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A mathematical analysis of a non-linear smoking model via fractional operators

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Abstract

Smoking is one of the most significant public health hazards that adversely affects all organs in the body and has a detrimental effect on general health. In this work, we investigate a mathematical smoking model by considering a singular and non-local Caputo operator as well as a non-singular modified Atangana-Baleanu Caputo derivative. We propose a Yang transform decomposition technique, which combines the Yang transform with the Adomian decomposition method, to obtain the analytical solution of the model. The existence of a unique solution to the model is established using the Lipschitz condition and fixed-point theory. The local asymptotic stability of the equilibrium point is also discussed. Furthermore, graphical analysis is carried out in order to demonstrate the impact of fractional order.

Keywords. Smoking model, Yang transform decomposition technique, Caputo fractional derivative, Modified Atangana-Baleanu Caputo derivative.
2010 Mathematics Subject Classification. 26A33, 33E12, 44A10, 92B05.

1. Introduction

Smoking is becoming an extremely hazardous habit globally, especially among young people. Approximately 21 million teenagers worldwide, comprising 15 million boys and 6 million girls, between the ages of 13 and 15, report being current smokers. Globally, 8% of boys and 3% of girls, or 6% of teenagers between the ages of 13 and 15 on average, smoke cigarettes [34]. There were 1.1 billion tobacco users worldwide in 2000, and that number is expected to stay there until at least 2025. Smoking causes more deaths from cardiovascular heart disease and stroke than from all other diseases. Smoking damages your immune system and causes inflammation throughout your body. Type 2 diabetes is also a result of smoking and might be more difficult to manage. Smokers are significantly more likely to suffer from heart disease, stroke, and lung cancer than non-smokers. Annually, smoking causes more fatalities than the total number of the following causes: HIV infection, alcohol use, motor vehicle injuries, and events involving firearms. E-cigarettes contain nicotine, a highly addictive substance that can harm a child's developing brain. The smoke that emanates from people smoking tobacco products into restaurants, offices, homes, and other enclosed locations is known as second-hand smoke. There's no such thing as a safe second-hand smoking exposure level. Every year, second-hand smoke prematurely claims the lives of over 1.3 million people due to severe respiratory and cardiovascular illnesses such lung cancer and coronary heart disease. The Institute for Health Metrics and Evaluation (IHME) in their annual "Global Burden of Disease 2021" [12] studied that high blood pressure, smoking and high blood sugar were the three leading risk factors globally for early death and poor health worldwide in 2021. Figure 1, shows that the IHME in their Global burden of disease study 2021, estimates that around 6.18 million people die prematurely from smoking. A risk factor is a condition or behavior that increases the likelihood of developing a given disease or injury, or an outcome such as death. Numerous initiatives have been undertaken worldwide, such as the World Health Organization's "Global Action Plan for the Prevention and Control of Non-communicable Diseases 2013-2020" [36] and

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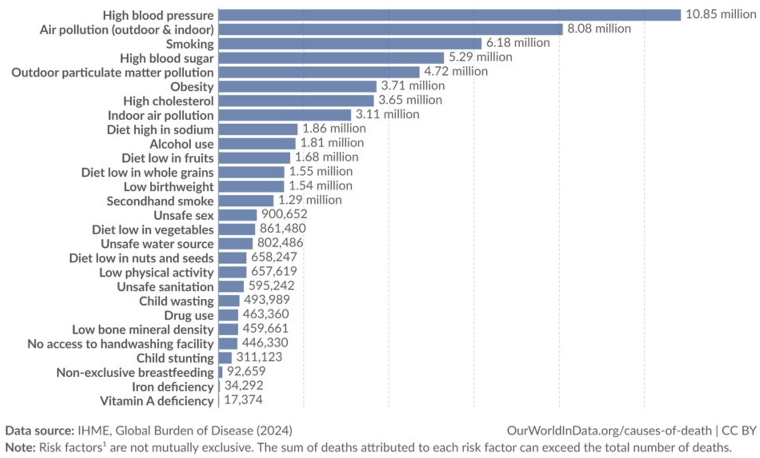


FIGURE 1. Global deaths by risk factor in 2021, with wide uncertainties, from the IHME's Global Burden of Disease Study [13].

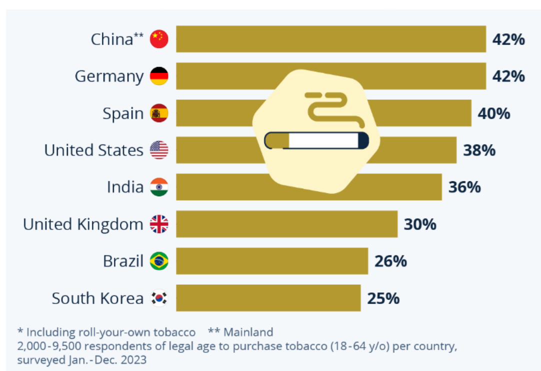


FIGURE 2. Smoking prevalence in selected countries, based on surveys of 2,000 to 9,500 legal-age respondents (18-64 years old) per country [9].

social campaigns to educate people about the risk factors of smoking. Tax increases on cigarettes and other tobacco products are associated with a considerable decrease in tobacco usage, according to research conducted globally. High-income country studies predict that a 10% rise in cigarette pricing will, over the medium term, cut general smoking by 2.5% to 5%, and possibly by twice this amount over the longer term [14]. Despite the abundant evidence of the detrimental consequences of smoking, the habit is still highly prevalent in several countries (See, Figure 2).

Giving up smoking can lengthen your life and reduce your risk of smoking-related diseases. In an effort to protect human life, numerous countries, mathematicians, and physicians are working to reduce smoking [26, 36]. This is why researchers have been trying to create various smoking models that work effectively and give the best representation of the phenomenon of cigarette smoking.

Mathematical modeling is a captivating area of research which uses equations to express a wide range of natural phenomena. Mathematical modeling has evolved into more valuable in the domain of mathematics because it permits us to transform the real world issues into mathematical language, more specifically into equations and empoly the suitable processes to forecast results. In recent years, mathematical modeling together with fractional calculus has given way for a thorough investigation of the vigorous features of numerous biological and physical models [5, 21, 29].

Over the last few decades, an enormous spike has been seen in the study of fractional calculus. More specifically, fractional calculus has been identified as an effective tool for understanding biological processes associated with memory. In biological system analysis, fractional derivatives have proven to be more successful and effective compared to integer-order derivatives. Several researchers have recently studied the fractional modeling of infectious and non-infectious diseases using a variety of fractional operators, including Riemann-Liouville, Caputo, Atangana-Baleanu Caputo, and Hilfer-fractional derivatives [11, 25]. In [19], authors have proposed the modified Homotopy analysis transform method to compute the analytical solution of the fractional giving up smoking model. In [27], authors considered non-integer order smoking model using Grünwald-Letnikov derivative with an iterative scheme. Singh et al. [28] used the non-singular Caputo-Fabrizio fractional derivative to study the giving up smoking model. The qualitative behaviour of the smoking model is discussed in [30]. In order to analyze the smoking model, Veeresha et al. [32] recently employed the q-homotpy analysis transform method in Caputo form. Furthermore, the smoking model is examined in the Atangana-Baleanu, Caputo, and Caputo-Fabrizio forms in [22] using the Natural transform decomposition approach.

Since smoking is one of the biggest health issues the world is facing at present, Smokers will often live 12–13 years shorter lives than non-smokers. Yang recently [37] introduced the concept of the Yang transform and derived the solution to a steady heat transfer equation. The Yang transform technique has been widely used in recent years by researchers to solve a wide range of fractional and partial differential equations due to its efficiency, accuracy, and straightforward approach [2, 20]. When it comes to solving nonlinear ordinary, partial, and fractional differential equations, the Adomian decomposition method (ADM) is strong and effective. It is shown in [24, 33] that the ADM is preferable to Picard's iterated technique and variational iteration technique. In this work, we thoroughly examine the fractional form of smoking model in Caputo sense as well as in modified Atangana-Baleanu Caputo (MABC) sense. This Caputo fractional derivative, which is singular and non-local, was initially described by Michele Caputo in his article [7] in 1967. The MABC derivative [3] is an extension of the Atangana-Baleanu Caputo derivative across a larger space, whose kernel has an integrable singularity at the origin. To determine the approximate solution of the model, we applied the Yang transform decomposition method, which is a spectacular amalgam of ADM and Yang transform.

