Research Paper
Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. 14, No. 2, 2026, pp. 567-580
DOI:10.22034/cmde.2024.61477.2651

A novel approach to fractional kinetic equations involving Srivastava polynomial and multi-index Bessel function

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Abstract

In the present work, the generalized fractional kinetic equations (FKE) incorporating the composition of Multi-Index Bessel function and Srivastava polynomial are expressed with their fractional derivatives. Moreover, by employing the idea of the Laplace transform, solutions are obtained in terms of the Mittag-Leffler function. Finally, a numerical and graphical interpretation of the outcome is displayed.

Keywords. Generalized fractional kinetic equation, Srivastava Polynomial, Multi-Index Bessel function, Fractional derivative, Laplace transform, Mittag-Leffler function.

2010 Mathematics Subject Classification. 26A33, 33E12, 33E20, 44A99.

1. Introduction

The importance of special functions as a tool for mathematical analysis is widely acknowledged by physicists and engineers (see [26]). Various special functions are used to describe the solution of many real-world problems with the involvement of differential equations of different orders. Many researchers have used a variety of special functions in defining the problems of various domains as well as identifying the solutions of a variety of differential equations of integer order [1, 4, 7, 8, 27] and fractional order [14, 25, 31, 34, 35]. Fractional order calculus (FOC), which involves the integration and differentiation under an arbitrary order, emerges in different realms of engineering, management, and applied sciences. During the past few decades, fractional differential equations (FDEs) have received more attention and importance and have been demonstrated to be significant tools in the modeling of various types of phenomena in different areas like economy, electrochemistry, viscoelasticity, porous media, electromagnetic, engineering, and physics. Therefore, it is significant to study the differential equations of integer/fractional order to enhance the research visibility and the applications in the relevant areas. In this order, the current manuscript contains a study on a specific fractional differential equation, the fractional kinetic equation.

2. Fractional Kinetic Equation

The kinetic equation is a system of differential equations where the rate of change of the chemical composition of a star is defined for all orders in terms of the reaction rates for production and destruction. Due to its significance in the realm of applied science such as dynamical systems, mathematical physics, control theory, astrophysics, and various engineering problems, fractional kinetic equations [FKEs] and their solution have received more attention from several researchers. The extension and generalization of FKE containing a variety of special functions have been found in various works ([3, 6, 9, 12, 13, 17, 19]).

Received: 01 May 2024; Accepted: 23 December 2024.
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To the enormous significance of kinetic equations and to explore their applications in mathematics and science, Haubold and Mathai [11] established a fractional generalization of the kinetic equation involving the rate of change of reaction $\varsigma = \varsigma_t$, rate of destruction $Q = Q(\varsigma)$ and rate of production $P = P(\varsigma)$ as follows:

$$\frac{d\varsigma}{dt} = -Q(\varsigma_t) + P(\varsigma_t), \tag{2.1}$$

where $\varsigma = (\varsigma_t)$ represent a function, explained by $\varsigma_t(t^*) = \varsigma(t - t^*),\ t^* > 0$.

By neglecting the spatial fluctuations and inhomogeneities in the amount ς_t, the authors have taken the particular case of (2.1), which is represented in the form of the differential equation such as:

$$\frac{d\varsigma_i}{dt} = -c_i\varsigma_i(t), \tag{2.2}$$

with the initial condition $\varsigma_i(t = 0) = \varsigma_0$, the number of density of a species i at the time $t = 0$ and $c_i > 0$.

By leaving out the index i and solving (2.2), they obtained

$$\varsigma(t) - \varsigma = -c\ {}_0D_t^{-1}\varsigma(t). \tag{2.3}$$

where c is a constant.

The operator ${}_0D_t^{-1}$ is the specific case of Riemann-Liouville fractional integral operator ${}_0D_t^{-v}\varsigma(t)$, which is defined as

$${}_0D_t^{-v}f(x) = \frac{1}{\Gamma(v)}\int_0^t (t-u)^{v-1}f(u)du,\ x > 0, \Re(v) > 0. \tag{2.4}$$

For more details, one may refer to [18].

The generalized form of (2.3) was expressed in the following way

$$\varsigma(t) - \varsigma_0 = -c^v{}_0D_t^{-v}\varsigma(t), \tag{2.5}$$

and, its solution was given as

$$\varsigma(t) = \varsigma_0\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma(\nu j+1)}(ct)^{\nu j}. \tag{2.6}$$

Furthermore, the most generalized form of FKE given by Saxena and Kalla [28] as

$$\varsigma(t) - \varsigma_0 f(t) = -c^v\ {}_0D_t^{-v}\varsigma(t), \quad \Re(v) > 0, \quad c > 0, \tag{2.7}$$

where c is constant and $f(t) \in L(0, \infty)$.

For the present work, the following functions and results are required.

(a) **The Srivastava polynomial [24] is mentioned as**

$$s_w^p(\xi) = \sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}A_{w,r}(\xi)^r,\ p \in \mathbb{N},\ w \in \mathbb{N}_0, \tag{2.8}$$

where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and the coefficients $A_{w,r}(w, r \in \mathbb{N}_0)$ are arbitrary real or complex constants.

