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A mathematical study on infinite boundary value problem for MHD flow of a micropolar nanofluid

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Abstract

In this paper, the modified q-Homotopy analysis method (q-HAM) is employed to study the problem of magnetohydrodynamic (MHD) flow of nanofluid under buoyancy effects semi-analytically. The approximate analytic expressions of dimensionless velocity, dimensionless angular velocity, dimensionless temperature and dimensionless concentration profiles are given explicitly. We can also derive the approximate analytical expressions for skin friction coefficient, Nusselt Number, and sherwood number. The graphical representation for numerous physical factors involved in the model are provided. This method is also extended to resolve various nonlinear problems in the applied sciences.

Keywords. Micropolar nanofluid, Buoyancy effect, MHD fluid flow, Non-linear boundary value problem, q-Modified Homotopy analysis method.
1991 Mathematics Subject Classification. 34E05,34E10,34E15,34E20.

1. Introduction

Micropolar nanofluids enhancement of heat transfer is critical problem in many industrial applications. Hayat et al. [15] investigated micropolar nanofluid using Buongiorno model. Eringen [11, 12] was the researcher who discussed the concept of micropolar fluid and compressive investigation of micropolar. Lukaszewick [24] authored a comprehensive book about the micropolar nanofluid and the polar fluid, which encompass the spherical or rigid particles. Some common instances of micropolar fluid are liquid crystals, liquids of bubby and animal blood. Damseh et al. [10] studied the chemical reaction of micropolar fluid in the presence of heat generation on the stretching sheet which is directly proportional to the skin friction coefficient. Stretching and Shrinking sheet was discussed by many authors in their studies [9, 30].

Chamkha et al. [5] investigated the MHD flow of a micropolar fluid. Magyari et al. [27, 28] found that the temperature decreased as the heat absorption parameter was increased. The development of micropolar fluid is discussed by many researchers in their articles [2, 14, 32, 34, 40, 42]. This work has been extended by numerous researchers with the existence of heat source and magnetic fields for energy extraction, nanofluid and magnetohydrodynamic. These studies have provided valuable insights into the stretching flow problem under various conditions, such as flow over porous surfaces [6, 13, 26, 38]. Numerous studies by mathematicians and scientists [7, 25, 29, 31, 33, 35] have sought to understand various problems related to MHD fluids, heat and mass transfer, slip effects, etc., and the flow of heat transfer characteristic over a stretching permeable surface can be examined.

The two models that support the transport phenomena are: the first is the Buongiorno model [3], which represents a two-phase model, and the second model was developed by Tiwari et al. [39], which represents a single-phase model and it described the thermophysical properties as a function of nanoparticles. The concepts of the Nusselt number and entropy generation were discussed by Chamkha et al. [5].

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Nowadays, many authors solved non-linear differential equations by using various numerical methods such as the Runge-Kutta method, the shooting method, finite difference scheme, etc. Nanofluids exhibit higher thermal conductivity as compared to their base fluid [8]. Choi [31] was the first to introduce the concept of enhancing heat transport by dispersing solid particles nanometer-sized in a base fluid (water).

In this article, approximate analytical solutions of MHD flow over a stretching sheet in the presence of radiation are obtained by using suitable semi-analytical method, namely the q- Modified Homotopy analysis method (q-MHAM). The physical factors regarding dimensionless velocity, angular velocity, temperature and concentration are graphically illustrated to show their influences.

2. Mathematical Formulation of the problem

Consider the micropolar nanofluid over a vertical linearly shrinking surface with the effect of magnetic field. All assumptions about the problem is shown in Figure 1. The field velocities in the $x-$ and $y-$direction are denoted by u and v respectively. Here, N, T, and C represent the micro rotation, temperature and volume friction of nanoparticles within the boundary layer, respectively. Meanwhile, T_w indicates the temperature of the fluid and C_w denotes the volume friction of nanoparticles at the wall. In the case when both parameters are away from the wall, they are represented by T_∞ and C_∞, respectively. It can be assumed that the uniform intensity of magnetic force acts normally on the plane of the surface. Along with all mentioned conditions, the governing equations of continuity, momentum, angular momentum, temperature and concentration are given in the Buongiorno model [3] as below:

$$\bigtriangledown V = 0, \tag{2.1}$$

$$\begin{aligned}\rho_f \frac{dV}{dt} &= - \bigtriangledown P + (\mu_f + k) \bigtriangledown^2 Vk + (\bigtriangledown \times N) + (J \times B) \\ &\quad + [1 - C_\infty](\rho_f)_\infty \beta(T - T\infty) - (\rho_p - (\rho_f)_\infty)(C - C_\infty),\end{aligned} \tag{2.2}$$

$$\rho_f j \frac{dN}{dt} = \gamma \bigtriangledown^2 N - K(2N - \bigtriangledown \times V), \tag{2.3}$$

$$(\rho C)_p \frac{dT}{dt} = k \bigtriangledown^2 T + (\rho C)_f [D_B \bigtriangledown C \bigtriangledown T + \frac{D_T}{T_\infty}(\bigtriangledown T)^2], \tag{2.4}$$

$$\frac{dC}{dt} = D_B \bigtriangledown^2 C + \frac{D_T}{T_\infty} \bigtriangledown^2 T, \tag{2.5}$$

where the velocity vector is $V = [u(x,y), v(x,y), 0]$, the micro rotation vector is N, applied magnetic field strength is $B = \sigma^*(V \times B)$, micro-rotation viscosity coefficient is k, the gravitational acceleration vector is $g = [0, g, 0]$, and the micro-inertia density is j. Furthermore, μ_f, p, ρ_p, ρ_f, k, D_B, D_T, $(\rho c)_p$, and $(\rho c)_f$ denote the base fluid viscosity, pressure, the nanoparticles material densities, densities of base fluid, the thermal conductivity, Brownian diffusion coefficient, thermophoretic diffusion coefficient, the effective heat capacity of the nanoparticles material, and the effective heat capacity of the base fluid, respectively.

Applying scale analysis, resulting boundary layer equations are obtained below [36]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (v_f + \frac{k}{\rho})\frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho_f}\frac{\partial N}{\partial y} - \frac{\sigma^* B_0^2 u}{\rho_f} + \frac{1}{\rho_f}[(1 - C_\infty)\rho_{f\infty}\beta(T - T_\infty) - (\rho_p - \rho_{f\infty})(C - C_\infty)], \tag{2.7}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{1}{\rho_j}[\gamma\frac{\partial^2 N}{\partial y^2} - k(2N + \frac{\partial u}{\partial y})], \tag{2.8}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_f\frac{\partial^2 T}{\partial y^2} + \tau_w\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{2.9}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{2.10}$$

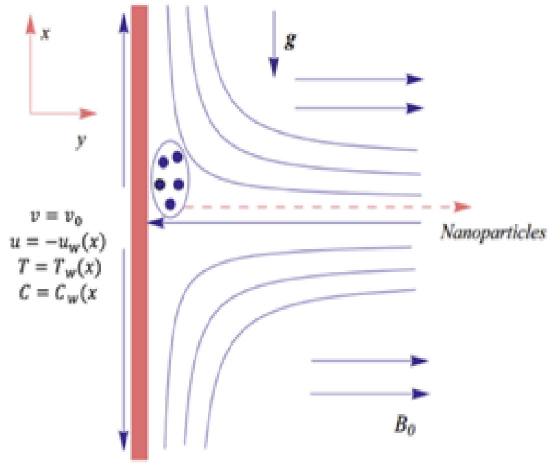

FIGURE 1. Diagram of flow problem and coordinate system.

subject to boundary conditions

$$\begin{aligned} &v = v_0;\ u = -u_w(x);\ N = -m\frac{\partial u}{\partial y}T = T_w;\ \ C = C_w\ \ at\ \ y = 0,\\ &u \to 0;\ N \to 0; T \to T_\infty;\ \ C \to C_\infty\ \ \ as\ y \to \infty. \end{aligned} \tag{2.11}$$

For the current problem, we introduce the following dimensionless similarity variables:

$$\begin{aligned} &\eta = y\sqrt{\frac{a}{v_f}},\ u = a\,x\,f'(\eta),\ \ v = -\sqrt{av_f}\,f(\eta),\ \ N = \sqrt{\frac{a}{v_f}}a\,x\,g(\eta),\\ &\theta(\eta) = \frac{(T - T_\infty)}{T_w - T_\infty},\ \ \ \pi(\eta) = \frac{(C - C_\infty)}{C_\infty}. \end{aligned} \tag{2.12}$$