The novelty of this work lies in applying the smoking model, we apply the Yang transform decomposition method. The primary purpose for using YTDM is its simple methodology, precision, and efficiency for dealing with fractional non-linear systems. In this method, requires no liberarlization or perturbation, and it essentially transforms the given problem into its simplest form. Using the Caputo and MABC operators, we impose fractional derivatives on the non-linear smoking model. As a result, we have a greater understanding about the way singular and non-singular operators affect the behaviour of each subclass within the population, as well as the way fractional order affects the disease. A comparison between these operators has been emphasised through simulations. This analytical technique investigates the smoking model efficiently. Many theoretical aspects of the model have been studied, such as its existence, uniqueness, equilibria, and reproduction number.

The complete work is structured and divided into the following sections: Section 2 provides fundamental definitions for the Yang transform and fractional calculus. In section 3, the integer-order smoking model is covered, and the flow chart of our proposed scheme is given. In section 4, Equilibrium points, reproduction number, and stability analysis of the model have been discussed. Section 5 is further divided into subsections 5.1 and 5.2. Subsection 5.1 presents a numerical solution to the fractional smoking model in Caputo form. It further includes the existence and uniqueness of the model solution. Subsection 5.2 contains a numerical solution to the fractional smoking model in MABC form. The uniqueness and existence of the model solution are also given. In section 6, the findings from the research are graphically analyzed and discussed. Section 7 presents the conclusions of our work.

2. Preliminaries

Definition 2.1. A real valued function f(t), $t > 0$ is said to be in space C_v, if there exists a real number $k > v$ such that $f(t) = t^k f_1(t)$, where $f_1(t) \in C[0, \infty)$, and it is said to be in the space C_v^n if $f^n \in C_v, n \in \mathbb{N}$.

Definition 2.2. The Riemann-Liouville fractional integral of a function $f(t) \in C_v (v \geq -1)$ and of order $\mu > 0$, is defined as follows [23]:

$$I_t^{\mu} f(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-y)^{\mu-1} f(y)\, dy, \tag{2.1}$$

$$I_t^0 f(t) = f(t). \tag{2.2}$$

Definition 2.3. The Caputo derivative of $f(t)$ of order μ, $p-1<\mu\le p$, $p\in\mathbb{N}$ is given as follows [7]:

$$^{C}D_t^{\mu} f(t) = \frac{1}{\Gamma(p-\mu)}\int_0^t (t-y)^{p-\mu-1} f^{(p)}(y)\,dy. \tag{2.3}$$

The properties of the Caputo fractional derivative are as follows [17, 23]:

$$(^{C}D_t^{\mu}\ I_t^{\mu} f)(t) = f(t), \tag{2.4}$$

$$(I_t^{\mu}\ {}^{C}D_t^{\mu} f)(t) = f(t) - \sum_{k=0}^{p-1} f^{(k)}(0^+)\frac{t^k}{k!}. \tag{2.5}$$

Definition 2.4. The Atangana-Baleanu fractional derivative of order $0<\mu<1$ of Caputo sense, with a non-singular kernel is defined as follows [4]:

$$^{ABC}D_t^{\mu}[f(t)] = \frac{B[\mu]}{1-\mu}\int_0^t E_{\mu}\left(\frac{-\mu(t-y)^{\mu}}{1-\mu}\right) f'(y)\,dy, \tag{2.6}$$

where $B[\mu]$ is a normalization function.

The MABC fractional derivative is an extension of ABC fractional derivative in a wider space. The kernel of the MABC derivative has an integrable singularity at the origin.

Definition 2.5. The MABC derivative of the function f(t) with order $\mu\in(0,1)$ defined as [3]:

$$^{MABC}D_t^{\mu}[f(t)] = \frac{B[\mu]}{1-\mu}\left[f(t) - E_{\mu}\left(\frac{-\mu\, t^{\mu}}{1-\mu}\right)f(0) - \frac{\mu}{1-\mu}\int_0^t (t-y)^{\mu-1}E_{\mu,\mu}\left(\frac{-\mu\,(t-y)^{\mu}}{1-\mu}\right)f(y)\,dy\right]. \tag{2.7}$$

Moreover, E_{μ} represents the Mittag-Leffler function [18] and $E_{\mu,\rho}$ is known as the two-parameter Mittag-Leffler function [35], defined as follows:

$$E_{\mu}(y) = \sum_{m=0}^{\infty}\frac{y^m}{\Gamma(\mu m+1)},\ (y,\mu\in\mathbb{C},\Re(\mu)>0),$$

and

$$E_{\mu,\rho}(y) = \sum_{m=0}^{\infty}\frac{y^m}{\Gamma(\mu m+\rho)},\ (y,\mu,\rho\in\mathbb{C},\Re(\mu)>0,\Re(\rho)>0).$$

Definition 2.6. The modified Atangana-Baleanu fractional integral defined as follows [3, 6]:

$$^{MAB}I_t^{\mu}[f(t)] = \frac{1-\mu}{B[\mu]}\left[f(t) + \frac{\mu}{1-\mu}\ {}_0^{RL}I_t^{\mu}[f(t)] - f(0) - f(0)\frac{\mu}{1-\mu}\frac{t^{\mu}}{\Gamma(\mu+1)}\right]. \tag{2.8}$$

Definition 2.7. The Yang transform of a function $f(t)$, where $t>0$, is given by [38]:

$$Y[f(t)] = \int_0^{\infty} e^{\frac{-t}{\xi}} f(t)\,dt, \tag{2.9}$$

where ξ represents the transformed variable.
The Yang transform of some useful functions is given as follows [8]:

$$Y[1] = \xi,$$
$$Y[t] = \xi^2,$$
$$Y[t^q] = \Gamma(q+1)\xi^{q+1}.$$

Definition 2.8. The Yang transform of Caputo derivative is given as [16, 37]:

$$Y[^{C}D_t^{\mu} f(t)] = \frac{Y[f(t)]}{\xi^{\mu}} - \sum_{k=0}^{n-1}\frac{f^k(0)}{\xi^{\mu-k-1}},\quad (n-1<\mu\le n). \tag{2.10}$$

Laplace Yang Duality: If the Laplace transform of the function $f(t)$ is $F(u)$, then the Yang transform of a function $f(t)$ is given as [8]:

$$Y[f(t)] = F(1/u).$$

Lemma 2.9. *For a continuous function $f(t)$, the Yang transform of the modified Atangana-Baleanu Caputo fractional derivative of $f(t)$ is given as*

$$Y[^{MABC}D_t^{\mu} f(t)] = \frac{B[\mu]\xi}{1-\mu+\mu\xi^{\mu}}\left(\frac{Y[f(t)]}{\xi} - f(0)\right). \tag{2.11}$$

Proof. The Laplace transform of the modified Atangana-Baleanu Caputo derivative is given as [3]

$$L[^{MABC}D_t^{\mu} f(t)] = \frac{B[\mu]}{1-\mu}\frac{\xi^{\mu}L[f] - \xi^{\mu-1}f(0)}{\xi^{\mu} + \frac{\mu}{1-\mu}}.$$

Using the Laplace-Yang Duality property, we have

$$Y[^{MABC}D_t^{\mu} f(t)] = \frac{B[\mu]}{1-\mu}\frac{(\frac{1}{\xi})^{\mu}Y[f] - (\frac{1}{\xi})^{\mu-1}f(0)}{(\frac{1}{\xi})^{\mu} + \frac{\mu}{1-\mu}},$$

$$Y[^{MABC}D_t^{\mu} f(t)] = \frac{B[\mu]}{1-\mu+\mu\xi^{\mu}}\{Y[f] - \xi f(0)\}.$$

After simplification, we get the desired result. □

3. Smoking Model and Proposed Methodology

To comprehend biological issues, it is both essential and beneficial to develop mathematical models and analyze their dynamical behaviours. The smoking mathematical model has been the subject of extensive analysis and investigation. The system consists of five non-linear differential equations that describes the smoking model is taken into account in this work. Let P(t) be the total population at time t. This population is further divided into five groups: Potential smokers (Non-smoker) L(t), Occasional smokers R(t), Smokers M(t), Temporarily quit smokers S(t), Permanently quit smokers A(t). The mathematical representation of the smoking model [1, 30] consists of a set of non-linear differential equations as follows:

$$\begin{cases} \frac{dL}{dt} = \Upsilon - \varpi LM - vL, \\ \frac{dR}{dt} = \varpi LM - \kappa_1 R - vR, \\ \frac{dM}{dt} = \kappa_1 R + \kappa_2 MS - (v+\varphi)M, \\ \frac{dS}{dt} = -\kappa_2 MS - vS + \varphi(1-\varsigma)M, \\ \frac{dA}{dt} = \varsigma\varphi M - vA. \end{cases} \tag{3.1}$$

In the above model, Υ represents the recruitment rate in potential smokers, ϖ denotes the effective contact rate between smokers and potential smokers, v indicates the rate of natural death, φ represents the rate of quitting smoking, $(1-\varsigma)$ represents the fraction of smokers who temporarily quit smoking, ς symbolizes the remaining fraction of smoking who successfully gave up smoking, κ_1 stands for the rate of transition from occassional smokers to regular smokers, κ_2 indicates the interaction ratio between smokers and temporary quitters who resume smoking.