(b) **The multi-index Bessel function [5] is defined as:**

$$\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(z) = \sum_{\mu=0}^{\infty}\frac{(\alpha)_{\mu k}}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!}(-z)^{\mu},\ m \in \mathbb{N}. \tag{2.9}$$

where $\tau_i,\ \zeta_i\ (i = 1, 2, \ldots, m);\ \alpha \in \mathbb{C},\ \Re(\alpha) > 0,\ \Re(\zeta_i) > -1, \sum_{i=1}^{m}\Re(\zeta_i) > max\{0; \Re(k) - 1\}, k > 0$.

Here, the pochhammer symbol $(\alpha)_n$ is expressed as:

$$(\alpha)_n = \begin{cases} 1, & n = 0, \\ \alpha(\alpha+1)...(\alpha+n-1), & n \in \mathbb{N}. \end{cases} \tag{2.10}$$

(c) **Fractional derivative of the multi-index Bessel function.**

The fractional order derivative [20] of order λ of the function $f(t) = t^\beta$ is defined as:

$$ {}_0D_t^\lambda t^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\lambda+1)} t^{\beta-\lambda}; \qquad \Re(\beta) > -1, 0 < \Re(\lambda) < 1, t > 0. \tag{2.11}$$

So, in view of (2.9), we have

$$ {}_0D_t^\lambda \left(\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(t) \right) = \sum_{\mu=0}^{\infty} \frac{(-1)^\mu (\alpha)_{\mu k} \Gamma(\mu+1)}{\prod_{i=1}^m \Gamma(\tau_i\mu+\zeta_i+1)\mu!\; \Gamma(\mu-\lambda+1)} (t)^{\mu-\lambda}. \tag{2.12}$$

(d) **Fractional derivative of the composition of the multi-index Bessel function and the Srivastava polynomial.**

Using the definition of the fractional derivatives, we have

$$ {}_0D_t^\lambda \left(\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(t) s_w^p(t) \right) = \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^\mu (\alpha)_{\mu k} \Gamma(\mu+r+1)}{\prod_{i=1}^m \Gamma(\tau_i\mu+\zeta_i+1)\mu!\; \Gamma(\mu+r-\lambda+1)} A_{w,r} (t)^{\mu+r-\lambda}. \tag{2.13}$$

Using the Laplace transform ([29]) on (2.13), we have

$$ L\left[{}_0D_t^\lambda \left(\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(t) s_w^p(t) \right) \right] = \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^\mu (\alpha)_{\mu k} \Gamma(\mu+r+1)}{\prod_{i=1}^m \Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r} (s)^{-(\mu+r-\lambda+1)}. \tag{2.14}$$

Various authors have proposed many FKEs involving variety of special functions ([2, 15, 21, 23, 30, 32, 37]) and these equations are of great importance according to the concerned function involved therein in the related application domain.

In the present work, we proffer a generalized form of FKE involving the composition of Srivastava polynomial [24] and multi index Bessel function [10, 16, 22, 33, 38, 39] with their fractional derivatives for the dominance and concernment of the kinetic equation in several problems of astrophysics and obtain a solution by the approach of Laplace transform.

As the Srivastava polynomial is a generalized polynomial and can be reduced in various classical orthogonal polynomials and generalized hypergeometric polynomials (such as Hermite polynomial, Jacobi polynomials, Gagenbauer polynomials, Legendre polynomials, Tchebycheff polynomials, Laguerre polynomials, Bessel polynomials, Gould Hopper polynomials etc.). the application work of these polynomials in various domains can be seen in the related literature evidently. Specifically, the application of Bessel's function can be seen in wave propogation, cylinderical coordinate systems, rotational flows, heat conduction, vibration theory, diffusion problems etc. Therefore, it is evident that the composition of the duo (Srivastava polynomial and Bessel's function) may lead to many known and new applications through the FKEs in several domains of science, engineering and applied mathematics.

3. Main Results

In this section, some new FKEs involving the composition of multi-index Bessel function and the Srivastava polynomial are demonstrated and their solutions are calculated by the application of Laplace transform [29]. Further, the results are interpreted numerically and graphically by using various appropriate assignments of parametric values.

Theorem 3.1. *If $c > 0,\ v > 0,\ p \in \mathbb{N},\ A_{w,r}(w, r \in \mathbb{N}_0),\ \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \tau_i,\ \zeta_i\ (i = 1, 2, ..., m), m \in \mathbb{N};\ \alpha \in \mathbb{C},\ \Re(\alpha) > 0, \Re(\zeta) > -1,\ \sum_{i=1}^m \Re(\zeta_i) > max\{0; \Re(k) - 1\},\ k > 0$, then the solution of the FKE*

$$ \varsigma(t) - \varsigma_0 \left\{ \Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(t) s_w^p(t) \right\} = -c^v \; {}_0D_t^{-v} \varsigma(t), \tag{3.1}$$

is provided by

$$ \varsigma(t) = \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^\mu (\alpha)_{\mu k} \Gamma(\mu+r+1)}{\prod_{i=1}^m \Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r} (t)^{\mu+r} E_{v,\mu+r+1}(-c^v t^v), \tag{3.2}$$

where $E_{v,\mu+r+1}(.)$ is the Mittag-Leffler function [40].