By applying the similarity variables defined in Eq. (2.12) to the partial differential Equations (2.6)-(2.10), we obtain the following system of ordinary differential equations:

$$(1 + k)\,f^{'''} + f f'' - f'^2 + kg' - Mf' + \delta(\theta - N_r\phi), \tag{2.13}$$

$$\left(1 + \frac{k}{2}\right)g'' + fg' - gf' - 2Kg' - Kf'' = 0, \tag{2.14}$$

$$\frac{1}{Pr}\theta'' + f\theta' + N_b\phi'\theta' + N_t(\theta')^2 = 0, \tag{2.15}$$

$$\frac{1}{Sc}\phi'' + f\phi' + \frac{1}{Sc}\frac{N_t}{N_b}\theta'' = 0, \tag{2.16}$$

with the reduced boundary conditions given below:

$$\begin{aligned} &f(0) = f_w,\ f'(0) = -1,\ g(0) = -mf''(0),\ \theta(0) = 1,\ \phi(0) = 1,\\ &f'(\eta) \to 0,\ g(\eta) \to 0,\ \theta(\eta) \to 0,\ \ \phi(\eta) \to 0,\ \ as\ \ (\eta) \to \infty, \end{aligned} \tag{2.17}$$

where K is the micropolar material parameter, M is the Hartmann number, δ is the parameter of local mixed convection or Richardson number which depends on the x independent variable. Therefore, the problem can be considered as a local one. It is worth noting that $\delta < 0, \delta > 0$, and $\delta = 0$ indicate a cooled surface, a hot surface and a forced convection flow, respectively. N_r is the buoyancy parameter, P_r and S_c are the Prandtl and Schmidt numbers respectively. N_t and N_b are the thermophoresis and Brownian diffusion parameters respectively. Suction/blowing parameter is f_w, and

m is the micro-gyration parameter. The above all parameters can be expressed as:

$$K = \frac{k}{\mu},\ M = \frac{\sigma^* B_0^2}{\rho_f a},\ \delta = \frac{g\beta(1-C_\infty)(T_w - T_\infty)}{a^2 x},$$
$$N_r = \frac{(\rho_p - \rho_{f_\infty})C_\infty}{\rho_{f_\infty}\beta(1-C_\infty)(T_w - T_\infty)},\ Pr = \frac{v_f}{\alpha_f},\ Sc = \frac{v_f}{D_B},$$
$$N_t = \frac{\tau_w D_T (T_w - T\infty)}{v_f T_\infty},\ N_b = \frac{\tau_w D_B C_\infty}{v_f},\ f_w = -\frac{v_0}{\sqrt{a v_f}},$$

The physical quantities of interest are skin friction coefficient, local Nusselt number, and Sherwood number, which can be expressed as,

$$C_f = \frac{[(\mu + K)\frac{\partial u}{\partial y} + kN]_{y=0}}{\rho u^2{}_w},\ N_u = \frac{-x[\frac{\partial T}{\partial y}]_{y=0}}{(T_w - T_\infty)},\ S_h = \frac{-x(\frac{\partial C}{\partial y})_{y=0}}{C_\infty}, \tag{2.18}$$

Using Eq. (2.7) in Eq. (2.13), we get

$$C_f (Re_x)^{1/2} = (1 + (1-m)k) f"(0), N_u (Re)^{-1/2} = -\theta'(0) S_h (Re)^{-1/2} = -\phi'(0), \tag{2.19}$$

where $Re_x = \frac{ax^2}{v_f}$ is local Reynolds number.

3. Semi-analytical expressions of the dimensionless velocity, angular velocity, temperature and concentration profiles with the aid of q-MHAM

The semi-analytical Homotopy Analysis Method (HAM) is a non-perturbative technique that provides series solutions for nonlinear differential equations. The non-linearity in non- linear differential equation is a polynomial that contains the unknown function and its derivatives, as specified in previous studies. An innovative semi-analytical method, namely the "Homotopy analysis method", was introduced by Liao [17–22], . This provides possible ways to estimate the solution and from the infinite power series, we can attain the numerical values. A finite number of terms and the system of differential equations were solved, which examined the accuracy of this method (HAM). In this technique there is an auxiliary parameter h, that helps us to control and modify the convergence region of the solution series. Several issues in physical, chemical and engineering sciences can be solved by Homotopy analysis method.

Applying q-MHAM [1], [37] to dimensionless concentration, velocity, angular velocity and temperature profiles, we have derived the approximate analytical solutions.

The resulting approximate analytical solutions as follows:

$$\begin{aligned}
f'(\eta) = & -a f_w e^{(-a\eta)} - e^{(-\eta)}(f_w a - 1) - \left((h+n)\left(\frac{f_w a^4}{-a^3 + \frac{Ma}{K+1}} - \frac{f_w a - 1}{-1 + \frac{M}{K+1}}\right)\right) \\
& - \frac{M(h+n)}{K+1}\left(\frac{f_w a^2}{-a^3 + \frac{Ma}{K+1}} - \frac{-f_w a - 1}{-1 + \frac{M}{K+1}}\right) \\
& + \frac{h}{K+1}\left(-Km\left(\frac{-a_1(-f_w a_1^3) - 3a_1}{-a1^3 + \frac{Ma_1}{K+1}}\right) + \frac{\delta(-a_1 - 1)}{-(a_1+1)^3 + \frac{M(a_1+1)}{K+1}}\right) \\
& + \frac{h}{K+1}\left(\frac{\delta N_r(-a_1 - 1)}{-(a_1+1)^3 + \frac{M(a_1+1)}{K+1}}\right)\sqrt{K+1}\sqrt{\frac{M}{K+1}}e^{-\sqrt{\frac{M}{K+1}}} \\
& + (h+n)\left(\frac{f_w a^4 e^{-a\eta}}{-a^3 + \frac{Ma}{K+1}}\right) - \frac{M(h+n)}{K+1}\left(\frac{f_w a^2 e^{-a\eta}}{-a^3 + \frac{Ma}{K+1}} - \frac{e^{-\eta}(f_w a - 1)}{-1 + \frac{M}{K+1}}\right) \\
& + \frac{h}{K+1}\left(\frac{-f_w^2 a^3 e^{-a\eta}}{-a^3 + \frac{Ma}{K+1}} + \frac{f_w(f_w a - 1)e^{-\eta}}{-1 + \frac{M}{K+1}} + \frac{2f_w^2 a^3 e^{-2a\eta}}{-8a^3 + \frac{2Ma}{K+1}} + \frac{2e^{-2\eta}(f_w a - 1)^2}{-8 + \frac{2M}{K+1}}\right)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{h}{K+1}\left(\frac{2f_w a(f_w a-1)(-a-1)e^{-\eta(a+1)}}{-(a+1)^3+\frac{(a+1)M}{K+1}}-Km\left(\frac{-a_1e^{-a\eta}(-f_w a_1-3a_1)}{-a_1^3+\frac{Ma_1}{K+1}}\right)-\right)\\
&-Km\left(\frac{-e^{\eta}(f_w a_1-2)}{-1+\frac{M}{K+1}}+\frac{\delta(-a_1-1)e^{-\eta(a_1+1)}}{-(a_1+1)^3}-\frac{\delta N_r(-a_1-1)e^{-\eta(a_1+1)}}{-(a_1+1)^3+\frac{M(a_1+1)}{K+1}}\right),
\end{aligned}
\tag{3.1}
$$

$$
\begin{aligned}
g(\eta)=&-m(f_w a^2e^{-a\eta}-(f_w a-1)e^{-\eta})+\frac{(n+h)m}{1+\frac{K}{2}}\left(\frac{f_w a^4}{a^2-\frac{2K}{1+\frac{k}{2}}}-\frac{a(f_w a-1)}{a^2-\frac{2aK}{1+\frac{k}{2}}}\right)\\
&+2\left(\frac{Km(n+h)}{1+\frac{k}{2}}\left(\frac{f_w a}{a^2-\frac{2K}{1+\frac{k}{2}}}-\frac{(f_w a-1)}{a^2-\frac{2aK}{1+\frac{k}{2}}}\right)\right)-mh\left(\frac{-f_w^2a-f_w a(f_w a-1)}{4a^2-\frac{2K}{1+\frac{k}{2}}}\right)\\
&+(f_w a^4(f_w a-1)a^3+a(f_w a-1)^2)\left(\frac{1}{(a+1)^2-\frac{2K}{1+\frac{k}{2}}}+\frac{1}{(a)^2-\frac{2K}{1+\frac{k}{2}}}\right)\\
&+\frac{hm}{1+\frac{K}{2}}\left(\frac{f_w a^2-2f_w a-1}{(a)^2-\frac{2K}{1+\frac{k}{2}}}+\frac{f_w a-1}{1-\frac{2K}{1+\frac{K}{2}}}\right)-\frac{hK}{1+\frac{K}{2}}\left(\frac{f_w a^2}{(a)^2-\frac{2K}{1+\frac{k}{2}}}-\frac{f_w a-1}{1-\frac{2K}{1+\frac{K}{2}}}\right)\\
&e^{\sqrt{2}\sqrt{\frac{K}{1+\frac{K}{2}}x}}-\left(\frac{m(n+h)}{1+\frac{k}{2}}\right)\left(\frac{f_w a^4e^{-a\eta}}{a^2-\frac{2K}{1+\frac{k}{2}}}-\frac{a(f_w a-1)e^{-a\eta}}{a^2-\frac{2aK}{1+\frac{k}{2}}}\right)++2\left(\frac{Km(n+h)}{1+\frac{k}{2}}\right)\\
&\left(\frac{f_w a^2e^{-a\eta}}{a^2-\frac{2K}{1+\frac{k}{2}}}-\frac{(f_w a-1)e^{-a\eta}}{a^2-\frac{2aK}{1+\frac{k}{2}}}\right)-mh\left(\frac{e^{-2a\eta(-f_w)^2a^3-af_w(f_w a-1)}}{4a^2-\frac{2K}{1+\frac{K}{2}}}\right)\\
&-mh\left(\frac{e^{-\eta(a+1)f_w^4(f_w a-1)a^3+a(f_w a-1)^2}}{(a+1)^2-\frac{2K}{1+\frac{K}{2}}}-\right)-mh\left(\frac{e^{-\eta(a+1)f_w^4(f_w a-1)a^3+a(f_w a-1)^2}}{(a+1)^2-\frac{2K}{1+\frac{K}{2}}}\right)\\
&+\frac{hm}{1+\frac{K}{2}}\left(\frac{(f_w a^2-2f_w a-1)e^{-a\eta}}{(a)^2-\frac{2K}{1+\frac{k}{2}}}+\frac{(f_w a-1)e^{-\eta}}{1-\frac{2K}{1+\frac{K}{2}}}\right)\\
&-\frac{hK}{1+\frac{K}{2}}\left(\frac{(f_w a^2)e^{-a\eta}}{(a)^2-\frac{2K}{1+\frac{k}{2}}}-\frac{(f_w a-1)e^{-\eta}}{1-\frac{2K}{1+\frac{K}{2}}}\right),
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
\theta(\eta)=&\,e^{-\eta(q+1)}-n-h-h\left(\frac{-f_w P_r(q+1)}{(a+q+1)^2}+\frac{(f_w a-1)P_r(q+1)}{(q+1)^2}-\frac{(f_w a-1)P_r}{q+1}\right)\\
&-h\left(\frac{N_b(q+1)^2}{(2q+2)^2}-\frac{N_t}{4}\right)+(n+h)e^{-\eta(q+1)}+h\left(\frac{f_w P_r(q+1)e^{-\eta(a+q+1)}}{(a+q+1)^2}\right)\\
&+h\left(\frac{(f_w a-1)P_r(q+1)e^{-\eta(q+2)}}{(q+2)^2}-\frac{(f_w a-1)P_r e^{-\eta(q+1)}))}{(q+1)}+\frac{b(q+1)^2e{-}\eta(2q+2)}{(2q+2)^2}\right)\\
&-h\left(\frac{N_te^{-2\eta(q+1)}}{4}\right),
\end{aligned}
\tag{3.3}
$$

