithm (TDMA) as a test. The specific values of constants, initial conditions, and boundary conditions used for this numerical computation can be found in Table 1. These parameter values have been selected based on data for Silicon provided by Song et al. [28]. The numerical computations are carried out for $l_1 = 1$ and $l_2 = 1$.

At $t = 0$:

$$u(x,y,t) = 0, \quad v(x,y,t) = 0, \quad m(x,y,t) = 0, \quad \text{and} \quad T(x,y,t) = T_0 \sin\left(\frac{\pi x}{l_1}\right)\sin\left(\frac{\pi y}{l_2}\right).$$

The boundary conditions are defined as:

$$u(x,y,t) = 0, \quad v(x,y,t) = 0, \quad N(x,y,t) = 0, \quad \text{and} \quad T(x,y,t) = 0,$$

where Γ represents the boundary of the body.

Figures 2–9 display the distributions of displacement components $u(x,y,t)$ and $v(x,y,t)$, temperature $T(x,y,t)$, and carrier density $N(x,y,t)$ in three-dimensional space. Each of these unknown quantities has been computed using recurrence formulas and the Tridiagonal Matrix Algorithm (TDMA) with the assistance of MATLAB software.

This simulation provides a visual representation of how these variables evolve within the given physical system under the specified initial and boundary conditions. It facilitates a deeper understanding of the behavior and interactions of these field variables throughout the studied domain over time.

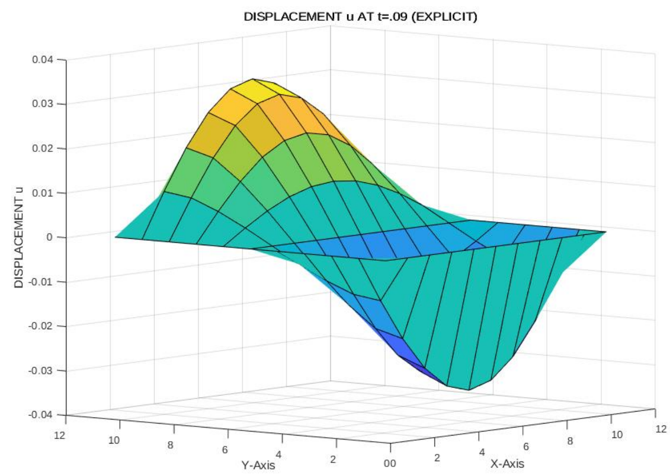

FIGURE 2. Horizontal displacement u by explicit method at t=0.09.

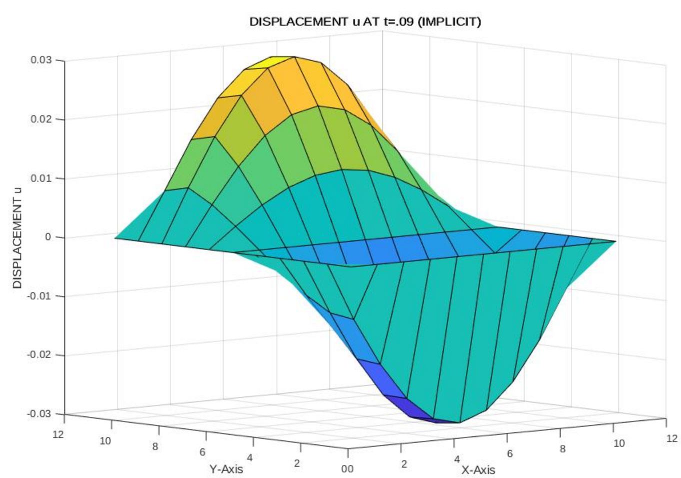

FIGURE 3. Horizontal displacement u by implicit method at t=0.09.

TABLE 1. Material constant.