Proof. By employing the Laplace transform on (3.1), we get

$$\begin{aligned}\bar{\varsigma}(s) &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(\mu+r+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(s)^{-(\mu+r+1)}(1+c^v s^{-v})^{-1} \\ &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(\mu+r+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(s)^{-(\mu+r+1)} \sum_{n=0}^{\infty}(-c^v s^{-v})^n \\ &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(\mu+r+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r} \sum_{n=0}^{\infty}(-c^v)^n(s)^{-(\mu+r+1+vn)}. \end{aligned} \tag{3.3}$$

Now taking the inverse Laplace transform of (3.3), we have

$$\begin{aligned}\varsigma(t) &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(\mu+r+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(t)^{\mu+r} \sum_{n=0}^{\infty} \frac{(-c^v t^v)^n}{\Gamma(vn+\mu+r+1)} \\ &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(\mu+r+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(t)^{\mu+r} E_{v,\mu+r+1}(-c^v t^v).\end{aligned}$$

□

Theorem 3.2. *If* $c > 0,\ v > 0, p \in \mathbb{N}, A_{w,r}(w, r \in \mathbb{N}_0),\ \mathbb{N}_0 = \mathbb{N} \cup \{0\},\ \tau_i,\ \zeta_i\ (i = 1, 2, ..., m),\ m \in \mathbb{N},\ \alpha \in \mathbb{C}, \Re(\alpha) > 0, \Re(\zeta) > -1, \sum_{i=1}^{m} \Re(\zeta_i) > max\{0; \Re(k) - 1\}, k > 0,$ *then the solution of the FKE*

$$\varsigma(t) - \varsigma_0\left\{\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(c^v t^v) s_w^p(c^v t^v)\right\} = -d^v\ {}_0D_t^{-v}\varsigma(t), \tag{3.4}$$

is given by

$$\varsigma(t) = \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(ct)^{v(\mu+r)} E_{v,(v(\mu+r)+1)}(-d^v t^v). \tag{3.5}$$

Proof. By employing the Laplace transform on (3.5), we get

$$\begin{aligned}\bar{\varsigma}(s) &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(s)^{-(v(\mu+r)+1)}(c)^{v(\mu+r)}(1+d^v s^{-v})^{-1} \\ &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(c)^{v(\mu+r)} \sum_{n=0}^{\infty}(-d^v s^{-v})^n(s)^{-(v(\mu+r)+1)} \\ &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(c)^{v(\mu+r)} \sum_{n=0}^{\infty}(-d^v)^n(s)^{-(v(\mu+r)+1+vn)}.\end{aligned}$$

Now, taking inverse Laplace transform, we obtain

$$\begin{aligned}\varsigma(t) &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(c)^{v(\mu+r)} \sum_{n=0}^{\infty} \frac{(-d^v)^n(t)^{v(\mu+r)+vn}}{\Gamma(v(\mu+r)+vn+1)} \\ &= \varsigma_0 \sum_{\mu=0}^{\infty} \sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!} \frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(ct)^{v(\mu+r)} E_{v,(v(\mu+r)+1)}(-d^v t^v).\end{aligned}$$

□

Theorem 3.3. *If $c>0$, $v>0, p\in\mathbb{N}, A_{w,r}$ $(w,r\in\mathbb{N}_0)$, $\mathbb{N}_0=\mathbb{N}\cup\{0\}$, τ_i, ζ_i $(i=1,2,...,m)$, $m\in\mathbb{N}$, $\alpha\in\mathbb{C}$, $\Re(\alpha)>0$, $\Re(\zeta)>-1$, $\sum_{i=1}^{m}\Re(\zeta_i)>max\{0;\Re(k)-1\}, k>0$, then the solution of FKE*

$$\varsigma(t)-\varsigma_0\Big({}_0D_t^{\lambda}\Big(\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(t)s_w^p(t)\Big)\Big)=-c^v\ {}_0D_t^{-v}\varsigma(t). \tag{3.6}$$

is given by

$$\varsigma(t)=\varsigma_0\sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(\mu+r+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!}A_{w,r}(t)^{\mu+r-\lambda}E_{v,\mu+r-\lambda+1}(-c^vt^v). \tag{3.7}$$

Proof. By employing the Laplace transform on (3.6), we get

$$\bar{\varsigma}(s)=\varsigma_0\sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(\mu+r+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!}A_{w,r}(s)^{-(\mu+r-\lambda+1)}(1+c^vs^{-v})^{-1}.$$

By applying the similar procedure and calculation as we have used in Theorem 3.1 and 3.2, and taking the inverse Laplace transform, we get

$$\varsigma(t)=\varsigma_0\sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(\mu+r+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!}A_{w,r}(t)^{\mu+r-\lambda}E_{v,\mu+r-\lambda+1}(-c^vt^v).$$