$$
\phi(\eta)=e^{-\eta(w+1)}-n-h-h\left(c\left(\frac{-f_w(w+1)}{(a+w+1)^2}+\frac{(f_w a-1)(w+1)}{(w+2)^2}-\frac{(f_w a-1)}{w+1}\right)\right)
$$

$$-\frac{hN_t}{N_b}+(n+h)e^{-\eta(w+1)}+h\left(c\left(\frac{-f_w(w+1)e^{-\eta(a+w+1)}}{(a+w+1)^2}+\frac{(f_wa-1)(w+1)e^{-\eta(w+2)}}{(w+1)^2}\right)\right)$$
$$-hc\left(\frac{(f_wa-1)e^{-\eta(w+1)}}{w+1}\right)+\frac{N_{te^{-\eta(w+1)}}}{N_b}, \tag{3.4}$$

$$a=m\,\delta K\,M\,P_r\,N_t\,f_w,\;\; a_1=m\,\delta K\,M\,P_r\,N_t\,f_w\,N_b\,S_c\,N_r,\; q=m\,\delta K\,M\,P_r\,N_t\,f_w\,N_b\,S_c\,N_r$$
$$w=m\,\delta K\,M\,P_r\,N_t\,f_w\,N_b\,S_c\,N_r,\;\; b_1=\frac{1}{1+\frac{K}{2}},\;\; B_1=c_1=c_3=c_6=0 \tag{3.5}$$

$$A_1=-B_1-C_1-(n+h)\left(\frac{-a^3}{-a^3+\frac{Ma}{K+1}}+\frac{f_wa-1}{-1+\frac{M}{K+1}}\right)+\frac{(n+h)m}{K+1}\left(\frac{-f_wa}{-a^3+\frac{Ma}{K+1}}+\frac{f_wa-1}{-1+\frac{M}{K+1}}\right)$$
$$-\frac{h}{K+1}\left(\frac{f_w^2a^2}{-a^3+\frac{Ma}{K+1}}-\frac{f_w(f_wa-1)}{-1+\frac{M}{K+1}}-\frac{f_w^2a^2}{-8a^3+\frac{2Ma}{K+1}}\right)$$
$$-\frac{h}{K+1}\left(\frac{2af_w(f_wa-1)}{-(a+1)^3+\frac{M(a+1)}{K+1}}-Km\left(\frac{-f_wa_1^3-3a_1+1}{-a_1^3+\frac{Ma_1}{K+1}}+\frac{f_wa_1-2}{-1+\frac{M}{K+1}}\right)\right)$$
$$\frac{h}{K+1}\left(\frac{\delta}{-(a_1+1)^3+\frac{M(a_1+1)M}{K+1}}-\frac{\delta N_r}{-(a_1+1)^3+\frac{(a_1+1)M}{K+1}}\right), \tag{3.6}$$

$$C_1=\frac{\sqrt{K+1}}{M}\left((h+n)\left(\frac{a^4}{-a^3+\frac{Ma}{K+1}}-\frac{f_wa-1}{-1+\frac{M}{K+1}}\right)\right)$$
$$-\frac{\sqrt{K+1}}{M}\frac{M(h+n)}{K+1}\left(\frac{f_wa^2}{-a^3+\frac{Ma}{K+1}}-\frac{f_wa-1}{-1+\frac{M}{K+1}}\right)$$
$$+\frac{\sqrt{K+1}}{M}\frac{h}{K+1}\left(\frac{-f_w^2a^3}{-a^3+\frac{Ma}{K+1}}+\frac{f_w(f_wa-1)}{-1+\frac{M}{K+1}}+\frac{2f_w^2a^3}{-8a^3+\frac{2Ma}{K+1}}+\frac{2f_wa(f_wa-1)(-a-1)}{-(a+1)^3+\frac{M(a+1)}{K+1}}\right)$$
$$-\frac{Kmh}{K+1}\left(\frac{-a_1(-f_wa_1^3)-3a_1}{-a_1^3+\frac{Ma_1}{K+1}}-\frac{f_wa_1-2}{-1+\frac{M}{K+1}}\right)+\frac{h}{K+1}\left(\frac{\delta(a_1-1)}{-(a_1+1)^3+\frac{M(a_1+1)}{K+1}}\right), \tag{3.7}$$

$$c_2=-(n+h)m\left(\frac{a^4}{a^2-2Kb_1}-\frac{a(af_w-1)}{a^2-2Kb_1}\right)+2Km(n+h)b_1\left(\frac{f_wa^2}{a^2-2Kb_1}-\frac{f_wa-1}{a^2-2Kb_1}\right)$$
$$-mhb_1\left(\frac{-f_w^2a^3-f_wa(f_wa-1)}{4a^2-2Kb_1}+\frac{f_w^4(f_wa-1)a^3+a(f_wa-1)^2}{(a+1)^2-2Kb_1}\right)$$
$$-mhb_1\left(\frac{f_w^4(f_wa-1)a^3+a(f_wa-1)^2}{a^2-2Kb_1}\right)+mb_1\left(\frac{f_wa^2-2f_wa-1}{a^2-2Kb_1}+\frac{f_wa-1}{1-2Kb_1}\right)$$
$$-mhb_1\left(-Kb_1\left(\frac{f_wa^2}{a^2-2b_1}-\frac{f_wa-1}{1-2Kb_1}\right)\right), \tag{3.8}$$

$$c_4=n-h-h\left(\frac{-f_wP_r(a_1+1)}{(a+q+1)^2}+\frac{(f_wa-1)P_r(q+1)}{(q+1)^2}-\frac{(f_wa-1)P_r}{(q+1)}\right)$$
$$-h\left(\frac{(f_wa-1)P_r}{(q+1)}+\frac{N_b(q+1)^2}{(2q+2)^2}-\frac{N_t}{4}\right), \tag{3.9}$$

$$c_5 = -n - h - h\left(S_c\left(\frac{-f_w(a_1+1)}{(a+w+1)^2}\right) + \frac{(f_w a-1)(w+1)}{(w+2)^2} - \frac{f_w a-1}{w+1}\right) + \frac{N_t}{N_b}\Bigg), \tag{3.10}$$

$$\begin{aligned}C_f = {} & (1+(1-m)K)(f_w a+1+\frac{1}{\sqrt{(K+1)}}(C_1)(n+h)\left(\frac{-f_w a^3}{-a^3+\frac{Ma}{K+1}} + \frac{f_w a-1}{-1+\frac{M}{K+1}}\right) \\ & - \frac{(n+h)m}{K+1}\left(\frac{-f_w a}{-a^3+\frac{Ma}{K+1}} + \frac{f_w a-1}{-1+\frac{M}{K+1}}\right) + \frac{h}{K+1}\left(\frac{f_w^2 a^4}{-a^3+\frac{Ma}{K+1}} - \frac{f_w(f_w a-1)}{-1+\frac{M}{K+1}}\right) \\ & + \frac{h}{K+1}\left(\frac{4f_w^2 a^4}{-8a^3+\frac{2Ma}{K+1}} - \frac{4(f_w^2 a-1)^2}{-8+\frac{2M}{K+1}} + \frac{2af_w(f_w a-1)(-a-1)^2}{-(a+1)^2+\frac{M(a+1)}{K+1}}\right) \\ & - \frac{hKm}{K+1}\left(\frac{-f_w a_1^3-3a_1+a_1^2}{-(a+1)^3+\frac{M(a+1)}{K+1}} + \frac{f_w a_1-2}{-1+\frac{M}{K+1}}\right) + \frac{\delta(-a_1-1)^2}{-(a_1+1)^3+\frac{M(a_1+1)M}{K+1}} \\ & - \frac{\delta(-a_1-1)^2 N_r}{-(a_1+1)^3+\frac{M(a_1+1)M}{K+1}},\end{aligned} \tag{3.11}$$

$$\begin{aligned}Nu = {} & w+1-(h+n)(-q-1) - h\left(\frac{-f_w P_r(q+1)(-a-q-1)}{(a+q+1)^2} + \frac{N_b(q+1)^2(-2q-2)}{(2q+2)^2}\right) \\ & - h\left(\frac{N_t(-2q-2)}{4}\right),\end{aligned} \tag{3.12}$$

$$\begin{aligned}Sh = {} & w+1-(n+h)(-w-1) - h\left(Sc\left(\frac{-f_w(w+1)(-a-w-1)}{(a+w+1)^2}\right)\right) \\ & + hSc\left(\left(\frac{(f_w a-1)(w+1)(-w-2)}{(w+2)^2} - \frac{(-f_w a-1)(-w-1)}{w+1} + \frac{N_t(-q-1)}{N_b}\right)\right).\end{aligned} \tag{3.13}$$