Material Constant			
SNo	Quantity	Symbol	unit
1	Lames constant	λ	$36.9 \times 10^9 N/m^2$
2	Lames constant	μ	$54.6 \times 10^9 N/m^2$
3	Density	ρ	$2.331 \times 10^3 Kg/m^3$
4	Specific Heat	C_e	$695 JKg^{-1}K^{-1}$
5	Diffusion	D_e	$1.12eV$
6	Temperature	T	$300K$

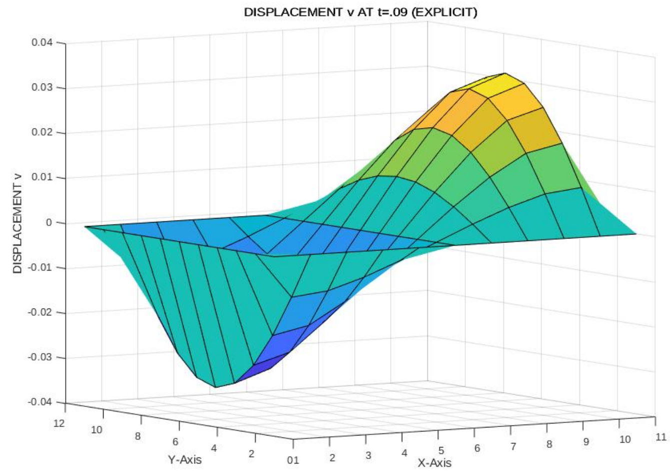

FIGURE 4. Vertical displacement v by explicit method at t=0.09.

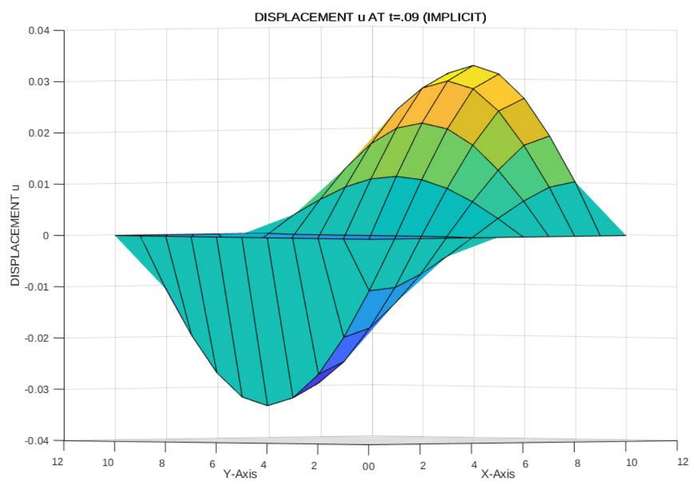

FIGURE 5. Vertical displacement v by implicit method at t=0.09.

5. Conclusion

In this study, a two-dimensional semi-infinite semiconducting medium with thermally insulated and stress-free boundary conditions has been investigated using finite difference techniques. The governing coupled equations describing elastic displacements, carrier density, and temperature fields were solved numerically by formulating both explicit and implicit finite difference schemes. The primary objective was to analyze wave propagation characteristics and to compare the effectiveness and reliability of the two numerical approaches.

The comparative analysis reveals an unexpected but important similarity between the explicit and implicit finite difference solutions for all physical fields considered. This suggests that, for the present problem and chosen discretization parameters, both schemes are equally effective and reliable. The results validate the applicability of finite difference methods in analyzing coupled thermoelastic and carrier transport phenomena in semiconducting media. The

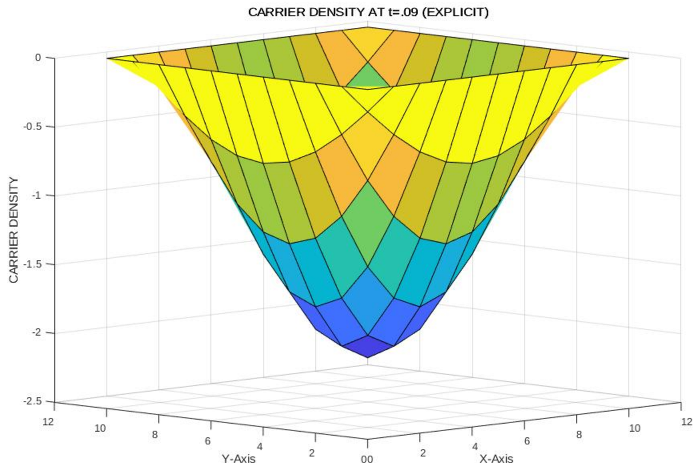

FIGURE 6. Carrier Density N by Explicit method at t=0.09.