The flow chart (See, Figure 3) of our proposed methodology is given as follows:

4. Equilibrium Points and Stability Analysis

In this section, we investigate the equilibrium points, reproduction number and stability analysis of the smoking model (3.1).

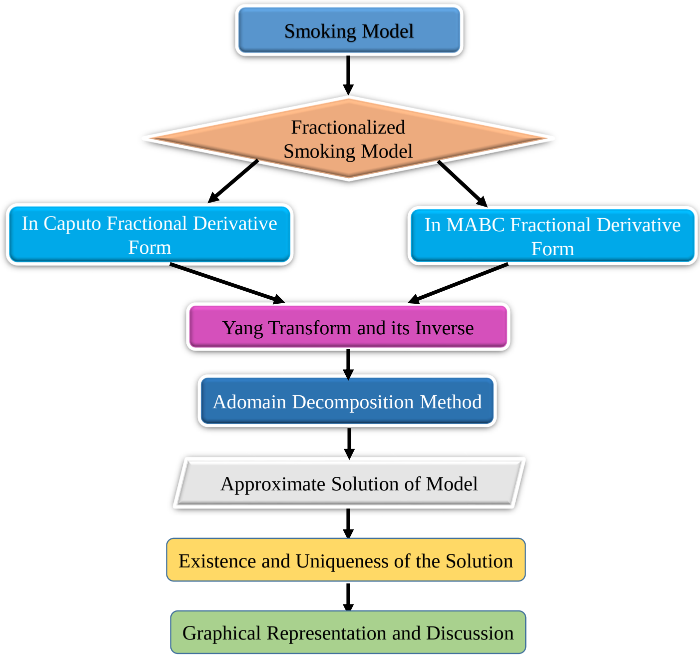


FIGURE 3. Flow chart of proposed methodoloy.

4.1. **Equilibrium Points.** The system (3.1) has two equilibria, namely disease-free equilibria and endemic equilibria. The disease free equilibrium can be defined as the point at which there is no disease in the population. To find the equilibrium points, we consider

$$\frac{d}{dt}L = \frac{d}{dt}R = \frac{d}{dt}M = \frac{d}{dt}S = \frac{d}{dt}A = 0,$$

The disease-free equilibrium is obtained when $R^0 = M^0 = 0$,

$$\Upsilon - vL^0 = 0.$$

Therefore,

$$L^0 = \frac{\Upsilon}{v}.$$

We have, the disease-free equilibrium point $E_0 = (\frac{\Upsilon}{v}, 0, 0, 0, 0)$.

The endemic equilibrium point is

$$E^* = (L^*, R^*, M^*, S^*, A^*),$$

where,

$$L^* = \frac{\Upsilon}{\varpi M^* + v}, \ R^* = \frac{\varpi \Upsilon M^*}{(\varpi M^* + v)(\kappa_1 + v)}, \ S^* = \frac{\varphi(1-\varsigma)M^*}{\kappa_2 M^* + v}, \ A^* = \frac{\varsigma\varphi M^*}{v}.$$

4.2. **Reproduction Number.** Reproduction number $\mathbf{R}_0$ is the average number of secondary infections produced by one infected individual introduced in a susceptible population throughout the individual duration of infectiousness. We calculate the reproduction number of the model by applying the next-generation matrix technique [31]. For the model (3.1), the infected compartments are R and M. At the disease-free equilibrium matrices F and V are

$$F = \begin{bmatrix} 0 & \frac{\varpi\Upsilon}{v} \\ 0 & 0 \end{bmatrix}, \quad and \quad V = \begin{bmatrix} (\kappa_1 + v) & 0 \\ -\kappa_1 & (v+\varphi) \end{bmatrix}, FV^{-1} = \begin{bmatrix} \frac{\varpi\kappa_1\Upsilon}{v(v+\varphi)(\kappa_1+v)} & \frac{\varpi\Upsilon}{v(v+\varphi)} \\ 0 & 0 \end{bmatrix}.$$

So, FV^{-1} has eigenvalues 0 and $\mathbf{R}_0$, where

$$\mathbf{R}_0 = \frac{\varpi\kappa_1\Upsilon}{v(v+\varphi)(\kappa_1+v)}.$$

4.3. **Stability Analysis.**

Theorem 4.1. *The disease free equilibrium is locally asymptotically stable if $\boldsymbol{R}_0 < 1$ and unstable if $\boldsymbol{R}_0 > 1$.*

Proof. For the smoking model (3.1) at $E_0 = (\frac{\Upsilon}{v}, 0, 0, 0, 0)$, the Jacobian matrix is given by

$$J = \begin{bmatrix} -v & 0 & \frac{-\varpi\Upsilon}{v} & 0 & 0 \\ 0 & -(\kappa_1+v) & \frac{\varpi\Upsilon}{v} & 0 & 0 \\ 0 & \kappa_1 & -(v+\varphi) & 0 & 0 \\ 0 & 0 & \varphi(1-\varsigma) & -v & 0 \\ 0 & 0 & \varsigma\varphi & 0 & -v \end{bmatrix}.$$

At equilibrium point $E_0 = (\frac{\Upsilon}{v}, 0, 0, 0, 0)$, the system is stable if all the eigenvalues have negative real parts.
The characteristic equation is

$$(-v-\lambda)^3(\lambda^2 + (v+\varphi+\kappa_1+v)\lambda + ((v+\varphi)(\kappa_1+v) - \frac{\varpi\kappa_1\Upsilon}{v})) = 0. \tag{4.1}$$

We have the quadratic equation

$$(\lambda^2 + (v+\varphi+\kappa_1+v)\lambda + ((v+\varphi)(\kappa_1+v) - \frac{\varpi\kappa_1\Upsilon}{v})) = 0,$$

where, sum of the roots $= -(v+\varphi+\kappa_1+v) < 0$. It follows that the roots of the quadratic equation have negative real parts if product of the roots $= ((v+\varphi)(\kappa_1+v) - \frac{\varpi\kappa_1\Upsilon}{v}) > 0$. Therefore, all the roots of Equation (4.1) have negative real parts if

$$(v+\varphi)(\kappa_1+v) - \frac{\varpi\kappa_1\Upsilon}{v} > 0,$$

if

$$\frac{\varpi\kappa_1\Upsilon}{v(v+\varphi)(\kappa_1+v)} < 1,$$

i.e,

$$\mathbf{R}_0 < 1.$$

Therefore, the disease-free equilibrium is locally asymptotically stable for $\mathbf{R}_0 < 1$, and unstable otherwise. □

5. Smoking Model in Fractional Orders

This section explores the dynamics of smoking behaviour through fractional calculus. This approach allows for a more nuanced understanding of the memory and hereditary properties inherent in smoking patterns. The Caputo derivative provides a more accurate depiction of real-world scenarios. Additionally, the MABC derivative offers a new perspective, increasing the model's flexibility and applicability. Together, these derivatives provide a comprehensive framework for analyzing and predicting smoking behavior over time, facilitating more effective public health interventions.

5.1. **Fractionalized Smoking Model in Caputo Form and its Solution.** We employ the Caputo fractional order derivative to examine smoking model in system (3.1). In the system, dimensional inconsistency has been prevented by taking into account a parameter γ (see [6, 10]). Therefore, the fractional differential system in Caputo form is given by

$$\begin{cases} \frac{1}{\gamma^{1-\mu}} \ {}^C D_t^\mu L = \Upsilon - \varpi LM - vL, \\ \frac{1}{\gamma^{1-\mu}} \ {}^C D_t^\mu R = \varpi LM - \kappa_1 R - vR, \\ \frac{1}{\gamma^{1-\mu}} \ {}^C D_t^\mu M = \kappa_1 R + \kappa_2 MS - (v+\varphi)M, \\ \frac{1}{\gamma^{1-\mu}} \ {}^C D_t^\mu S = -\kappa_2 MS - vS + \varphi(1-\varsigma)M, \\ \frac{1}{\gamma^{1-\mu}} \ {}^C D_t^\mu A = \varsigma\varphi M - vA, \end{cases} \tag{5.1}$$

with the initial conditions L(0) $= L_0$, R(0) $= R_0$, M(0) $= M_0$, S(0) $= S_0$, A(0) $= A_0$.