4. Results and discusssion

In this section, the graphical view for the dimensionless velocity $f'(\eta)$, dimensionless angular velocity $g(\eta)$, dimensionless temperature $\theta(\eta)$, and dimensionless concentration $\phi(\eta)$ were discussed. Furthermore, the semi-analytical findings using q-MHAM are compared with previous work done by Lund et al. [23] using fourth-order Runge-Kutta method, which shows a good agreement. Figure 1 illustrates the schematic diagram of the problem and coordinate system. Figures 2–7 depict that the dimensionless velocity $f'(\eta)$ versus dimensionless coordinate η. From Figures 2–3, it indicates that when the micropolar material K raises, the dimensionless velocity falls for some specified values of parameters. From Figures 4 and 5 it shows that when the parameter of local fixed convection δ increases, the dimensionless velocity decrease in some particular values of the other parameters. From Figures 6 and 7, the dimensionless velocity decreases by increasing the micro-gyration parameter m. Figure 8 depicts that dimensionless suction parameter f_w versus dimensionless skin friction $f''(0)$. From this figure, while increasing the micropolar material K, the dimensionless skin friction factor also raises in particular values for remaining dimensionless parameters.

Figures 9–12 demonstrates the effect of dimensionless angular velocity $g(\eta)$ versus dimensionless coordinate η. Figures 9 and 10 clearly show that the dimensionless angular velocity increases by raising the micropolar material parameter K. Similarly, Figures 11 and 12 reveal that an increase in the micro-gyration parameter m leads to an increase in the angular velocity.

Figures 13–17 show that the dimensionless temperature $\theta(\eta)$ versus dimensionless coordinates η . From Figures 13 and 14, it is predicted that when the dimensionless local mixed convection parameter δ increases, the temperature also increases for some parameter. Figure 15 indicates that the dimensionless temperature increases as the thermophoresis parameter N_t increases. As seen in Figure 16, the dimensionless temperature increases with an increase in the Brownian

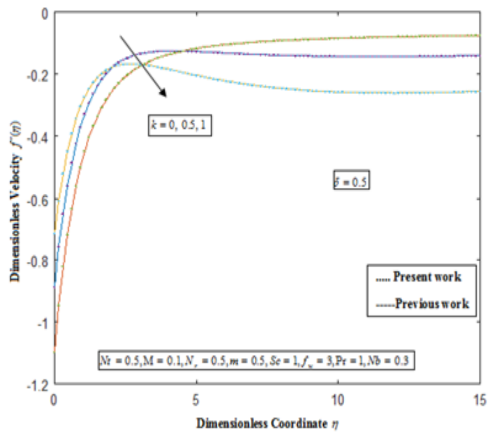

FIGURE 2. Dimensionless coordinate η versus the dimensionless velocity $f'(\eta)$ using the Eq. (3.1) for different values of micropolar material K and for some particular points of $f_w, Pr, m, M, N_b, N_t, N_r, Sc,$ and δ.

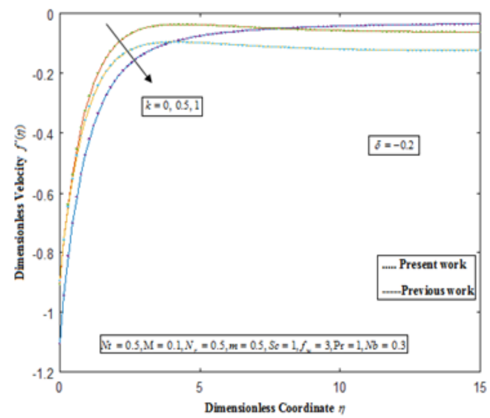

FIGURE 3. Dimensionless coordinate η versus the dimensionless velocity $f'(\eta)$ using the Eq. (3.1) for different values of parameter of micropolar material K and in some stable values of $f_w, Pr, m, M, N_b, N_t, N_r, Sc,$ and δ.

motion parameter N_b, for the fixed values of the other parameters. Figure 17 demonstrates that the dimensionless Prandtl number increases, the temperature. Figure 18 indicates that the dimensionless suction parameter f_w with the dimensionless local Nusselt number $-\theta'(0)$. From this Figure it is evident that when the dimensionless micropolar material parameter K increases, the dimensionless local nusselt number also increases for the given parameters.

Figures 19–23 display that the dimensionless concentration $\phi(\eta)$ versus dimensionless coordinate η. Figure 19 shows that the concentration decreases as the thermophoresis parameter N_t increases. Figures 20 and 21 indicate that the concentration decreases with an increase in the buoyancy ratio N_r for the fixed values of the other parameters. Figure 22 illustrates that the dimensionless concentration increases with dimensionless Brownian motion parameter N_b. Figure 23 reveals that the concentration decreases as the Schmidt number Sc increases. Figure 24 shows dimensionless suction parameter f_w with the dimensionless local Sherwood number $-\phi'(0)$. From this figure, it is observed that the local Sherwood number increases with the micropolar material parameter K.

5. Conclusion

The mathematical analysis of the dimensionless velocity $f'(\eta)$, dimensionless angular velocity $g(\eta)$, dimensionless temperature $\theta(\eta)$ and dimensionless concentration profile $\phi(\eta)$ in micropolar nanofluids over a vertical permeable shrinking surface was provided and the corresponding semi-analytical expressions were derived by using the q- modified Homotopy analysis method for all values of the dimensionless parameters. The results demonstrate excellent agreement with previous numerical studies, validating the present approach. Furthermore, approximate analytical expressions of local skin friction, nusselt number, and local sherwood number. The study demonstrates the validity and considerable potential of the q-MHAM for solving such nonlinear boundary value problems.

In conclusion, the key findings are summarized as follows:

- By varying the value of micropolar material parameter K, the local Nusselt number and local Sherwood number increases, but the coefficient of skin friction decreases.
- By increasing the value of Brownian diffusion parameter N_b, dimensionless temperature and dimensionless concentration increases.

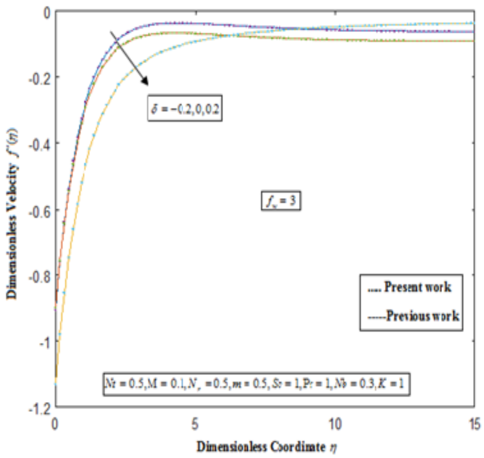

FIGURE 4. Dimensionless coordinate η versus the dimensionless velocity $f'(\eta)$ using the Eq. (3.1) for numerous values of local fixed convection δ and some specified values of $f_w, Pr, m, M, N_b, N_t, N_r, S_c,$ and K.

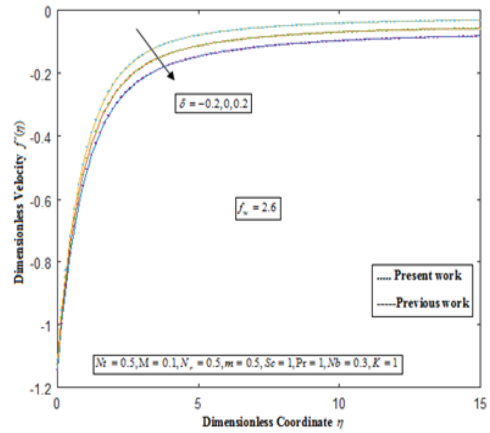

FIGURE 5. Dimensionless coordinate η versus the dimensionless velocity $f'(\eta)$ using the Eq. (3.1) for varying the amounts of micro-gyration parameter m and certain values of $Pr, \delta, M, f_w, N_b, N_t, N_r, Sc,$ and K.

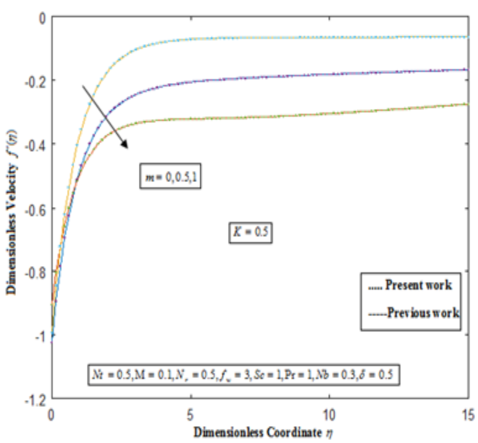

FIGURE 6. Dimensionless coordinate η versus the dimensionless velocity $f'(\eta)$ using the Eq. (3.1) for distinct values of micro-gyration parameter m and some fixed values of $Pr, \delta, M, f_w, N_b, N_t, N_r, Sc,$ and K.