□

Theorem 3.4. *If $c>0$, $v>0, p\in\mathbb{N}$, $A_{w,r}(w,r\in\mathbb{N}_0)$, $\mathbb{N}_0=\mathbb{N}\cup\{0\}$, τ_i, ζ_i $(i=1,2,...,m)$, $m\in\mathbb{N}$, $\alpha\in\mathbb{C},\Re(\alpha)>0,\Re(\zeta)>-1,\sum_{i=1}^{m}\Re(\zeta_i)>max\{0;\Re(k)-1\},k>0$, then the solution of the FKE*

$$\varsigma(t)-\varsigma_0\Big\{{}_0D_t^{\lambda}\Big(\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(c^vt^v)s_w^p(c^vt^v)\Big)\Big\}=-d^v\ {}_0D_t^{-v}\varsigma(t), \tag{3.8}$$

is given by

$$\varsigma(t)=\varsigma_0\sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!}A_{w,r}(ct)^{v(\mu+r)}t^{-\lambda}E_{v,v(\mu+r)-\lambda+1}(-d^vt^v). \tag{3.9}$$

Proof. By employing the Laplace transform on Equation (3.8), we get

$$\begin{aligned}\bar{\varsigma}(s)&=\varsigma_0\sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!}A_{w,r}(s)^{-(v(\mu+r)-\lambda+1)}(c)^{v(\mu+r)}(1+d^vs^{-v})^{-1}\\&=\varsigma_0\sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!}A_{w,r}(s)^{-(v(\mu+r)-\lambda+1)}(c)^{v(\mu+r)}\sum_{n=0}^{\infty}(-d^vs^{-v})^n\\&=\varsigma_0\sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!}A_{w,r}(c)^{v(\mu+r)}\sum_{n=0}^{\infty}(-d^v)^n(s)^{-(v(\mu+r)-\lambda+1+vn)}.\end{aligned}$$

Now, taking inverse Laplace transform, and simplifying, we get

$$=\varsigma_0\sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\frac{(-1)^{\mu}(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^{m}\Gamma(\tau_i\mu+\zeta_i+1)\mu!}A_{w,r}(ct)^{v(\mu+r)}t^{-\lambda}E_{v,v(\mu+r)-\lambda+1}(-d^vt^v).$$

□

4. Particular Cases

(1) Putting $d = c$ in Theorem 3.2, the FKE is reduced to the following form

$$\varsigma(t) - \varsigma_0\left\{\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(c^v t^v) s_w^p(c^v t^v)\right\} = -c^v\ {}_0D_t^{-v}\varsigma(t) \tag{4.1}$$

and its solution is given by

$$\varsigma(t) = \varsigma_0 \sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!}\frac{(-1)^\mu(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^m \Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(ct)^{v(\mu+r)} E_{v,(v(\mu+r)+1)}(-c^v t^v). \tag{4.2}$$

(2) Putting $c = 1$ in Theorem 3.2, the FKE is reduced into the following form

$$\varsigma(t) - \varsigma_0\left\{\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(t^v) s_w^p(t^v)\right\} = -d^v\ {}_0D_t^{-v}\varsigma(t), \tag{4.3}$$

and its solution is given by

$$\varsigma(t) = \varsigma_0 \sum_{\mu=0}^{\infty}\sum_{\mu=0}^{[w/p]} \frac{(-w)_{pr}}{r!}\frac{(-1)^\mu(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^m \Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(t)^{v(\mu+r)} E_{v,(v(\mu+r)+1)}(-d^v t^v). \tag{4.4}$$

(3) Putting $d = c$ in Theorem 3.4, the FKE is reduced into the following form

$$\varsigma(t) - \varsigma_0\left\{ {}_0D_t^\lambda\left(\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(c^v t^v) s_w^p(c^v t^v)\right)\right\} = -c^v\ {}_0D_t^{-v}\varsigma(t), \tag{4.5}$$

and its solution is given by

$$\varsigma(t) = \varsigma_0 \sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!}\frac{(-1)^\mu(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^m \Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(ct)^{v(\mu+r)} t^{-\lambda} E_{v,v(\mu+r)-\lambda+1}(-c^v t^v). \tag{4.6}$$

(4) Substituting $c = 1$ in Theorem 3.4, the FKE is reduced into the following form

$$\varsigma(t) - \varsigma_0\left\{ {}_0D_t^\lambda\left(\Im_{(\zeta_i)_m,k}^{(\tau_i)_m,\alpha}(t^v) s_w^p(t^v)\right)\right\} = -d^v\ {}_0D_t^{-v}\varsigma(t), \tag{4.7}$$

and the solution is obtained as

$$\varsigma(t) = \varsigma_0 \sum_{\mu=0}^{\infty}\sum_{r=0}^{[w/p]} \frac{(-w)_{pr}}{r!}\frac{(-1)^\mu(\alpha)_{\mu k}\Gamma(v(\mu+r)+1)}{\prod_{i=1}^m \Gamma(\tau_i\mu+\zeta_i+1)\mu!} A_{w,r}(t)^{v(\mu+r)-\lambda} E_{v,v(\mu+r)-\lambda+1}(-d^v t^v). \tag{4.8}$$

(5) If we put $k = 0, \tau = m = 1,\ \zeta = v, z = \frac{t^2}{4}$ in (2.9), then the Multi-index Bessel function is reduced in well-known Bessel function [26] as

$$\Im_{v,0}^{1,\alpha}\left(\frac{t^2}{4}\right) = \left(\frac{2}{t}\right)^v \Im_v(t); t, v \in \mathbb{C}, t \neq 0, \Re(v) > -1, \tag{4.9}$$

where $\Im_v(t)$ is the well known Bessel function

$$\Im_v(t) = \sum_{\rho=0}^{\infty} \frac{(-1)^\rho}{\rho!\ \Gamma(v+\rho+1)}\left(\frac{t}{2}\right)^{v+2\rho}. \tag{4.10}$$