5.1.1. *Yang Transform Decomposition Method.* Firstly, by applying the Yang transform in the system of Equations (5.1), we get

$$\begin{cases} \frac{1}{\xi^\mu}[Y[L(t)] - \xi L(0)] = Y[\gamma^{1-\mu}\{\Upsilon - \varpi LM - vL\}], \\ \frac{1}{\xi^\mu}[Y[R(t)] - \xi R(0)] = Y[\gamma^{1-\mu}\{\varpi LM - \kappa_1 R - vR\}], \\ \frac{1}{\xi^\mu}[Y[M(t)] - \xi M(0)] = Y[\gamma^{1-\mu}\{\kappa_1 R + \kappa_2 MS - (v+\varphi)M\}], \\ \frac{1}{\xi^\mu}[Y[S(t)] - \xi S(0)] = Y[\gamma^{1-\mu}\{-\kappa_2 MS - vS + \varphi(1-\varsigma)M\}], \\ \frac{1}{\xi^\mu}[Y[A(t)] - \xi A(0)] = Y[\gamma^{1-\mu}\{\varsigma\varphi M - vA\}]. \end{cases} \tag{5.2}$$

Then applying the Yang inverse transform in the system of Equations (5.2), we get

$$\begin{cases} L(t) = Y^{-1}\big[\xi L(0) + \xi^\mu Y\big[\gamma^{1-\mu}\{\Upsilon - \varpi LM - vL\}\big]\big], \\ R(t) = Y^{-1}\big[\xi R(0) + \xi^\mu Y\big[\gamma^{1-\mu}\{\varpi LM - \kappa_1 R - vR\}\big]\big], \\ M(t) = Y^{-1}\big[\xi M(0) + \xi^\mu Y\big[\gamma^{1-\mu}\{\kappa_1 R + \kappa_2 MS - (v+\varphi)M\}\big]\big], \\ S(t) = Y^{-1}\big[\xi S(0) + \xi^\mu Y\big[\gamma^{1-\mu}\{-\kappa_2 MS - vS + \varphi(1-\varsigma)M\}\big]\big], \\ A(t) = Y^{-1}\big[\xi A(0) + \xi^\mu Y\big[\gamma^{1-\mu}\{\varsigma\varphi M - vA\}\big]\big]. \end{cases} \tag{5.3}$$

Now, the Adomian decomposition polynomials gives the nonlinear terms as follows:

$$LM = \sum_{k=0}^{\infty} C_k, \quad MS = \sum_{k=0}^{\infty} D_k, \tag{5.4}$$

where C_k and D_k are both Adomian polynomials. The unknown functions are given by the infinite series as follows:

$$L(t) = \sum_{k=0}^{\infty} L_k, \ R(t) = \sum_{k=0}^{\infty} R_k, \ M(t) = \sum_{k=0}^{\infty} M_k, \ S(t) = \sum_{k=0}^{\infty} S_k, \ A(t) = \sum_{k=0}^{\infty} A_k. \tag{5.5}$$

Using (5.4) and (5.5) in Eq. (5.3), we have

$$\begin{cases} \sum_{k=0}^{\infty} L_k = Y^{-1}[L_0\xi] + Y^{-1}\big[\xi^\mu Y\big\{\gamma^{1-\mu}\big[\Upsilon - \varpi\sum_{k=0}^{\infty}\sum_{j=0}^{k} L_j M_{k-j} - v\sum_{k=0}^{\infty} L_k\big]\big\}\big], \\ \sum_{k=0}^{\infty} R_k = Y^{-1}[R_0\xi] + Y^{-1}\big[\xi^\mu Y\big\{\gamma^{1-\mu}\big[\varpi\sum_{k=0}^{\infty}\sum_{j=0}^{k} L_j M_{k-j} - \kappa_1\sum_{k=0}^{\infty} R_k - v\sum_{k=0}^{\infty} R_k\big]\big\}\big], \\ \sum_{k=0}^{\infty} M_k = Y^{-1}[M_0\xi] + Y^{-1}\big[\xi^\mu Y\big\{\gamma^{1-\mu}\big[\kappa_1\sum_{k=0}^{\infty} R_k + \kappa_2\sum_{k=0}^{\infty}\sum_{j=0}^{k} M_j S_{k-j} - (v+\varphi)\sum_{k=0}^{\infty} M_k\big]\big\}\big], \\ \sum_{k=0}^{\infty} S_k = Y^{-1}[S_0\xi] + Y^{-1}\big[\xi^\mu Y\big\{\gamma^{1-\mu}\big[-\kappa_2\sum_{k=0}^{\infty}\sum_{j=0}^{k} M_j S_{k-j} - v\sum_{k=0}^{\infty} S_k + \varphi(1-\varsigma)\sum_{k=0}^{\infty} M_k\big]\big\}\big], \\ \sum_{k=0}^{\infty} A_k = Y^{-1}[A_0\xi] + Y^{-1}\big[\xi^\mu Y\big\{\gamma^{1-\mu}\big[\varsigma\varphi\sum_{k=0}^{\infty} M_k - v\sum_{k=0}^{\infty} A_k\big]\big\}\big]. \end{cases} \tag{5.6}$$

From Eq. (5.6), we get

$${}^C L_0 = 40, \quad {}^C R_0 = 10, \quad {}^C M_0 = 20, \quad {}^C S_0 = 10, \quad {}^C A_0 = 5.$$

$$\begin{cases} {}^{C}L_1 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{\Upsilon - \varpi L_0M_0 - vL_0\}\big]\big], \\ {}^{C}R_1 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{\varpi L_0M_0 - \kappa_1R_0 - vR_0\}\big]\big], \\ {}^{C}M_1 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{\kappa_1R_0 + \kappa_2M_0S_0 - (v+\varphi)M_0\}\big]\big], \\ {}^{C}S_1 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{-\kappa_2M_0S_0 - vS_0 + \varphi(1-\varsigma)M_0\}\big]\big], \\ {}^{C}W_1 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{\varsigma\varphi M_0 - vA_0\}\big]\big], \end{cases} \tag{5.7}$$

and

$$\begin{cases} {}^{C}L_2 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{\Upsilon - \varpi(L_0M_1 + L_1M_0) - vL_1\}\big]\big], \\ {}^{C}R_2 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{\varpi(L_0M_1 + L_1M_0) - \kappa_1R_1 - vR_1\}\big]\big], \\ {}^{C}M_2 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{\kappa_1R_1 + \kappa_2(M_0S_1 + M_1S_0) - (v+\varphi)M_1\}\big]\big], \\ {}^{C}S_2 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{-\kappa_2(M_0S_1 + M_1S_0) - vS_1 + \varphi(1-\varsigma)M_1\}\big]\big], \\ {}^{C}A_2 = Y^{-1}\big[\xi^{\mu}Y\big[\gamma^{1-\mu}\{\varsigma\varphi M_1 - vA_1\}\big]\big]. \end{cases} \tag{5.8}$$

Using the above equations, we get the series solution

$$\begin{cases} {}^{C}L(t) = {}^{C}L_0 + {}^{C}L_1 + {}^{C}L_2 + \dots, \\ {}^{C}R(t) = {}^{C}R_0 + {}^{C}R_1 + {}^{C}R_2 + \dots, \\ {}^{C}M(t) = {}^{C}M_0 + {}^{C}M_1 + {}^{C}M_2 + \dots, \\ {}^{C}S(t) = {}^{C}S_0 + {}^{C}S_1 + {}^{C}S_2 + \dots, \\ {}^{C}A(t) = {}^{C}A_0 + {}^{C}A_1 + {}^{C}A_2 + \dots. \end{cases} \tag{5.9}$$

5.1.2. *Existence and Uniqueness of the Solution.* In this section, we will show the system (5.1) has a unique solution. Here, we present the existence and uniqueness of smoking model solution in Caputo form, as shown by the technique described in [15]. Applying the fractional integral and using initial condition, we have

$$L(t) = L(0) + \frac{\gamma^{1-\mu}}{\Gamma(\mu)}\int_0^t (t-\zeta)^{\mu-1}[\Upsilon - \varpi LM - vL]\, d\zeta, \tag{5.10}$$

$$R(t) = R(0) + \frac{\gamma^{1-\mu}}{\Gamma(\mu)}\int_0^t (t-\zeta)^{\mu-1}[\varpi LM - \kappa_1R - vR]\, d\zeta, \tag{5.11}$$

$$M(t) = M(0) + \frac{\gamma^{1-\mu}}{\Gamma(\mu)}\int_0^t (t-\zeta)^{\mu-1}[\kappa_1R + \kappa_2MS - (v+\varphi)M]\, d\zeta, \tag{5.12}$$

$$S(t) = S(0) + \frac{\gamma^{1-\mu}}{\Gamma(\mu)}\int_0^t (t-\zeta)^{\mu-1}[-\kappa_2MS - vS + \varphi(1-\varsigma)M]\, d\zeta, \tag{5.13}$$

$$A(t) = A(0) + \frac{\gamma^{1-\mu}}{\Gamma(\mu)}\int_0^t (t-\zeta)^{\mu-1}[\varsigma\varphi M - vA]\, d\zeta. \tag{5.14}$$