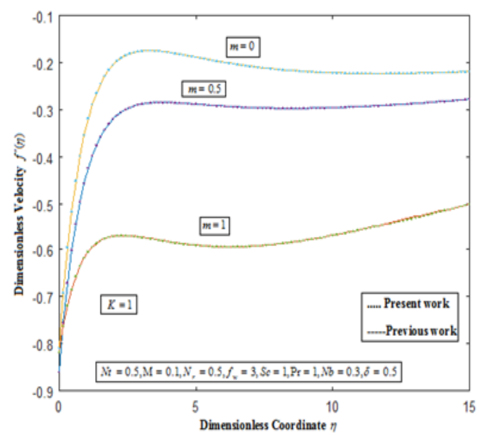

FIGURE 7. Dimensionless coordinate η versus the dimensionless velocity $f'(\eta)$ using the Eq. (3.1) for varying values of the micro-gyration parameter m and in some fixed values of $Pr, \delta, M, f_w, N_b, N_t, N_r, Sc,$ and K.

APPENDIX-A

BASIC CONCEPT OF Q-MODIFIED HOMOTOPY ANALYSIS METHOD (Q-MHAM)

Consider the following differential equation:

$$N[u(x,t)] - f(x,t) = 0, \tag{A.1}$$

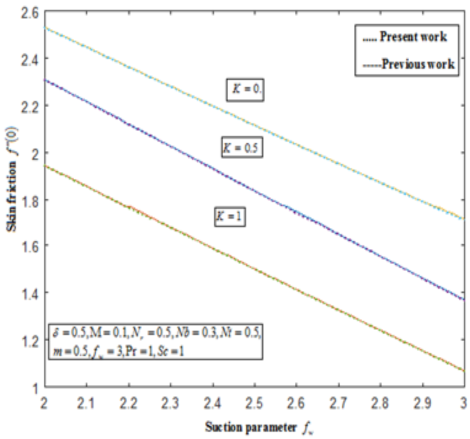

FIGURE 8. Dimensionless suction parameter f_w versus Dimensionless skin friction f"(0) using the Eq. (3.1) for distinct values of micropolar material K and for fixed values of $f_w, Pr, m, M, N_b, N_t, N_r, Sc,$ and δ.

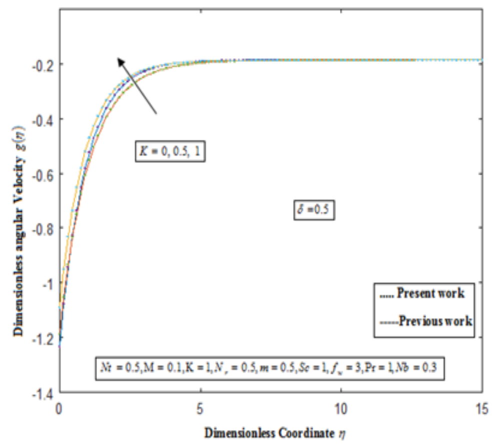

FIGURE 9. Dimensionless coordinate η versus the dimensionless angular velocity $g(\eta)$ using the Eq. (3.2) by varying the values of micropolar material parameter K and fixing the values of $P_r, \delta, M, f_w, N_b, N_t, N_r, Sc,$ and m.

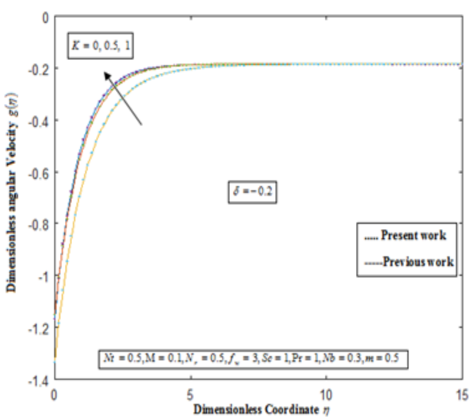

FIGURE 10. Dimensionless coordinate η versus the dimensionless angular velocity $g(\eta)$ using the Eq. (3.2) by different values of micropolar material parameter K and fixed points of $Pr, \delta, M, f_w, N_b, N_t, Sc, m,$ and N_r.

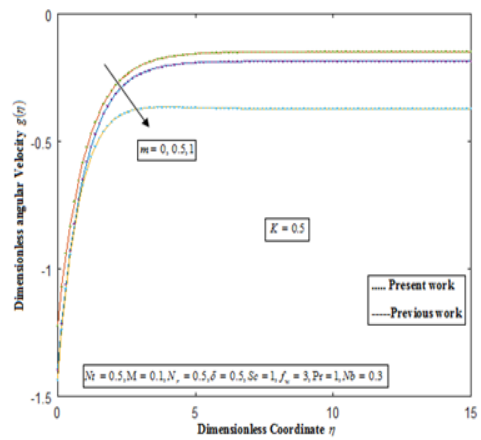

FIGURE 11. Dimensionless coordinate η versus the dimensionless angular velocity $g(\eta)$ using the Eq. (3.2) for distinct values of microgyration parameter m and by fixing values of $Pr, M, f_w, K, N_b, N_t, N_r,$ and Sc.

where N is a non-linear operator, (x, t) denotes independent variables, $f(x, t)$ is a known function and $u(x, t)$ is an unknown function.

Let us construct the so-called deformation equation of order zero:

$$(1 - nq)L[\phi(x, t; q) - u_0(x, t)] = q\, h\, H(x, t)(N[\phi(x, t; q)] - f(x, t)), \tag{A.2}$$

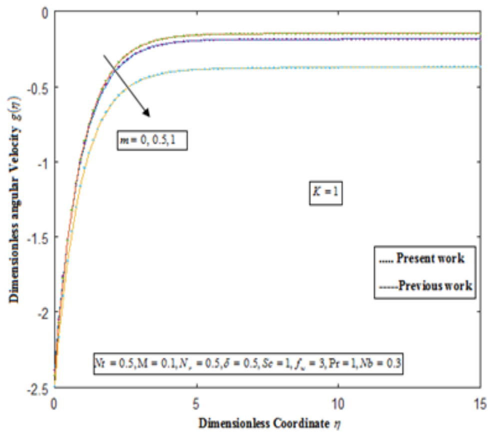

FIGURE 12. Dimensionless coordinate η versus the dimensionless angular velocity $g(\eta)$ using the Eq. (3.2) for different values of micro-gyration parameter m and in some fixing values of $Pr, M, f_w, K, N_b, N_t, N_r,$ and Sc.

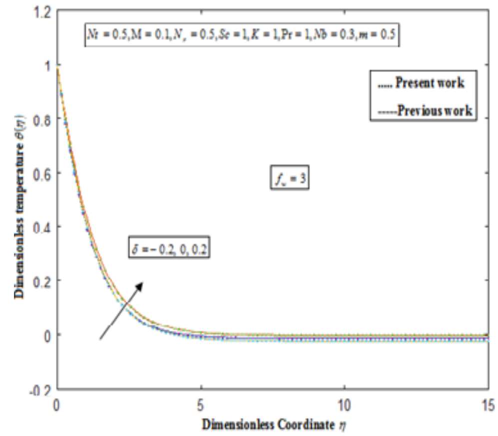

FIGURE 13. Dimensionless coordinate η versus the dimensionless temperature $\theta(\eta)$ using the Eq. (3.3) for varying the values of local fixed convection parameter δ and certain fixed values of $Pr, m, M, K, N_t, N_b, N_r,$ and Sc.

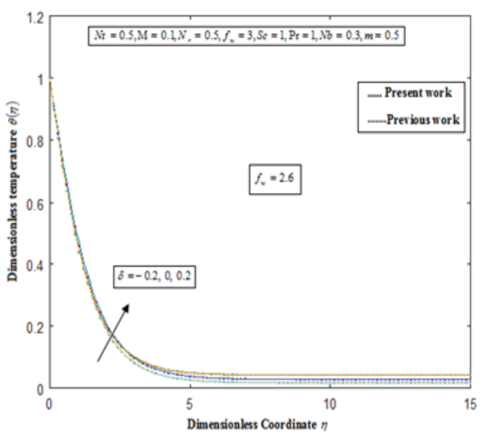

FIGURE 14. Dimensionless coordinate η versus the dimensionless temperature $\theta(\eta)$ using the Eq. (3.3) for various values of parameter of local fixed convection δ and for specified certain values of $Pr, m, M, K, N_t, N_b, N_r,$ and Sc.

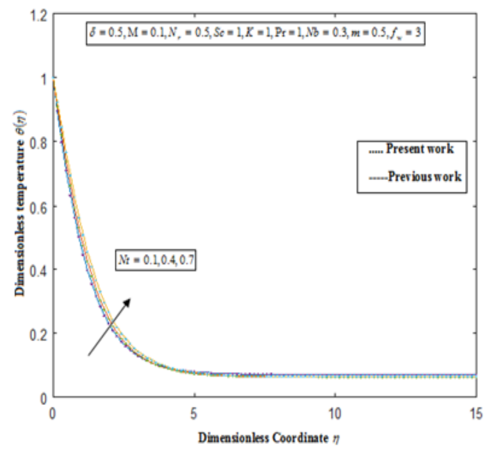

FIGURE 15. Dimensionless coordinate η versus the dimensionless temperature $\theta(\eta)$ using the Eq. (3.3) for distinct values of thermophoresis parameter N_t and for specified values of $K, Pr, Sc, M, m, N_b, N_r, f_w,$ and δ.

where n≥1, $q \in [0, \frac{1}{n}]$ denotes the so-called embedded parameter, L is an auxiliary linear operator with the property L(f)=0 when $f = 0$, $h \neq 0$ is an auxiliary parameter, $H(x,t)$ denotes a non-zero auxiliary function. It is obvious that

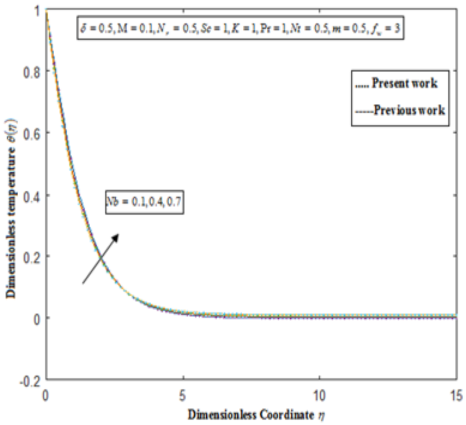

FIGURE 16. Dimensionless coordinate η versus the dimensionless temperature $\theta(\eta)$ using the Eq. (3.3) for different values of Brownian diffusion parameter N_b and by fixing values of $K, Pr, Sc, M, m, N_t, N_r, f_w$, and δ.