Hence

$$\Im_{v,0}^{1,\alpha}(t) = \sum_{\rho=0}^{\infty} \frac{(-1)^\rho}{\rho!\ \Gamma(v+\rho+1)}(t)^\rho. \tag{4.11}$$

Further, using (4.11) in Theorem 3.1-3.4, we get the results as

Corollary 4.1. *From Theorem 3.1, the FKE is reduced as*

$$\varsigma(t)-\varsigma_0\Big(\Im_{v,0}^{1,\alpha}(t)s_w^p(t)\Big)=-c^v\ {}_0D_t^{-v}\varsigma(t), \tag{4.12}$$

and its solution is

$$\varsigma(t)=\varsigma_0\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\sum_{\rho=0}^{\infty}\frac{(-1)^\rho\Gamma(\rho+r+1)}{\rho!\Gamma(v+\rho+1)}(t)^{\rho+r}E_{v,\rho+r+1}(-c^vt^v). \tag{4.13}$$

Corollary 4.2. *From Theorem 3.2, the FKE is reduced as*

$$\varsigma(t)-\varsigma_0\Big(\Im_{v,0}^{1,\alpha}(c^vt^v)s_w^p(c^vt^v)\Big)=-d^v\ {}_0D_t^{-v}\varsigma(t), \tag{4.14}$$

and its solution is

$$\varsigma(t)=\varsigma_0\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\sum_{\rho=0}^{\infty}\frac{(-1)^\rho\Gamma(v(\rho+r)+1)}{\rho!\Gamma(v+\rho+1)}(ct)^{v(\rho+r)}E_{v,v(\rho+r)+1}(-d^vt^v). \tag{4.15}$$

Corollary 4.3. *From Theorem 3.3, the FKE is reduced as*

$$\varsigma(t)-\varsigma_0\ {}_0D_t^{\lambda}\Big(\Im_{v,0}^{1,\alpha}(t)s_w^p(t)\Big)=-c^v\ {}_0D_t^{-v}\varsigma(t), \tag{4.16}$$

and its solution is

$$\varsigma(t)=\varsigma_0\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\sum_{\rho=0}^{\infty}\frac{(-1)^\rho\Gamma(\rho+r+1)}{\rho!\Gamma(v+\rho+1)}(t)^{\rho+r-\lambda}E_{v,\rho+r-\lambda+1}(-c^vt^v). \tag{4.17}$$

Corollary 4.4. *From Theorem 3.4, the FKE is reduced as*

$$\varsigma(t)-\varsigma_0\ {}_0D_t^{\lambda}\Big(\Im_{v,0}^{1,\alpha}(c^vt^v)s_w^p(c^vt^v)\Big)=-d^v\ {}_0D_t^{-v}\varsigma(t), \tag{4.18}$$

and its solution is

$$\varsigma(t)=\varsigma_0\sum_{r=0}^{[w/p]}\frac{(-w)_{pr}}{r!}\sum_{\rho=0}^{\infty}\frac{(-1)^\rho\Gamma(v(\rho+r)+1)}{\rho!\Gamma(v+\rho+1)}(ct)^{v(\rho+r)}(t)^{-\lambda}E_{v,v(\rho+r)-\lambda+1}(-d^vt^v). \tag{4.19}$$

(6) By using suitable parametric values, the Srivastava polynomial $s_w^p(.)$ reduces to unity, then, the reduced result is comparable with the known result due to Suthar et al. [36].

More specific cases of the outcomes stated in Theorems 3.1-3.4 may be obtained by suitable parametric values, but we do not put them down here due to lack of space.

5. Numerical and Graphical Interpretation of Results

For different assignments of parametric values, the numerical results for Theorems 3.1–3.4 are exhibited in Tables 1–4. Further, the behaviour of the results is presented by the 2D and 3D graphs related to the theorems in Figures 1–4, respectively.

6. Conclusion

The concept of fractional calculus extends the concept of integer-order calculus in a deeper way to understand various phenomenon of real-world problems and several basic concepts of science. Recently, research related to the area of fractional calculus has played a crucial role in numerous disciplines, including control systems, elasticity, electric drives, circuit systems, continuum mechanics, heat transfer, quantum mechanics, fluid mechanics, signal analysis, biomathematics, biomedicine, social systems, and bioengineering.