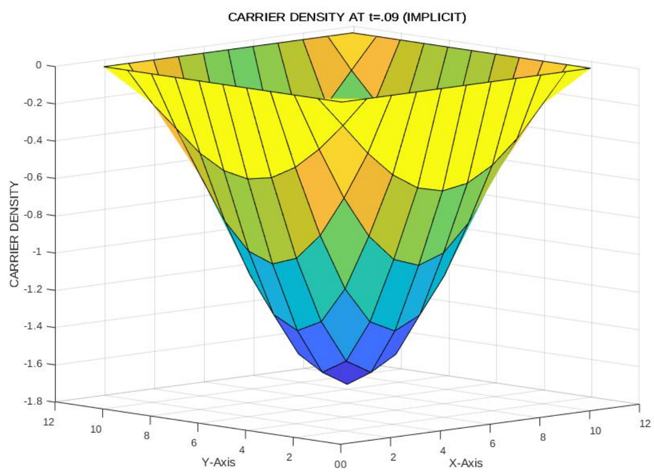

FIGURE 7. Carrier Density N by implicit method at t=0.09.

present formulation can be extended to more complex boundary conditions, material parameters, or higher-dimensional problems relevant to modern semiconductor and microelectronic applications.

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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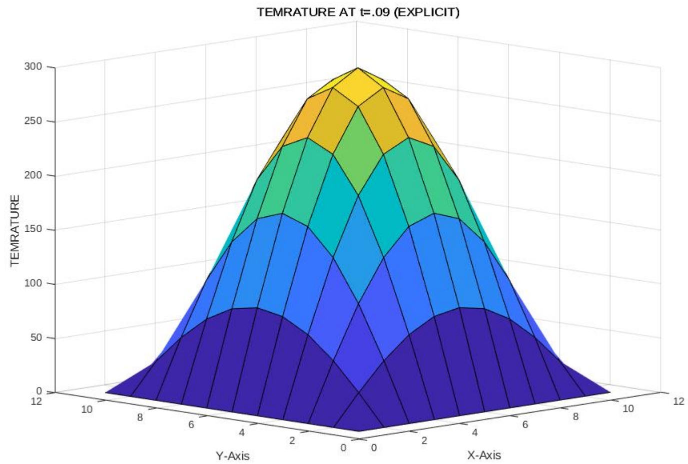

FIGURE 8. Temperature T by explicit method at t=0.09.

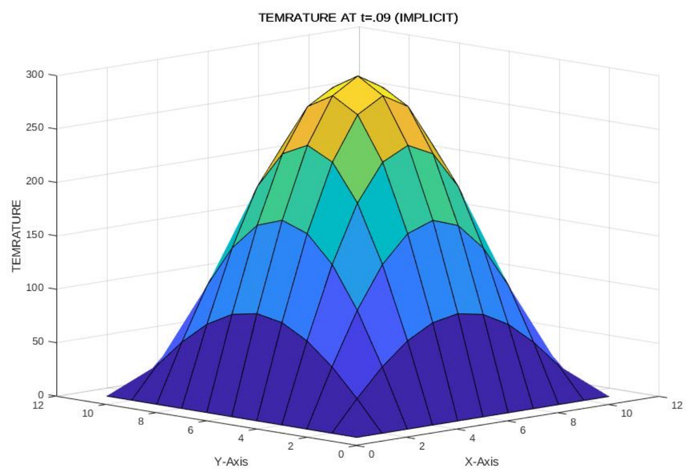

FIGURE 9. Temperature T by implicit method at t=0.09.

Acknowledgment

This section should come before the References and should be unnumbered. Funding information may also be included here.

TABLE 2. Displacement u (Explicit) at $t = 0.09$.

Horizontal Displacement u (Explicit)											
y \ x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	-0.008	-0.015	-0.020	-0.023	-0.024	-0.023	-0.019	-0.014	-0.007	0
0.2	0	-0.007	-0.014	-0.019	-0.022	-0.024	-0.022	-0.019	-0.014	-0.007	0
0.3	0	-0.005	-0.010	-0.014	-0.016	-0.017	-0.016	-0.014	-0.010	-0.005	0
0.4	0	-0.003	-0.005	-0.007	-0.009	-0.009	-0.009	-0.007	-0.005	-0.003	0
0.5	0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
0.6	0	0.003	0.005	0.007	0.009	0.009	0.009	0.007	0.005	0.003	0
0.7	0	0.005	0.010	0.014	0.016	0.017	0.016	0.014	0.010	0.005	0
0.8	0	0.007	0.014	0.019	0.022	0.024	0.022	0.019	0.014	0.007	0
0.9	0	0.007	0.014	0.019	0.023	0.024	0.023	0.020	0.015	0.008	0
1.0	0	0	0	0	0	0	0	0	0	0	0

TABLE 3. Displacement u (Implicit) at $t = 0.09$.