We have the following Kernels,

$$\begin{aligned} \Theta_1(t,L) &= \Upsilon - \varpi LM - vL, \\ \Theta_2(t,R) &= \varpi LM - \kappa_1R - vR, \\ \Theta_3(t,M) &= \kappa_1R + \kappa_2MS - (v+\varphi)M, \\ \Theta_4(t,S) &= -\kappa_2MS - vS + \varphi(1-\varsigma)M, \\ \Theta_5(t,A) &= \varsigma\varphi M - vA. \end{aligned}$$

Theorem 5.1. *Θ_1, Θ_2, Θ_3, Θ_4 and Θ_5 satisfy the Lipschitz condition. For each kernel $\Theta_j, j = 1\dots,5$, there exists $\aleph_j$ $j = 1,\dots,5$ such that*

$$\|\Theta_j(t,z) - \Theta_j(t,z_1)\| \le \aleph_j\|z(t) - z_1(t)\|,$$

where z is the state vector presented as $\{L,R,M,S,A\}$ and are contractions for $0 \le \aleph_j < 1$.

Proof. Firstly we show that Θ_1 satisfies the Lipschitz condition. let L and L_1 be two functions. then

$$\|\Theta_1(t, L) - \Theta_1(t, L_1)\| = \|(\Upsilon - \varpi LM - vL) - (\Upsilon - \varpi L_1 M - vL_1)\|. \tag{5.15}$$

Using Cauchy's inequality, we have

$$\|\Theta_1(t, L) - \Theta_1(t, L_1)\| \leq \|v + \varpi M(t)\| \quad \|L(t) - L_1(t)\|,$$
$$\|\Theta_1(t, L) - \Theta_1(t, L_1)\| \leq \quad \aleph_1 \|L(t) - L_1(t)\|, \tag{5.16}$$

where

$$\|v + \varpi M(t)\| \leq \aleph_1.$$

So, Θ_1 satisfies the Lipschitz condition and contraction for $0 \leq \aleph_1 < 1$. In a similar way, we can prove this for other kernels also.

Now, we take recursive formula

$$L_l(t) = \frac{\gamma^{1-\mu}}{\Gamma(\mu)} \int_0^t \Theta_1(\zeta, L_{l-1})(t - \zeta)^{\mu-1} \, d\zeta.$$

Now, consider the successive difference of two terms

$$L_l(t) - L_{l-1}(t) = \frac{\gamma^{1-\mu}}{\Gamma(\mu)} \int_0^t (t - \zeta)^{\mu-1} (\Theta_1(\zeta, L_{l-1}) - \Theta_1(\zeta, L_{l-2})) \, d\zeta. \tag{5.17}$$

By taking the norm on both sides of Eq. (5.17) and further solving it, we get

$$\begin{aligned} \|L_l(t) - L_{l-1}(t)\| &\leq \frac{\gamma^{1-\mu}}{\Gamma(\mu)} \int_0^t |(t - \zeta)^{\mu-1}| \, \|(\Theta_1(\zeta, L_{l-1}) - \Theta_1(\zeta, L_{l-2}))\| \, d\zeta \\ &\leq \aleph_1 \gamma^{1-\mu} \|(L_{l-1} - L_{l-2})(t)\| \left| \frac{t^\mu}{\Gamma(\mu + 1)} \right|. \end{aligned} \tag{5.18}$$

Therefore, we get

$$\|L_l(t) - L_{l-1}(t)\| \leq \aleph_1 \gamma^{1-\mu} \left| \frac{t^\mu}{\Gamma(\mu + 1)} \right| \, \|(L_{l-1}(t) - L_{l-2}(t))\|. \tag{5.19}$$

Similarly, we can show this for all the other kernels. □

Theorem 5.2. *[15] The Caputo fractional model given by Eq. (5.1) has a solution if we can find η satisfying the inequality*

$$\frac{\gamma^{1-\mu} \, \eta^\mu}{\Gamma(\mu + 1)} \aleph_j < 1, \qquad j = 1, 2, \ldots, 5.$$

Proof. Since equations are bounded and kernels satisfy Lipschitz condition, as a result of recursive technique and from Eq. (5.19), we have

$$\|L_l(t)\| \leq \|L(0)\| \left\{ \frac{\gamma^{1-\mu} \, \eta^\mu}{\Gamma(\mu + 1)} \aleph_1 \right\}^l. \tag{5.20}$$

Suppose for now that the following are satisfied:

$$L(t) - L(0) = L_l(t) - P_l(t).$$

Thus, we have

$$\|P_l(t)\| = \frac{\gamma^{1-\mu}}{\Gamma\mu} \int_0^t \|(\Theta_1(\zeta, L_l) - \Theta_1(\zeta, L_{l-1}))\| \quad |(t - \zeta)^{\mu-1}| \, d\zeta,$$

$$\|P_l(t)\| \leq \frac{\gamma^{1-\mu} \, t^\mu}{\Gamma(\mu + 1)} \aleph_1 \|(L_l - L_{l-1})\|.$$

Recursively repeating the process, we get

$$\|P_l(t)\| \leq \left\{ \frac{\gamma^{1-\mu}\ t^{\mu}}{\Gamma(\mu+1)} \right\}^l \aleph_1^l \|(L_1 - L_0)\|.$$

Taking $t = \eta$

$$\|P_l(t)\| \leq \left\{ \frac{\gamma^{1-\mu}\ \eta^{\mu}}{\Gamma(\mu+1)} \right\}^l \aleph_1^l \|(L_1 - L_0)\|.$$

Thus at applying the limit to both sides as $l \to \infty$, we see that $\|P_l(t)\| \to 0$ for

$$\frac{\gamma^{1-\mu}\ \eta^{\mu}}{\Gamma(\mu+1)} \aleph_1 < 1.$$

In a similar way, we can prove this for other kernels. Theorems 5.1 and 5.2 guarantes the existence of solution of model by the Banach fixed point theorem. □

Theorem 5.3. *[16] The Caputo fractionalized smoking model has a unique solution if*

$$\left| \frac{\gamma^{1-\mu}\ \eta^{\mu}}{\Gamma(\mu+1)} \right| \aleph_j < 1, \qquad j = 1, 2, \ldots, 5.$$

Proof. We suppose that there is another set of solution for the Eq. (5.10),

$$L(t) - L_1(t) = \frac{\gamma^{1-\mu}}{\Gamma(\mu)} \int_0^t (t-\zeta)^{\mu-1} (\Theta_1(\zeta, L) - \Theta_1(\zeta, L_1))\, d\zeta, \tag{5.21}$$

taking the norm on both side of Eq. (5.21), we get

$$\|L(t) - L_1(t)\| = \frac{\gamma^{1-\mu}}{\Gamma(\mu)} \int_0^t |(t-\zeta)^{\mu-1}|\ \|(\Theta_1(\zeta, L) - \Theta_1(\zeta, L_1))\|\, d\zeta,$$

$$\|L(t) - L_1(t)\| \leq \left| \frac{\gamma^{1-\mu}\ t^{\mu}}{\Gamma(\mu+1)} \right| \aleph_1\ \|(L - L_1)\|.$$

Since,

$$1 - \frac{\gamma^{1-\mu}\ t^{\mu}}{\Gamma(\mu+1)} \aleph_1 > 0.$$

Therefore, we have

$$L(t) = L_1(t),$$

similarly,

$$R(t) = R_1(t),$$

$$M(t) = M_1(t),$$

$$S(t) = S_1(t),$$

and

$$A(t) = A_1(t).$$

Therefore, the system has a unique solution. □

5.2. **Fractionalized Smoking Model in Modified Atangana-Baleanu Caputo Form and its Solution.** We employ the modified Atangana-Baleanu Caputo fractional order derivative to examine smoking model in system (3.1). In the system, dimensional inconsistency has been prevented by taking into account a parameter γ (see [6, 10]). The fractional differential system is given by

$$\begin{cases} \frac{1}{\gamma^{1-\mu}} \ {}^{MABC}D_t^{\mu} L = \Upsilon - \varpi LM - vL, \\ \frac{1}{\gamma^{1-\mu}} \ {}^{MABC}D_t^{\mu} R = \varpi LM - \kappa_1 R - vR, \\ \frac{1}{\gamma^{1-\mu}} \ {}^{MABC}D_t^{\mu} M = \kappa_1 R + \kappa_2 MS - (v+\varphi)M, \\ \frac{1}{\gamma^{1-\mu}} \ {}^{MABC}D_t^{\mu} S = -\kappa_2 MS - vS + \varphi(1-\varsigma)M, \\ \frac{1}{\gamma^{1-\mu}} \ {}^{MABC}D_t^{\mu} A = \varsigma\varphi M - vA, \end{cases} \tag{5.22}$$

with the initial conditions $L(0) = L_0,\ R(0) = R_0,\ M(0) = M_0,\ S(0) = S_0,\ A(0) = A_0$.