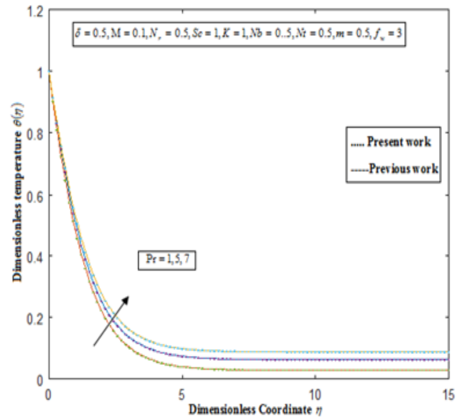

FIGURE 17. Dimensionless coordinate η versus the dimensionless temperature $\theta(\eta)$ using the Eq. (3.3) for different values of the Prandtl number Pr and fixing some values of $K, P_r, Sc, M, m, N_t, N_r, f_w$, and δ.

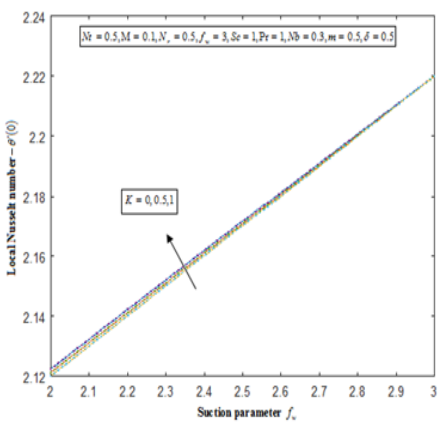

FIGURE 18. Dimensionless suction parameter f_w versus the dimensionless local Nusselt number $-\theta'(0)$ using the Eq. (3.12) for different values of micropolar material parameter K and fixing values of the other dimensionless parameters $P_r, \delta, M, f_w, N_b, N_t, N_r, Sc$, and m.

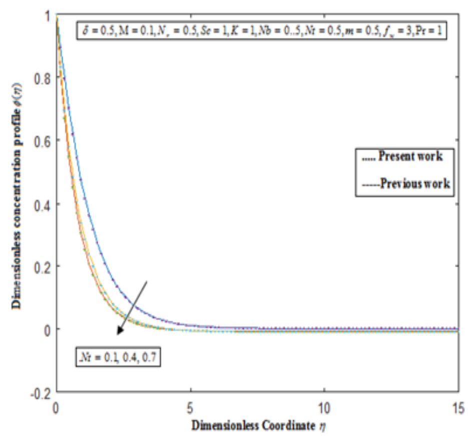

FIGURE 19. Dimensionless coordinate η with the dimensionless concentration profile $\phi(\eta)$ using the Eq. (3.4) for various values of thermophoresis parameter N_t and some fixed values of $K, Pr, Sc, M, m, N_b, N_t, f_w$, and δ.

$q = 0$ and $q = \frac{1}{n}$ when Eq. (A.2) becomes:

$$\phi(x, t; 0) = u_0(x, t) \quad \phi\left(x, t; \frac{1}{n}\right) = u(x, t), \tag{A.3}$$

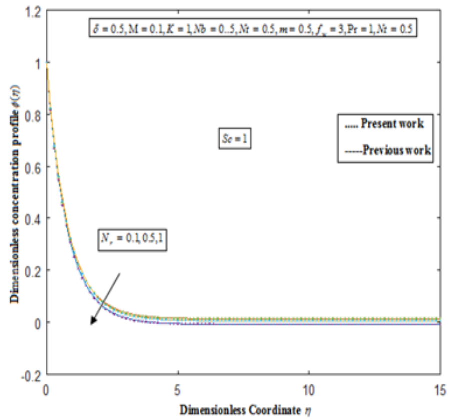

FIGURE 20. Dimensionless coordinate η with the dimensionless concentration profile $\phi(\eta)$ using the Eq. (3.4) for various values of buoyancy parameter N_r and fixing values of other parameters $K, Pr, Sc, M, m, N_b, N_t, f_w$, and δ.

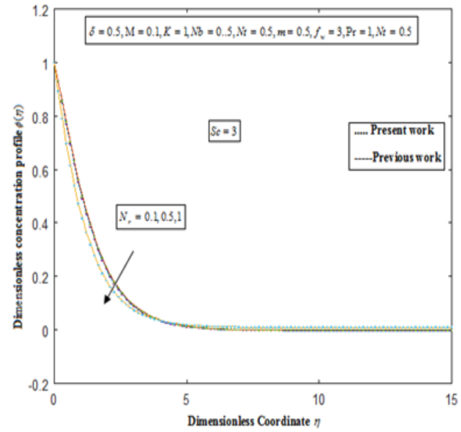

FIGURE 21. Dimensionless coordinate η vs the dimensionless concentration profile $\phi(\eta)$ using the Eq. (3.4) for distinct values of thermophoresis parameter N_t and fixing values of $K, Pr, S_c, M, m, N_b, N_r, f_w$, and δ.

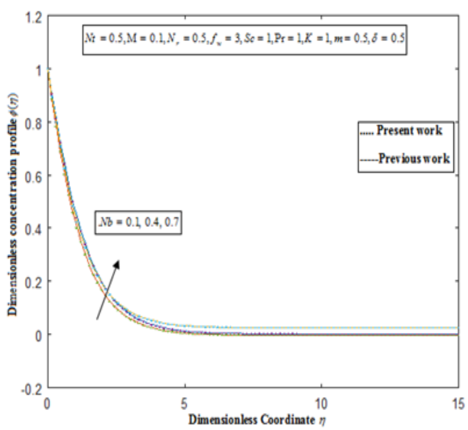

FIGURE 22. Dimensionless coordinate η with the dimensionless concentration profile $\phi(\eta)$ using the Eq. (3.4) for distinct values of Brownian diffusion parameter N_b and fixing values of $Pr, K, S_c, M, m, N_r, N_t, f_w$, and δ.

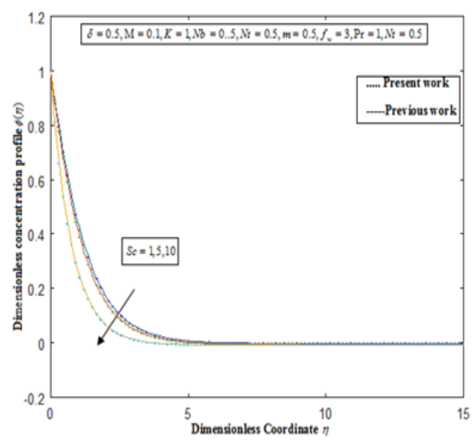

FIGURE 23. Dimensionless coordinate η vs the dimensionless concentration profile $\phi(\eta)$ using the Eq. (3.4) for various values of Schmidt number Sc and some fixed values of $Pr, K, N_b, M, m, N_r, N_t, f_w$, and δ.

respectively. Thus as q increases from 0 to $\frac{1}{n}$, the solution $\phi(x, t; q)$ varies from the initial guess $u_0(x, t)$ to the solution $u(x, t)$. Having the freedom to choose $u_0(x, t),\ L,\ h,\ H(x, t)$ we can assume that all of them can be properly chosen

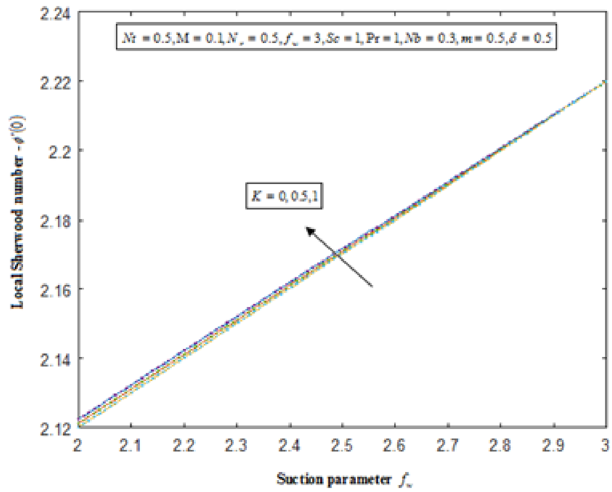

FIGURE 24. Dimensionless suction parameter f_w vs the dimensionless local Sherwood number $-\phi'(0)$ using the Eq. (3.13) different values of micropolar material parameter K and fixing values of $Pr, N_b, M, m, N_r, N_t, f_w,$ and δ.