Horizontal Displacement u (Implicit)											
y \ x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	-0.008	-0.015	-0.020	-0.023	-0.024	-0.023	-0.019	-0.014	-0.007	0
0.2	0	-0.007	-0.014	-0.019	-0.022	-0.024	-0.022	-0.019	-0.014	-0.007	0
0.3	0	-0.005	-0.010	-0.014	-0.016	-0.017	-0.016	-0.014	-0.010	-0.005	0
0.4	0	-0.003	-0.005	-0.007	-0.009	-0.009	-0.009	-0.007	-0.005	-0.003	0
0.5	0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
0.6	0	0.003	0.005	0.007	0.009	0.009	0.009	0.007	0.005	0.003	0
0.7	0	0.005	0.010	0.014	0.016	0.017	0.016	0.014	0.010	0.005	0
0.8	0	0.007	0.014	0.019	0.022	0.024	0.022	0.019	0.014	0.007	0
0.9	0	0.007	0.014	0.019	0.023	0.024	0.023	0.020	0.015	0.008	0
1.0	0	0	0	0	0	0	0	0	0	0	0

Table 4. Displacement v (Explicit) at $t = 0.09$.

Vertical Displacement v (Explicit)											
$y \backslash x$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	-0.008	-0.007	-0.005	-0.003	0	0.003	0.005	0.007	0.008	0
0.2	0	-0.016	-0.014	-0.010	-0.005	0	0.005	0.010	0.014	0.016	0
0.3	0	-0.021	-0.019	-0.014	-0.007	0	0.007	0.014	0.019	0.021	0
0.4	0	-0.025	-0.023	-0.016	-0.009	0	0.009	0.016	0.023	0.025	0
0.5	0	-0.027	-0.024	-0.017	-0.009	0	0.009	0.017	0.024	0.027	0
0.6	0	-0.025	-0.023	-0.016	-0.009	0	0.009	0.016	0.023	0.025	0
0.7	0	-0.021	-0.019	-0.014	-0.007	0	0.007	0.014	0.019	0.021	0
0.8	0	-0.016	-0.014	-0.010	-0.005	0	0.005	0.010	0.014	0.016	0
0.9	0	-0.008	-0.007	-0.005	-0.003	0	0.003	0.005	0.007	0.008	0
1.0	0	0	0	0	0	0	0	0	0	0	0

Table 5. Displacement v (Implicit) at $t = 0.09$.

Vertical Displacement v (Implicit)											
$y \backslash x$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	-0.008	-0.007	-0.005	-0.003	0	0.003	0.005	0.007	0.008	0
0.2	0	-0.016	-0.014	-0.010	-0.005	0	0.005	0.010	0.014	0.016	0
0.3	0	-0.021	-0.019	-0.014	-0.007	0	0.007	0.014	0.019	0.021	0
0.4	0	-0.025	-0.023	-0.016	-0.009	0	0.009	0.016	0.023	0.025	0
0.5	0	-0.027	-0.024	-0.017	-0.009	0	0.009	0.017	0.024	0.027	0
0.6	0	-0.025	-0.023	-0.016	-0.009	0	0.009	0.016	0.023	0.025	0
0.7	0	-0.021	-0.019	-0.014	-0.007	0	0.007	0.014	0.019	0.021	0
0.8	0	-0.016	-0.014	-0.010	-0.005	0	0.005	0.010	0.014	0.016	0
0.9	0	-0.008	-0.007	-0.005	-0.003	0	0.003	0.005	0.007	0.008	0
1.0	0	0	0	0	0	0	0	0	0	0	0

TABLE 6. Carrier Density N (Explicit) at $t = 0.09$.

Carrier Density N (Explicit) at $t = 0.09$											
y \ x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	-0.180	-0.343	-0.472	-0.555	-0.584	-0.555	-0.472	-0.343	-0.180	0
0.2	0	-0.343	-0.652	-0.898	-1.056	-1.110	-1.056	-0.898	-0.652	-0.343	0
0.3	0	-0.472	-0.898	-1.236	-1.453	-1.528	-1.453	-1.236	-0.898	-0.472	0
0.4	0	-0.555	-1.056	-1.453	-1.708	-1.796	-1.708	-1.453	-1.056	-0.555	0
0.5	0	-0.584	-1.110	-1.528	-1.796	-1.888	-1.796	-1.528	-1.110	-0.584	0
0.6	0	-0.555	-1.056	-1.453	-1.708	-1.796	-1.708	-1.453	-1.056	-0.555	0
0.7	0	-0.472	-0.898	-1.236	-1.453	-1.528	-1.453	-1.236	-0.898	-0.472	0
0.8	0	-0.343	-0.652	-0.898	-1.056	-1.110	-1.056	-0.898	-0.652	-0.343	0
0.9	0	-0.180	-0.343	-0.472	-0.555	-0.584	-0.555	-0.472	-0.343	-0.180	0
1.0	0	0	0	0	0	0	0	0	0	0	0