5.2.1. *Yang Transform Decomposition Method.* Firstly, by applying the Yang transform in the system of Equations (5.22), we get

$$\begin{cases} \frac{B[\mu]\xi}{1-\mu+\mu\xi^{\mu}} \left[\frac{Y[L(t)]}{\xi} - L(0) \right] = Y[\gamma^{1-\mu}\{\Upsilon - \varpi LM - vL\}], \\ \frac{B[\mu]\xi}{1-\mu+\mu\xi^{\mu}} \left[\frac{Y[R(t)]}{\xi} - R(0) \right] = Y[\gamma^{1-\mu}\{\varpi LM - \kappa_1 R - vR\}], \\ \frac{B[\mu]\xi}{1-\mu+\mu\xi^{\mu}} \left[\frac{Y[M(t)]}{\xi} - M(0) \right] = Y[\gamma^{1-\mu}\{\kappa_1 R + \kappa_2 MS - (v+\varphi)M\}], \\ \frac{B[\mu]\xi}{1-\mu+\mu\xi^{\mu}} \left[\frac{Y[S(t)]}{\xi} - S(0) \right] = Y[\gamma^{1-\mu}\{-\kappa_2 MS - vS + \varphi(1-\varsigma)M\}], \\ \frac{B[\mu]\xi}{1-\mu+\mu\xi^{\mu}} \left[\frac{Y[A(t)]}{\xi} - A(0) \right] = Y[\gamma^{1-\mu}\{\varsigma\varphi M - vA\}]. \end{cases} \tag{5.23}$$

Operating the inverse of Yang transform in system of Equations (5.23), we get

$$\begin{cases} L(t) = Y^{-1}\left[\xi L(0) + \frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{\Upsilon - \varpi LM - vL\} \right] \right], \\ R(t) = Y^{-1}\left[\xi R(0) + \frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{\varpi LM - \kappa_1 R - vR\} \right] \right], \\ M(t) = Y^{-1}\left[\xi M(0) + \frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{\kappa_1 R + \kappa_2 MS - (v+\varphi)M\} \right] \right], \\ S(t) = Y^{-1}\left[\xi S(0) + \frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{-\kappa_2 MS - vS + \varphi(1-\varsigma)M\} \right] \right], \\ A(t) = Y^{-1}\left[\xi A(0) + \frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{\varsigma\varphi M - vA\} \right] \right]. \end{cases} \tag{5.24}$$

The nonlinear terms and infinite series solution of Adomian decomposition polynomials are given in Eqs. (5.4) and (5.5). Substituting the values of (5.4) and (5.5) in Eq. (5.24), we have

$$\begin{cases} \sum_{k=0}^{\infty} L_k = Y^{-1}[L_0\xi] + Y^{-1}\left[\frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{\Upsilon - \varpi \sum_{k=0}^{\infty}\sum_{j=0}^{k} L_j M_{k-j} - v\sum_{k=0}^{\infty} L_k\} \right] \right], \\ \sum_{k=0}^{\infty} R_k = Y^{-1}[R_0\xi] + Y^{-1}\left[\frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{\varpi \sum_{k=0}^{\infty}\sum_{j=0}^{k} L_j M_{k-j} - \kappa_1 \sum_{k=0}^{\infty} R_k - v\sum_{k=0}^{\infty} R_k\} \right] \right], \\ \sum_{k=0}^{\infty} M_k = Y^{-1}[M_0\xi] + Y^{-1}\left[\frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{\kappa_1 \sum_{k=0}^{\infty} R_k + \kappa_2 \sum_{k=0}^{\infty}\sum_{j=0}^{k} M_j S_{k-j} - (v+\varphi)\sum_{k=0}^{\infty} M_k\} \right] \right], \\ \sum_{k=0}^{\infty} S_k = Y^{-1}[S_0\xi] + Y^{-1}\left[\frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{-\kappa_2 \sum_{k=0}^{\infty}\sum_{j=0}^{k} M_j S_{k-j} - v\sum_{k=0}^{\infty} S_k + \varphi(1-\varsigma)\sum_{k=0}^{\infty} M_k\} \right] \right], \\ \sum_{k=0}^{\infty} A_k = Y^{-1}[A_0\xi] + Y^{-1}\left[\frac{1-\mu+\mu\xi^{\mu}}{B[\mu]} Y\left[\gamma^{1-\mu}\{\varsigma\varphi \sum_{k=0}^{\infty} M_k - v\sum_{k=0}^{\infty} A_k\} \right] \right]. \end{cases} \tag{5.25}$$

From Eq. (5.25), we get

$${}^{MABC}L_0 = 40, \quad {}^{MABC}R_0 = 10, \quad {}^{MABC}M_0 = 20, \quad {}^{MABC}S_0 = 10, \quad {}^{MABC}A_0 = 5,$$

and

$$\begin{cases} {}^{MABC}L_1 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{\Upsilon - \varpi L_0 M_0 - vL_0\}\right]\right], \\ {}^{MABC}R_1 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{\varpi L_0 M_0 - \kappa_1 R_0 - vR_0\}\right]\right], \\ {}^{MABC}M_1 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{\kappa_1 R_0 + \kappa_2 M_0 S_0 - (v+\varphi)M_0\}\right]\right], \\ {}^{MABC}S_1 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{-\kappa_2 M_0 S_0 - vS_0 + \varphi(1-\varsigma)M_0\}\right]\right], \\ {}^{MABC}W_1 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{\varsigma\varphi M_0 - vA_0\}\right]\right], \end{cases} \tag{5.26}$$

and

$$\begin{cases} {}^{MABC}L_2 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{\Upsilon - \varpi(L_0 M_1 + L_1 M_0) - vL_1\}\right]\right], \\ {}^{MABC}R_2 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{\varpi(L_0 M_1 + L_1 M_0) - \kappa_1 R_1 - vR_1\}\right]\right], \\ {}^{MABC}M_2 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{\kappa_1 R_1 + \kappa_2(M_0 S_1 + M_1 S_0) - (v+\varphi)M_1\}\right]\right], \\ {}^{MABC}S_2 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{-\kappa_2(M_0 S_1 + M_1 S_0) - vS_1 + \varphi(1-\varsigma)M_1\}\right]\right], \\ {}^{MABC}A_2 = Y^{-1}\left[\frac{1-\mu+\mu\xi^\mu}{B[\mu]}Y\left[\gamma^{1-\mu}\{\varsigma\varphi M_1 - vA_1\}\right]\right]. \end{cases} \tag{5.27}$$

Using the above equations, we get the series solution

$$\begin{cases} {}^{MABC}L(t) = {}^{MABC}L_0 + {}^{MABC}L_1 + {}^{MABC}L_2 + \dots, \\ {}^{MABC}R(t) = {}^{MABC}R_0 + {}^{MABC}R_1 + {}^{MABC}R_2 + \dots, \\ {}^{MABC}M(t) = {}^{MABC}M_0 + {}^{MABC}M_1 + {}^{MABC}M_2 + \dots, \\ {}^{MABC}S(t) = {}^{MABC}S_0 + {}^{MABC}S_1 + {}^{MABC}S_2 + \dots, \\ {}^{MABC}A(t) = {}^{MABC}A_0 + {}^{MABC}A_1 + {}^{MABC}A_2 + \dots. \end{cases} \tag{5.28}$$