so that the solution $\phi(x,t;q)$ of Eq. (A.2) exists for all $q \in [0, \frac{1}{n}]$. Expanding in Taylor series, one has:

$$\phi(x,t;q) = u_0(x,t) + \sum_{m=1}^{+\infty} u_m(t,q)q^m, \tag{A.4}$$

where

$$u_m(x,t) = \frac{1}{m!}\frac{\partial^m \phi(x,t;q)}{\partial q^m}|q = 0. \tag{A.5}$$

Assume that $h, H(x,t), u_0(x,t), L$ are so properly chosen such that the series Eq. (A.4) converges at $q = \frac{1}{n}$ and

$$\phi(x,t;\frac{1}{n}) = u_0(x,t) + \sum_{m=1}^{+\infty} u_m(x,t)\left(\frac{1}{n}\right)^m. \tag{A.6}$$

Defining the vectors $u_r(x,t) = u_0(x,t),\ u_1(x,t).......u_r(x,t)$ Differentiating, Eq. (A.2) m-times with respect to x and then setting $q = 0$ and finally dividing them by $m!$, we have the so-called m^{th} order deformation equation:

$$L[u_m(x,t) - K_m u_{m-1}(x,t)] = h\ H(x,t)R_m(u_{m-1}(x,t)), \tag{A.7}$$

where

$$R_m(u_{m-1}(x,t)) = \frac{1}{(m-1)!}\frac{\partial^{m-1}(N[\phi(x,t;q) - f(x,t)])}{\partial q^m - 1}|q = 0, \tag{A.8}$$

and:

$$K_m = \begin{cases} 0, & m \leq 1, \\ n, & otherwise. \end{cases} \tag{A.9}$$

It should be emphasized that $u_m(x,t)$ for $m \geq$1 is governed by the linear Eq. (A.7) with linear boundary conditions that come from the original problem. Due to the existance of the factor $\left(\frac{1}{n}\right)^m$, more chances for convergence may occur or even much faster convergence can be obtained better than the standard HAM. It should be noted that the case of $n = 1$ in the Eq. (A.2) standard HAM can be reached.

Appendix-B

Approximate analytical expressions of the MHD micropolar nanofluid Eq. (2.13)-(2.16) by q-MHAM [17]-[22]

We described below that how the Eqs. (3.1)-(3.4) are derived in this paper. To find the solution of the Eqs. (2.13)-(2.16) we construct the Homotopy are as follows:

$$(1-nq)\left(f''' - \frac{M}{K+1}f'\right) - q\,h\left(f''' + \frac{1}{K+1}(ff'' - f'^2 + Kg' - Mf' + \delta(\theta - N_r\phi)\right) = 0, \tag{B.1}$$

$$(1-nq)\left(g'' - \frac{2Kg}{1+\frac{K}{2}}\right) - q\,h\left(g'' + \left(\frac{1}{1+\frac{K}{2}}\right)fg' - gf' - 2Kg - Kf''\right) = 0, \tag{B.2}$$

$$(1-nq)\theta'' - q\,h(\theta'' + P_r(f\theta' + N_b\theta'\phi' + N_t(\theta')^2)) = 0, \tag{B.3}$$

$$(1-nq)\phi'' - q\,h\left(\phi'' + S_c(f\phi') + \frac{N_t}{N_b}\theta''\right). \tag{B.4}$$

The initial approximation for Eq. (2.13) to (2.16) is as follows:

$$f_0(0) = f_w,\;\; \frac{df_0(0)}{d\eta} = -1,\;\; \frac{df_0(\infty)}{d\eta} = 0,\;\; f_1(0) = 0,\;\; \frac{df_1(0)}{d\eta} = 0,\;\; \frac{df_1(\infty)}{d\eta} = 0, \tag{B.5}$$

$$g_0(0) = -m\frac{df_0^2(0)}{d\eta},\;\; g_0(\infty) = 0,\;\; g_1 = 0,\;\; g_1(\infty) = 0, \tag{B.6}$$

$$\theta_0(0) = 1,\;\; \theta_0(\infty) = 0,\;\; \theta_1(0) = 0,\;\; \theta_1(\infty) = 0, \tag{B.7}$$

$$\phi_0(0) = 1,\;\; \phi_0(0) = 1,\;\; \phi_1(0) = 0,\;\; \phi_0(0) = 0. \tag{B.8}$$

The approximate analytical solutions of the Eqs. (2.13) to (2.16) are as follows:

$$f = f_0 + qf_1 + q^2 f_2 + ..., \tag{B.9}$$

$$g = g_0 + qg_1 + q^2 g_2 + ..., \tag{B.10}$$

$$\theta = \theta_0 + q\theta_1 + q^2\theta_2 + ..., \tag{B.11}$$

$$\phi = \phi_0 + q\phi_1 + q^2\phi_2 + ... \tag{B.12}$$

Substituting Eq. (B.9) into Eq. (B.1), (B.10) into Eq. (B.2), (B.11) into Eq. (B.3) and (B.12) into Eq. (B.4), respectively, we obtain

$$\begin{aligned}(1-nq)&\left(\frac{d^3(f_0+qf_1+....)}{d\eta^3} - \frac{M}{K+1}\left(\frac{d(f_0+qf_1+....)}{d\eta}\right)\right)\\ &- qh\left(\frac{d^3(f_0+qf_1+...)}{d\eta^3} + \frac{1}{K+1}\left((f_0+qf_1+...)\frac{d^2(f_0+qf_1+...)}{d\eta^2}\right)\right)\\ &- \frac{qh}{K+1}\left(K\frac{d(g_0+qg_1+...)}{d\eta} - M\frac{d(f_0+qf_1+...)}{d\eta} + \delta(\theta_0+q\theta_1+....)\right)\\ &- \left(\frac{qh}{K+1}N_r(\phi_0+q\phi_1+...)\right) = 0,\end{aligned} \tag{B.13}$$

$$\begin{aligned}(1-nq)&\left(\frac{d^2(g_0+qg_1+..)}{d\eta} - \frac{2K}{1+\frac{K}{2}}(g_0+qg_1+...)\right)\\ &- qh\left(\frac{d^2(g_0+qg_1+...)}{d\eta^2} + \frac{1}{1+\frac{K}{2}}\left((f_0+qf_1+...)\frac{d(g_0+qg_1+...)}{d\eta}\right)\right)\end{aligned}$$

$$- qh\left(\frac{1}{1+\frac{K}{2}}\left((g_0+qg_1+..)\frac{d(f_0+qf_1+...)}{d\eta)}\right) - 2K((g_0+qg_1+..))\right)$$
$$- \frac{qh}{1+\frac{K}{2}}\left(\frac{(d^2(f_0+qf_1+...)}{d\eta^2}\right) = 0, \tag{B.14}$$
$$(1-nq)\left(\frac{d^2(\theta_0+q\theta_1+..)}{d\eta^2}\right) - qh\left(\frac{d^2(\theta_0+q\theta_1+...)}{d^2\eta}\right)$$
$$+ P_r\left((f_0+qf_1+....)\left(\frac{d(\theta_0+q\theta_1+..)}{d\eta}\right)\right)$$
$$- qhP_r\left(N_b\frac{d(\phi_0+q\phi_1+...)}{d\eta}\left(\frac{d(\theta_0+q\theta_1+..)}{d\eta} + N_t\left(\frac{d(\theta_0+q\theta_1+...)}{d\eta}^2\right)\right)\right) = 0, \tag{B.15}$$

$$(1-nq)\left(\frac{d^2(\phi_0+q\phi_1+..)}{d\eta^2}\right) - qh\left(\frac{d^2(\phi_0+q\phi_1+...)}{d^2\eta}\right)$$
$$+ S_c\left((f_0+qf_1+....)\frac{d(\phi_0+q\phi_1+..)}{d\eta}\right) - \frac{qhN_t}{N_b}\left(\frac{d^2(\theta_0+q\theta_1+...)}{d\eta^2}\right) = 0. \tag{B.16}$$

By equating the coefficients of q^0 and q^1 into the Eqs. (B.13)–(B.16), we obtain the following differential equations:

$$q^0: \frac{d^3f_0}{d\eta^3} - \frac{M}{K+1}\frac{df_0}{d\eta} = 0, \tag{B.17}$$
$$q^1: \frac{d^3f_1}{d\eta^3} - \frac{M}{K+1}\frac{df_1}{d\eta} - n\frac{d^3f_0}{d\eta^3} + \frac{Mn}{K+1}\frac{df_0}{d\eta} - h$$
$$\left(\frac{d^3f_0}{d\eta^3} + \frac{1}{K+1}\left(f_0\frac{d^2f_0}{d\eta^2} - \left(\frac{df_0}{d\eta}\right)^2 + K\frac{dg_0}{d\eta} - M\frac{df_0}{d\eta}\right)\right) - h\left(\delta(\theta_0 - N_r\phi_0)\right) = 0, \tag{B.18}$$
$$q^0: \frac{d^2g_0}{d\eta^2} - \frac{2K}{1+\frac{K}{2}}g_0 = 0, \tag{B.19}$$
$$q^1: \frac{d^2g_1}{d\eta^2} - \frac{2Kg_1}{1+\frac{K}{2}} - n\frac{d^2g_0}{d\eta^2} - \frac{2Kng_0}{1+\frac{K}{2}}$$
$$h\left(\frac{d^2g_0}{d\eta^2} + \frac{1}{1+\frac{K}{2}}\left(f_0\frac{dg_0}{d\eta} - g_0\frac{df_0}{d\eta} - 2Kg_0 - K\frac{d^2f_0}{d\eta^2}\right)\right) = 0, \tag{B.20}$$
$$q^0: \frac{d^2\theta_0}{d\eta^2} = 0, \tag{B.21}$$
$$q^1: \frac{d^2\theta_1}{d\eta^2} - n\frac{d^2\theta_0}{d\eta^2} - h\left(\frac{d^2\theta_0}{d\eta^2} + P_r\left(f_0\frac{d\theta_0}{d\eta} + N_b\frac{d\phi_0}{d\eta}\frac{d\theta_0}{d\eta} + N_b\left(\frac{d\theta_0}{d\eta}^2\right)\right)\right) = 0, \tag{B.22}$$
$$q^0: \frac{d^2\phi_0}{d\eta^2} = 0, \tag{B.23}$$
$$q^1: \frac{d^2\phi_1}{d\eta^2} - n\frac{d^2\phi_0}{d\eta^2} - h\left(\frac{d^2\phi_0}{d\eta^2} + S_c\left(f_0\frac{d\phi_0}{d\eta}\right) + \frac{N_t}{N_b}\left(\frac{d^2\phi_0}{d\eta^2}\right)\right) = 0. \tag{B.24}$$