TABLE 7. Carrier Density N (Implicit) at $t = 0.09$.

Carrier Density N (Implicit) at $t = 0.09$											
y \ x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	-0.180	-0.343	-0.472	-0.555	-0.584	-0.555	-0.472	-0.343	-0.180	0
0.2	0	-0.343	-0.652	-0.898	-1.056	-1.110	-1.056	-0.898	-0.652	-0.343	0
0.3	0	-0.472	-0.898	-1.236	-1.453	-1.528	-1.453	-1.236	-0.898	-0.472	0
0.4	0	-0.555	-1.056	-1.453	-1.708	-1.796	-1.708	-1.453	-1.056	-0.555	0
0.5	0	-0.584	-1.110	-1.528	-1.796	-1.888	-1.796	-1.528	-1.110	-0.584	0
0.6	0	-0.555	-1.056	-1.453	-1.708	-1.796	-1.708	-1.453	-1.056	-0.555	0
0.7	0	-0.472	-0.898	-1.236	-1.453	-1.528	-1.453	-1.236	-0.898	-0.472	0
0.8	0	-0.343	-0.652	-0.898	-1.056	-1.110	-1.056	-0.898	-0.652	-0.343	0
0.9	0	-0.180	-0.343	-0.472	-0.555	-0.584	-0.555	-0.472	-0.343	-0.180	0
1.0	0	0	0	0	0	0	0	0	0	0	0

Table 8. Temperature T (Explicit) at $t = 0.09$.

Temperature T (Explicit) at $t = 0.09$											
y / x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	28.64	54.48	74.99	88.15	92.69	88.15	74.99	54.48	28.64	0
0.2	0	54.48	103.63	142.63	167.67	176.30	167.67	142.63	103.63	54.48	0
0.3	0	74.99	142.63	196.31	230.78	242.66	230.78	196.31	142.63	74.99	0
0.4	0	88.15	167.67	230.78	271.30	285.26	271.30	230.78	167.67	88.15	0
0.5	0	92.69	176.30	242.66	285.26	299.94	285.26	242.66	176.30	92.69	0
0.6	0	88.15	167.67	230.78	271.30	285.26	271.30	230.78	167.67	88.15	0
0.7	0	74.99	142.63	196.31	230.78	242.66	230.78	196.31	142.63	74.99	0
0.8	0	54.48	103.63	142.63	167.67	176.30	167.67	142.63	103.63	54.48	0
0.9	0	28.64	54.48	74.98	88.15	92.68	88.15	74.98	54.48	28.64	0
1.0	0	0	0	0	0	0	0	0	0	0	0

Table 9. Temperature T (Implicit) at $t = 0.09$.

Temperature T (Implicit) at $t = 0.09$											
y / x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	28.64	54.51	75.04	88.22	92.77	88.23	75.06	54.55	28.69	0
0.2	0	54.48	103.63	142.63	167.68	176.30	167.68	142.63	103.63	54.48	0
0.3	0	74.99	142.63	196.32	230.79	242.66	230.79	196.32	142.63	74.99	0
0.4	0	88.15	167.68	230.79	271.30	285.27	271.30	230.79	167.68	88.15	0
0.5	0	92.69	176.30	242.66	285.27	299.95	285.27	242.66	176.30	92.69	0
0.6	0	88.15	167.68	230.79	271.30	285.27	271.30	230.79	167.68	88.15	0
0.7	0	74.99	142.63	196.32	230.79	242.66	230.79	196.32	142.63	74.99	0
0.8	0	54.48	103.63	142.63	167.68	176.30	167.68	142.63	103.63	54.48	0
0.9	0	28.69	54.55	75.06	88.23	92.77	88.22	75.04	54.51	28.64	0
1.0	0	0	0	0	0	0	0	0	0	0	0

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