5.2.2. *Existence and Uniqueness of the Solution.* In this section, we will show the system (5.22) has a unique solution. Here, we present the existence and uniqueness of smoking model solution in modified Atangana-Baleanu Caputo form, as shown by the technique described in [6]. Applying the fractional integral on system of Equations (5.22) and using initial condition, we have

$$\begin{cases} L(t) - L(0) = \gamma^{1-\mu}\ {}^{MAB}I_t^\mu[\Upsilon - \varpi LM - vL], \\ R(t) - R(0) = \gamma^{1-\mu}\ {}^{MAB}I_t^\mu[\varpi LM - \kappa_1 R - vR], \\ M(t) - M(0) = \gamma^{1-\mu}\ {}^{MAB}I_t^\mu[\kappa_1 R + \kappa_2 MS - (v+\varphi)M], \\ S(t) - S(0) = \gamma^{1-\mu}\ {}^{MAB}I_t^\mu[-\kappa_2 MS - vS + \varphi(1-\varsigma)M], \\ A(t) - A(0) = \gamma^{1-\mu}\ {}^{MAB}I_t^\mu[\varsigma\varphi M - vA]. \end{cases} \tag{5.29}$$

Utilising the value of modified Atangana-Baleanu fractional integral, we get

$$\begin{aligned} L(t) = L(0) + \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\big[\Upsilon - \varpi L(t)M(t) - vL(t) + \frac{\mu}{1-\mu}\ {}^{RL}I_t^\mu(\Upsilon - \varpi L(t)M(t) - vL(t)) \\ - (\Upsilon - \varpi L(0)M(0) - vL(0))(1 + \frac{\mu}{1-\mu}\frac{t^\mu}{\Gamma(\mu+1)})\big]. \end{aligned} \tag{5.30}$$

For clarity, we consider

$$\Theta_1(t, L) = \Upsilon - \varpi LM - vL,$$

$$\begin{aligned}\Theta_2(t,R) &= \varpi LM - \kappa_1 R - vR,\\ \Theta_3(t,M) &= \kappa_1 R + \kappa_2 MS - (v+\varphi)M,\\ \Theta_4(t,S) &= -\kappa_2 MS - vS + \varphi(1-\varsigma)M,\\ \Theta_5(t,A) &= \varsigma\varphi M - vA.\end{aligned} \tag{5.31}$$

In Theorem (5.1), we have seen that Θ_1, Θ_2, Θ_3, Θ_4, and Θ_5 satisfy the Lipschitz condition. With Θ_1, Eq. (5.30) can be written as

$$\begin{aligned}L(t) = L(0) &+ \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}[\Theta_1(t,L(t)) + \frac{\mu}{1-\mu}\, {}^{RL}I_0^{\mu}[\Theta_1(t,L(t))]\\ &- \left(1+\frac{\mu}{1-\mu}\frac{t^{\mu}}{\Gamma(\mu+1)}\right)(\Upsilon - \varpi L(0)M(0) - vL(0))].\end{aligned} \tag{5.32}$$

Considering the recursive formula,

$$L_l(t) = \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}[\Theta_1(t,L_{l-1}(t)) + \frac{\mu}{1-\mu}\, {}^{RL}I_0^{\mu}[\Theta_1(t,L_{l-1}(t))] - \left(1+\frac{\mu}{1-\mu}\frac{t^{\mu}}{\Gamma(\mu+1)}\right)(\Upsilon - \varpi L(0)M(0) - vL(0))].$$

Now, we examine

$$N_l(t) = L_l(t) - L_{l-1}(t),$$

$$N_l(t) = \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}[\Theta_1(t,L_{l-1}(t)) - \Theta_1(t,L_{l-2}(t))] + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\, {}^{RL}I_0^{\mu}[\Theta_1(t,L_{l-1}(t)) - \Theta_1(t,L_{l-2}(t))],$$

we can write,

$$L_l(t) = \sum_{j=0}^{l} N_j(t),$$

taking the norm on both sides

$$\begin{aligned}\|N_l(t)\| &= \|L_l(t) - L_{l-1}(t)\|,\\ \|N_l(t)\| &= \left\|\frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}[\Theta_1(t,L_{l-1}(t)) - \Theta_1(t,L_{l-2}(t))] + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\, {}^{RL}I_0^{\mu}[\Theta_1(t,L_{l-1}(t)) - \Theta_1(t,L_{l-2}(t))]\right\|,\\ \|N_l(t)\| &\le \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\big\|[\Theta_1(t,L_{l-1}(t)) - \Theta_1(t,L_{l-2}(t))]\big\| + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\big\|\, {}^{RL}I_0^{\mu}[\Theta_1(t,L_{l-1}(t)) - \Theta_1(t,L_{l-2}(t))]\big\|.\end{aligned}$$

Since Θ_1 satisfies Lipschitz condition

$$\|L_l(t) - L_{l-1}(t)\| \le \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\aleph_1\|L_{l-1}(t) - L_{l-2}(t)\| + \aleph_1\frac{\mu\gamma^{1-\mu}}{B[\mu]}\big\|\, {}^{RL}I_0^{\mu}\big\|[\Theta_1(t,L_{l-1}(t)) - \Theta_1(t,L_{l-2}(t))]\big\|.$$

Therefore, we get

$$\|N_l(t)\| \le \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\aleph_1\|N_{l-1}(t)\| + \aleph_1\frac{\mu\gamma^{1-\mu}}{B[\mu]}\, {}^{RL}I_0^{\mu}\|N_{l-1}(t)\|. \tag{5.33}$$

Theorem 5.4. *[6]. The model has a solution if there exists $\aleph_j$ for every kernel Θ_j, $j = 1,\dots,5$ such that*

$$\left[\frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\aleph_j + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\aleph_j\, t\right] \le 1.$$

Proof. Since, $L(t)$ is bounded function. We have seen that the kernels satisfy the Lipschitz condition. Utilising the Equation (5.33) along with recursive technique, we get

$$\|N_l(t)\| \le \|L(t)\|\left[\frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\aleph_1 + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\aleph_1 t\right]^l.$$

Suppose for now that the following is satisfied:

$$L(t) - L(0) = N_l(t) - V_l(t),$$

we get

$$\|V_l(t)\| = \left\| \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}[\Theta_1(t, L(t)) - \Theta_1(t, L_{l-1}(t))] + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\ {}^{RL}I_0^{\mu}[\Theta_1(t, L(t)) - \Theta_1(t, L_{l-1}(t))] \right\|.$$
$$\|V_l(t)\| \leq \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\|[\Theta_1(t, L(t)) - \Theta_1(t, L_{l-1}(t))]\| + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\ \big\|^{RL}I_0^{\mu}\big\|[\Theta_1(t, L(t)) - \Theta_1(t, L_{l-1}(t))]\big\|,$$
$$\|V_l(t)\| \leq \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\aleph_1\|[L(t) - L_{l-1}(t)]\| + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\aleph_1 t\big\|[L(t) - L_{l-1}(t)]\big\|,$$

continuing the above process, we get

$$\|V_l(t)\| \leq \left[\frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]} + \frac{\mu\gamma^{1-\mu}}{B[\mu]}t\right]^{l+1}\aleph_1^{l+1},$$

as $l \to \infty$, we get $\|V_l(t)\| \to 0$. □

Theorem 5.5. *[6]. The modified Atangana-Baleanu Caputo fractionalized smoking model has a unique solution if*

$$\left(1 - \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\aleph_j - \frac{\mu\gamma^{1-\mu}}{B[\mu]}\ \aleph_j\ t\right) \geq 0, \qquad j = 1, 2, \ldots, 5. \tag{5.34}$$

Proof. For kernel Θ_1, let the system has another solution $L_1(t)$.

$$\begin{aligned}\|L(t) - L_1(t)\| &= \left\| \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}[\Theta_1(t, L(t)) - \Theta_1(t, L_1(t))] + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\ {}^{Rl}I_0^{\mu}[\Theta_1(t, L(t)) - \Theta_1(t, L_1(t))] \right\| \\ &\leq \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\|[\Theta_1(t, L(t)) - \Theta_1(t, L_1(t))]\| + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\ {}^{RL}I_0^{\mu}\big\|[\Theta_1(t, L(t)) - \Theta_1(t, L_1(t))]\big\|,\end{aligned}$$

using Lipschitz condition

$$\leq \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\aleph_1\|[L(t) - L_1(t)]\| + \frac{\mu\gamma^{1-\mu}}{B[\mu]}\aleph_1 t\big\|[L(t) - L_1(t)]\big\|. \tag{5.35}$$

Thus

$$\|L(t) - L_1(t)\|\left(1 - \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\aleph_1 - \frac{\mu\gamma^{1-\mu}}{B[\mu]}\ \aleph_1 t\right) \leq 0, \tag{5.36}$$

but

$$\left(1 - \frac{\gamma^{1-\mu}(1-\mu)}{B[\mu]}\aleph_1 - \frac{\mu\gamma^{1-\mu}}{B[\mu]}\ \aleph_1 t\right) \geq 0. \tag{5.37}$$