In this work, four new fractional kinetic equations (FKEs) are proferred and their solutions are obtained by the most popular transform, the Laplace transform. The importance of the Kinetic equations are very well-known in the science fraternity. In this sequence, the application of these equations can be applied in the gas laws (like Boyle's law, Charle's

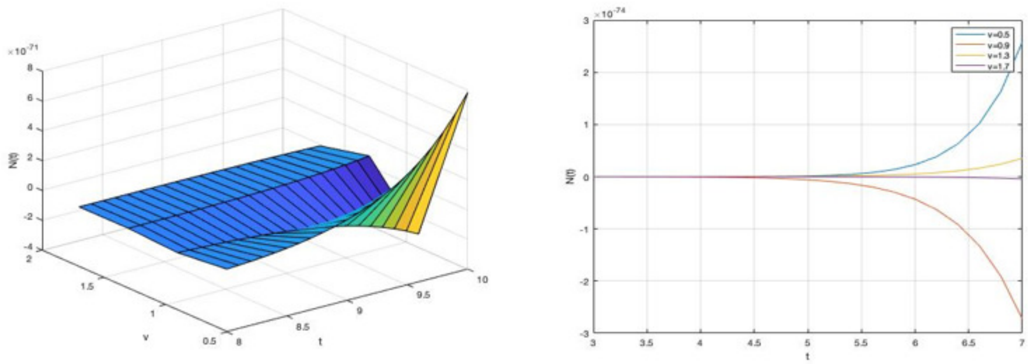

FIGURE 1. 3D and 2D graphs for $\zeta(t)$ corresponding to Theorem 3.1.

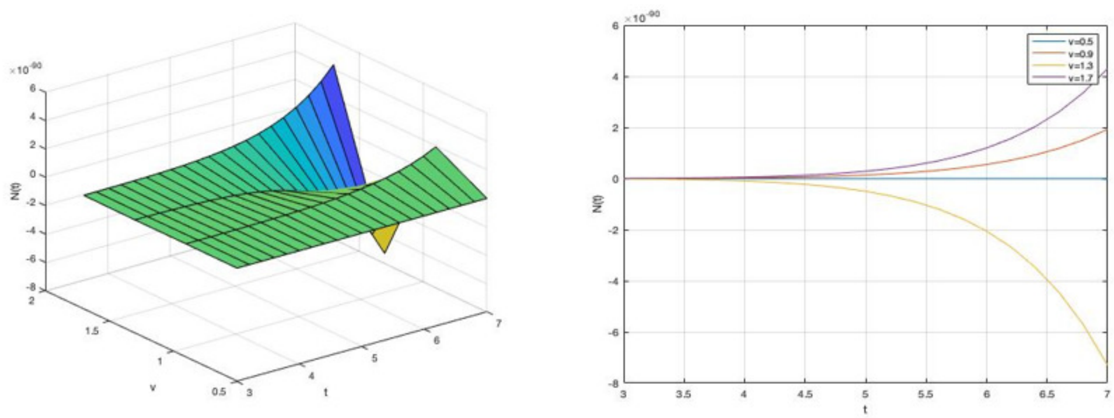

FIGURE 2. 3D and 2D graphs for $\zeta(t)$ corresponding to Theorem 3.2.

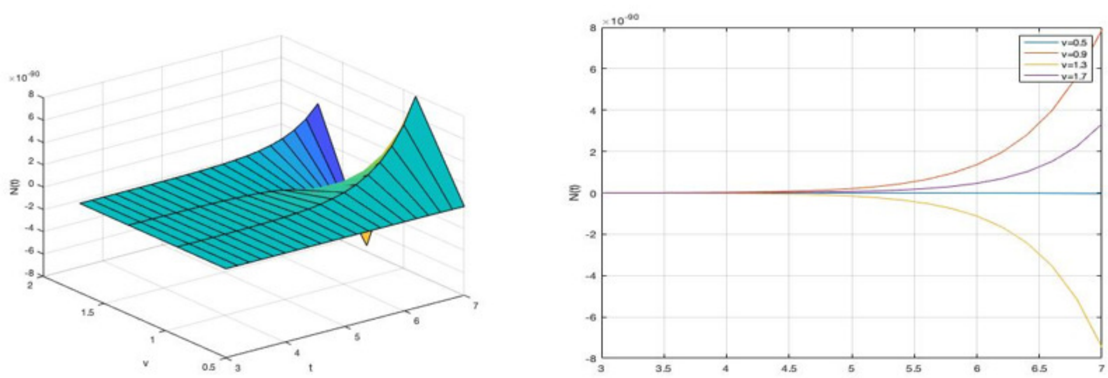

FIGURE 3. 3D and 2D graphs for $\zeta(t)$ corresponding to Theorem 3.3.

TABLE 1. Values of $\zeta(t)$ for t and v (Theorem 3.1).