For the modified q-MHAM solution, the initial guesses are chosen in the following form to satisfy Eqs. (B.5)–(B.8):
Solving Eqs. (B.17)–(B.24) and using the boundary condition (B.5)–(B.8), we obtain the following results:

$$f_0 = f_we^{-a\eta} - e^{-\eta}(f_wa - 1) + (f_wa - 1), \tag{B.25}$$

$$
\begin{aligned}
f_1 &= A_1 + B_1 e^{\sqrt{\frac{M}{K+1}}\eta} + C_1 e^{-\sqrt{\frac{M}{K+1}}\eta} + (h+n)\left(\frac{-a^3 f_w e^{-a\eta}}{-a^3+\frac{aM}{K+1}} + \frac{e^{-\eta}(f_w a-1)}{-1+\frac{M}{K+1}}\right)\\
&- \frac{M(h+n)}{K+1}\left(\frac{-af_w e^{-a\eta}}{-a^3+\frac{Ma}{K+1}} + \frac{(f_w a-1)e^{-\eta}}{\frac{M}{K+1}-1}\right) + \frac{h}{K+1}\left(\frac{f_w^2 a^2 e^{-a\eta}}{\frac{Ma}{K+1}-a^3} - \frac{f_w(f_w a-1)e^{-\eta}}{\frac{M}{K+1}-1}\right)\\
&- \frac{h}{K+1}\left(\frac{f_w^2 a^2 e^{-2a\eta}}{\frac{2M}{K+1}-8a^3} + \frac{e^{-2\eta}(f_w a-1)^2}{\frac{2M}{K+1}-8} - \frac{-2af_w(f_w a-1)e^{-\eta(a+1)}}{\frac{M(a+1)}{K+1}-(a+1)^3}\right)\\
&- \frac{Kmh}{K+1}\left(\frac{e^{-a\eta}(-f_w a^3-3a)}{-a^3+\frac{aM}{K+1}} + \frac{e^{-\eta}(f_w a_1-2)}{\frac{M}{K+1}-1}\right)\\
&+ \frac{h}{K+1}\left(\frac{\delta e^{-\eta(a_1+1)}}{\frac{M(a_1+1)}{(K+1))}-(a_1+1)^3} - \frac{\delta N_r e^{-\eta(a_1+1)}}{\frac{M(a_1+1)}{(K+1))}-(a_1+1)^3}\right),
\end{aligned}
\tag{B.26}
$$

$$
g_0 = -m(a^2 f_w e^{-a\eta} - (f_w a-1)e^{-\eta}), \tag{B.27}
$$

$$
\begin{aligned}
g_1 &= c_1 e^{\sqrt{(2Kb_1\eta)}} + c_2 e^{\sqrt{(2Kb_1\eta)}} - (n+h)m\left(\frac{a^4 f_w e^{-a\eta}}{a^2-2Kb_1} - \frac{(f_w a-1)e^{-\eta}}{1-2Kb_1}\right) + 2Kmb_1(n+h)\\
&\left(\frac{a^2 f_w e^{-a\eta}}{a^2-2Kb_1} - \frac{(f_w a-1)e^{-\eta}}{1-2Kb_1} + h\left(-mb_1\left(\frac{-f_w^2 a^3 e^{-2a\eta}}{4a^2-2Kb_1} + \frac{f_w(f_w a-1)e^{-\eta(a+1)}}{(a+1)^2-2Kb_1}\right)\right)\right)\\
&- mhb_1\left(\frac{(f_w a-1)f_w a^3 e^{-\eta(a+1)}}{(a+1)^2-2Kb_1} - \frac{(f_w a-1)^2 e^{-2\eta}}{4-2Kb_1}\right)\left(-\frac{a^3 f_w(f_w a-1)e^{-a\eta}}{a^2-2Kb_1} + \frac{(f_w a-1)^2}{1-2Kb_1}\right)\\
&+ mhb_1\left(\frac{-a^3 f_w^2 e^{-2a\eta}}{4a^2-2Kb_1} + \frac{a^2 f_w(f_w a-1)e^{-\eta(a+1)})}{(a+1)^2-2Kb_1} + \frac{af_w(f_w a-1)e^{-\eta(a+1)}}{(a+1)^2-2Kb_1} - \frac{(f_w a-1)^2 e^{-2\eta}}{4-2Kb_1}\right)\\
&- Khb_1\left(\frac{f_w a^2 e^{-a\eta}}{a^2-2Kb_1} - \frac{(f_w a-1)e^{-\eta}}{1-2Kb_1}\right),
\end{aligned}
\tag{B.28}
$$

$$
\theta_0 = e^{-\eta(q+1)}, \tag{B.29}
$$

$$
\begin{aligned}
\theta_1 &= c_3 + c_4\eta + (n+h)e^{-\eta(q+1)} + h\left(\frac{-f_w P_r(q+1)e^{-\eta(q+1)}}{(a+q+1)}^2 + \frac{(f_w a-1)P_r(q+1)e^{-\eta(q+2)}}{(q+2)^2}\right)\\
&+ h\left(\frac{-(f_w a-1)P_r e^{-\eta(q+1)}}{(q+1)} + \frac{b(w+1)(q+1)e^{-\eta(w+q+2)}}{(w+q+2)} - \frac{N_t e^{-2\eta(q+1)}}{4}\right),
\end{aligned}
\tag{B.30}
$$

$$
\phi_0 = e^{-\eta(w+1)}, \tag{B.31}
$$

$$
\begin{aligned}
\phi_1 &= c_5 + c_6\eta + (n+h)e^{-\eta(w+1)} + hS_c\left(\frac{-f_w(w+1)e^{-\eta(a+w+1)}}{(a+w+1)^2} + \frac{(f_w a-1)(w+1)e^{-\eta(w+2)}}{(w+2)^2}\right)\\
&+ hS_c\left(-\frac{(f_w a-1)e^{-\eta(w+1)}}{w+1}\right) + h\left(\frac{N_t e^{-\eta(w+1)}}{N_b}\right),
\end{aligned}
\tag{B.32}
$$

where the constant A_1, B_1, C_1, c_1 to c_6 are defined in the text Eqs. (3.5) to (3.10) respectively. According to q-MHAM, we conclude that

$$
f(\eta) = \lim_{q\to 1} f(\eta) = f_0(\eta) + f_1(\eta), \tag{B.33}
$$

$$
g(\eta) = \lim_{q\to 1} g(\eta) = g_0(\eta) + g_1(\eta), \tag{B.34}
$$

$$
\theta(\eta) = \lim_{q\to 1} \theta(\eta) = \theta_0(\eta) + \theta_1(\eta), \tag{B.35}
$$

$$\phi(\eta) = \lim_{q \to 1} \phi(\eta) = \phi_0(\eta) + \phi_1(\eta). \tag{B.36}$$

After substituting the values of $f_0, f_1, g_0, g_1, \theta_0, \theta_1, \phi_0, \phi_1$ into the above Eqs. we obtain the solutions.

Appendix:C

Nomenclature

Symbol	Meaning
(u, v)	Field velocities in the direction (x,y) respectively
V	Velocity vector
γ	Spin gradiant viscosity
J	Density of current
$f,\ g,\ \theta$	Dimensionless velocity, angular velocity, temperature
ϕ	Dimensionless concentration
K	Micropolar material parameter
δ	Local Mixed convection
N_r	Buoyancy ratio parameter
N_b	Brownian diffusion parameter
N_t	Thermophoresis parameter
m	Micro-gyration parameter
f_w	Suction parameter
m	Micropolar material parameter
$P_r,\ S_C$	Prandtl number, Schmidt number
η	Dimensionless coordinate
N	Micro rotation
T	Temperature
C	Volume friction
T_w	Temperature of fluid
T_∞, C_∞	Ambient temperature, Free stream concentration
g	Accelaration
B	Magnetic field strength
j	Micro inertia density
μ_f	Base fluid viscosity
P	Pressure
ρ_p	Nanoparticles material densities
ρ_f	Densities of base fluid
k	Thermal conductivity
D_B	Coefficient of Brownion motion
D_T	Thermophoretic diffusion
$(\rho C)_p$	Effective nano particle
$(\rho C)_f$	Effective base fluid
C_f, S_h	Skin friction coefficient, Sherwood number
$Re,\ M$	Reynolds number, Hartmann number
Nu	Local Nusselt number
μ	Viscosity Fluid
τ_w	Wall Temperature of vertical
$\bigtriangledown V,\ \bigtriangledown T$	Film velocity drop, Film temperature drop

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