t	Fix $v = 0.5$ $\varsigma(t)$	Fix $v = 0.9$ $\varsigma(t)$	Fix $v = 1.3$ $\varsigma(t)$	Fix $v = 1.7$ $\varsigma(t)$
3.0	4.37732E-80	-5.2813E-78	9.05974E-79	-1.1677E-79
3.2	1.23784E-79	-9.07797E-78	1.45437E-78	-1.82549E-79
3.4	3.25493E-79	-1.52669E-77	2.3028E-78	-2.82574E-79
3.6	8.04131E-79	-2.5197E-77	3.60506E-78	-4.3396E-79
3.8	1.88199E-78	-4.09138E-77	5.59123E-78	-6.62256E-79
4.0	4.20083E-78	-6.54977E-77	8.605E-78	-1.0056E-78
4.2	8.99268E-78	-1.03554E-76	1.31591E-77	-1.52088E-78
4.4	1.85476E-77	-1.61926E-76	2.00172E-77	-2.29301E-78
4.6	3.70016E-77	-2.50715E-76	3.03152E-77	-3.44862E-78
4.8	7.16352E-77	-3.84749E-76	4.57396E-77	-5.17654E-78
5.0	1.3497E-76	-5.8565E-76	6.87892E-77	-7.75835E-78
5.2	2.4809E-76	-8.84768E-76	1.03158E-76	-1.16137E-77
5.4	4.45832E-76	-1.32727E-75	1.54292E-76	-1.73676E-77
5.6	7.84735E-76	-1.97785E-75	2.30197E-76	-2.59508E-77
5.8	1.3551E-75	-2.92856E-75	3.42603E-76	-3.87476E-77
6.0	2.29902E-75	-4.30961E-75	5.08634E-76	-5.78152E-77
6.2	3.83693E-75	-6.30402E-75	7.53192E-76	-8.6207E-77
6.4	6.3065E-75	-9.1675E-75	1.11234E-75	-1.28449E-76
6.6	1.02186E-74	-1.32551E-74	1.63806E-75	-1.91237E-76
6.8	1.63378E-74	-1.90567E-74	2.40498E-75	-2.84459E-76
7.0	2.57953E-74	-2.72446E-74	3.51969E-75	-4.22686E-76

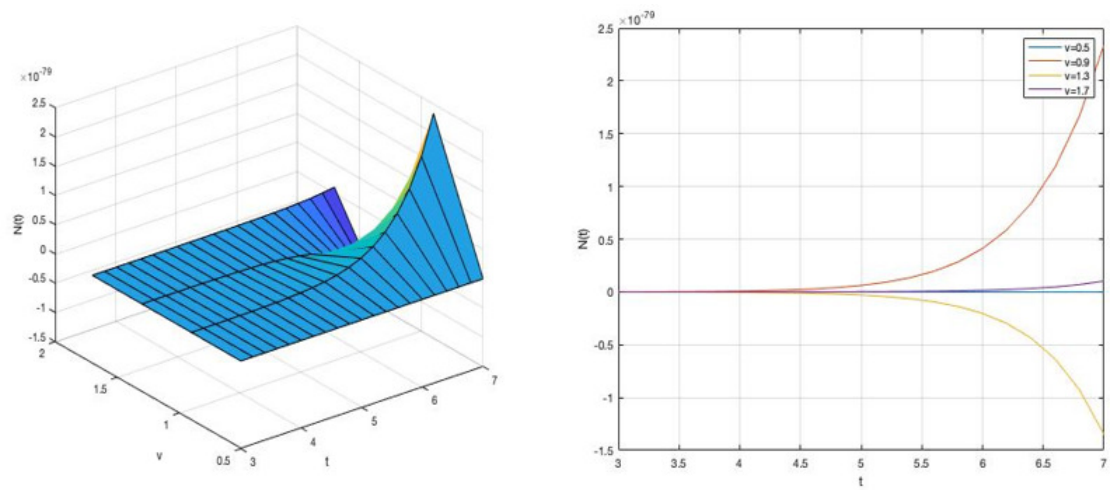

FIGURE 4. 3D and 2D graphs for $\zeta(t)$ corresponding to Theorem 3.4.

law, and Gay-Lussac's law), behaviour of gases, molecule energy, hydrodynamics, plasma physics, semiconductors, flying of airplanes, windmills, and hydropower plants, etc.

TABLE 2. Values of $\zeta(t)$ for ct and v (Theorem 3.2).

t	Fix $v = 0.5$ $\varsigma(t)$	Fix $v = 0.9$ $\varsigma(t)$	Fix $v = 1.3$ $\varsigma(t)$	Fix $v = 1.7$ $\varsigma(t)$
3.0	-1.275E-100	3.68761E-93	-1.24267E-92	6.61107E-93
3.2	-2.3605E-100	5.71453E-93	-1.95195E-92	1.04929E-92
3.4	-4.305E-100	8.65624E-93	-2.99223E-92	1.62369E-92
3.6	-7.6711E-100	1.28516E-92	-4.48958E-92	2.45712E-92
3.8	-1.3316E-99	1.87435E-92	-6.60957E-92	3.64563E-92
4.0	-2.2511E-99	2.69055E-92	-9.56756E-92	5.31479E-92
4.2	-3.7104E-99	3.80755E-92	-1.36416E-91	7.62732E-92
4.4	-5.9728E-99	5.31964E-92	-1.91881E-91	1.07926E-91
4.6	-9.4068E-99	7.34666E-92	-2.66612E-91	1.50782E-91
4.8	-1.45208E-98	1.00401E-91	-3.6637E-91	2.08243E-91
5.0	-2.20065E-98	1.35909E-91	-4.9842E-91	2.84611E-91
5.2	-3.27941E-98	1.82382E-91	-6.71897E-91	3.85307E-91
5.4	-4.81201E-98	2.42812E-91	-8.98238E-91	5.17128E-91
5.6	-6.96129E-98	3.20924E-91	-1.19173E-90	6.88577E-91
5.8	-9.9397E-98	4.21347E-91	-1.57014E-90	9.10254E-91
6.0	-1.40222E-97	5.49819E-91	-2.05557E-90	1.19534E-90
6.2	-1.95618E-97	7.13432E-91	-2.67539E-90	1.5602E-90
6.4	-2.70087E-97	9.20938E-91	-3.46347E-90	2.02507E-90
6.6	-3.69333E-97	1.18311E-90	-4.46162E-90	2.61501E-90
6.8	-5.0054E-97	1.51319E-90	-5.72145E-90	3.36091E-90
7.0	-6.72701E-97	1.92742E-90	-7.30648E-90	4.30085E-90

TABLE 3. Values of $\zeta(t)$ for t and v (Theorem 3.3).

t	Fix $v = 0.5$ $\varsigma(t)$	Fix $v = 0.9$ $\varsigma(t)$	Fix $v = 1.3$ $\varsigma(t)$	Fix $v = 1.7$ $\varsigma(t)$
3.0	-4.77098E-98	2.54316E-93	-2.43061E-93	1.0537E-93
3.2	-1.4655E-97	4.15714E-93	-3.78085E-93	1.60708E-93
3.4	-4.12126E-97	6.68562E-93	-5.82836E-93	2.43786E-93
3.6	-1.0748E-96	1.06028E-92	-8.92008E-93	3.68283E-93
3.8	-2.62752E-96	1.66131E-92	-1.3573E-92	5.54605E-93
4.0	-6.07431E-96	2.57564E-92	-2.05566E-92	8.33194E-93
4.2	-1.33752E-95	3.95586E-92	-3.1015E-92	1.24945E-92
4.4	-2.8218E-95	6.02464E-92	-4.66462E-92	1.87107E-92
4.6	-5.73197E-95	9.10478E-92	-6.99655E-92	2.799E-92
4.8	-1.1257E-94	1.36614E-91	-1.0469E-91	4.18368E-92
5.0	-2.14485E-94	2.03606E-91	-1.56298E-91	6.24925E-92
5.2	-3.97661E-94	3.01498E-91	-2.32839E-91	9.32937E-92
5.4	-7.19241E-94	4.4368E-91	-3.46094E-91	1.39204E-91
5.6	-1.27185E-93	6.48954E-91	-5.13248E-91	2.07601E-91
5.8	-2.20307E-93	9.43546E-91	-7.59256E-91	3.09433E-91
6.0	-3.74432E-93	1.3638E-90	-1.12019E-90	4.60925E-91
6.2	-6.25326E-93	1.95977E-90	-1.64797E-90	6.86079E-91
6.4	-1.02752E-92	2.79992E-90	-2.41692E-90	1.02033E-90
6.6	-1.6631E-92	3.97735E-90	-3.53293E-90	1.51585E-90
6.8	-2.65421E-92	5.61785E-90	-5.14603E-90	2.24928E-90
7.0	-4.18056E-92	7.89037E-90	-7.46774E-90	3.33289E-90

Table 4. Values of $\zeta(t)$ for t and v (Theorem 3.4).

t	Fix $v = 0.5$ $\varsigma(t)$	Fix $v = 0.9$ $\varsigma(t)$	Fix $v = 1.3$ $\varsigma(t)$	Fix $v = 1.7$ $\varsigma(t)$
3.0	-1.19912E-86	8.05353E-83	-4.25093E-83	4.06887E-84
3.2	-2.04285E-86	1.3078E-82	-6.74102E-83	6.3153E-84
3.4	-3.39262E-86	2.09108E-82	-1.0546E-82	9.67822E-84
3.6	-5.50809E-86	3.29959E-82	-1.63157E-82	1.4679E-83
3.8	-8.76304E-86	5.14752E-82	-2.50115E-82	2.20772E-83
4.0	-1.36881E-85	7.95063E-82	-3.80537E-82	3.29792E-83
4.2	-2.10269E-85	1.21716E-81	-5.75392E-82	4.89969E-83
4.4	-3.18093E-85	1.84844E-81	-8.65595E-82	7.24804E-83
4.6	-4.74451E-85	2.78638E-81	-1.29668E-81	1.06856E-82
4.8	-6.9845E-85	4.17113E-81	-1.93563E-81	1.57123E-82
5.0	-1.01572E-84	6.20278E-81	-2.88081E-81	2.30576E-82
5.2	-1.46031E-84	9.16503E-81	-4.27637E-81	3.37866E-82
5.4	-2.07711E-84	1.34574E-80	-6.33313E-81	4.94547E-82
5.6	-2.92472E-84	1.96388E-80	-9.3586E-81	7.23339E-82
5.8	-4.07913E-84	2.84852E-80	-1.38E-80	1.05742E-81
6.0	-5.63807E-84	4.10675E-80	-2.03055E-80	1.54527E-81
6.2	-7.72639E-84	5.88531E-80	-2.98117E-80	2.25763E-81
6.4	-1.05025E-83	8.38393E-80	-4.36658E-80	3.29779E-81
6.6	-1.41661E-83	1.18728E-79	-6.37989E-80	4.81632E-81
6.8	-1.89673E-83	1.67149E-79	-9.29664E-80	7.03263E-81
7.0	-2.52179E-83	8.05353E-83	-1.35083E-79	1.0266E-80

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