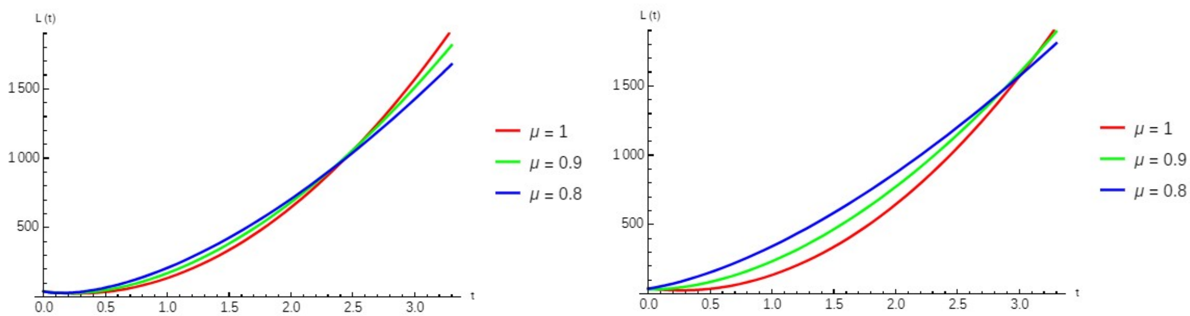
Therefore,

$$\|L(t) - L_1(t)\| = 0,$$
$$L(t) = L_1(t).$$

In a similar way, we can show this for other kernels also. We get the unique solution. □

Table 1. Description of specific values of parameters used in system (3.1).

Parameters	Descriptions	Values (unit: 1/time)
Υ	Recruitment rate in L	1
υ	Natural death rate	0.05
φ	Rate of quitting smoking	0.8
ϖ	Effective contact rate between M and L	0.14
ς	Remaining fraction of smoking who permanently quit smoking	0.1
κ_1	Rate of transition from occasional smokers to regular smokers	0.002
κ_2	The interaction ratio between M and S who resume smoking	0.0025



(a) Variation of L(t) for Caputo Operator. (b) Variation of L(t) for MABC Operator.

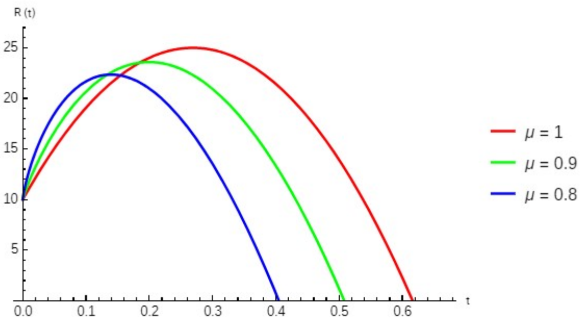
Figure 4. Approximate solution for potential smokers L(t) for $\mu = 1, 0.9, 0.8$.

6. Numerical Results and Discussion

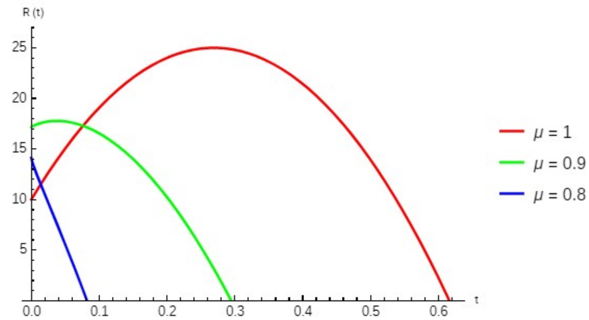
This section, presents the YTDM to give a numerical simulation for the smoking model. The graphs and tables show the dynamic behaviour of potential smokers, occasional smokers, smokers, temporarily quit smokers and permanently quit smokers for integer order as well as for fractional order. The behaviour was demonstrated using the fractional orders $\mu = 0.9$ and 0.8 and the integer order $\mu = 1$. The fractional order has a minor effect on the dynamics of the epidemic as shown in Figures 4(a)-8(b). This further shows that the method used to solve fractional differential equations provides accurate approximations for fractional models. For the computational simulation of the smoking model, the parameter values from Table 1 are used.

In Figures 4(a) and 4(b), it can be seen that the potential smokers, i.e., non-smokers, increase with the increase in time t and increases with a decrease in the value of μ. As shown in Table 2, for fractional order, as the time increases, the number of potential smokers is increasing rapidly with non-singular MABC operator as compared to Caputo operator. One can observe that the number of occasional smokers increases rapidly in the beginning, then declines rapidly, and the same behaviour occurs for temporarily quit smokers. Further, occasional and temporarily quit smokers decrease as the value of μ goes down. The number of smokers who smoke occasionally declines over time and tends to zero, showing the accuracy of the proposed method for smoking model. In Figures 6(a) and 6(b), we can see that the smokers rise with time and rise when the μ decreases. Figures 8(a) and 8(b) illustrate how the number of persons who give up permanently increases over time at first with the increase in the value of μ and then begins to decline over time.

It is crucial to note that the outcomes depend on the value of the fractional order as well as the singular and non-singular operators. The approximate solutions obtained by our method are compared with the existing methods like q-HATM [32], LADM [1] and NTDM [22] in the literature in Tables 2, 3, 4, 5, and 6. The graphical and tabular results show that the fractional derivative provides better information to analyze the smoking model. MATHEMATICA software was used for numerical computations and constructions of figures.

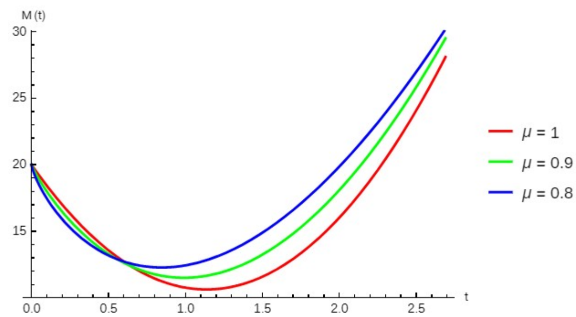


(a) Variation of R(t) for Caputo Operator.

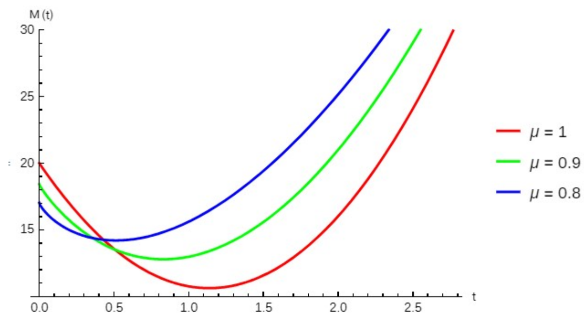


(b) Variation of R(t) for MABC Operator.

Figure 5. Approximate solution for occasional smokers R(t) for $\mu = 1$, 0.9, 0.8.

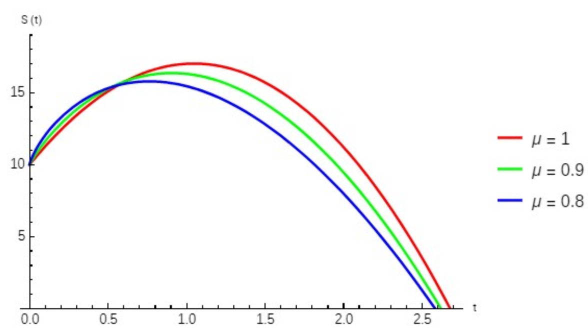


(a) Variation of M(t) for Caputo Operator.

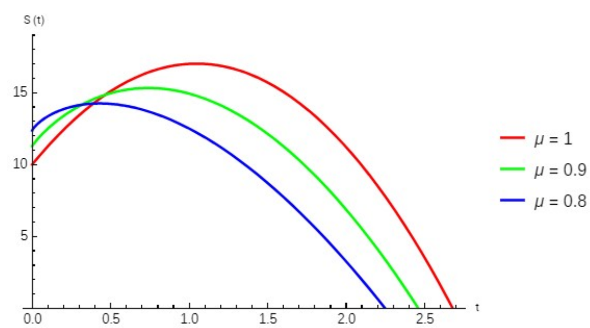


(b) Variation of M(t) for MABC Operator.

Figure 6. Approximate solution for smokers M(t) for $\mu = 1$, 0.9, 0.8.



(a) Variation of S(t) for Caputo Operator.

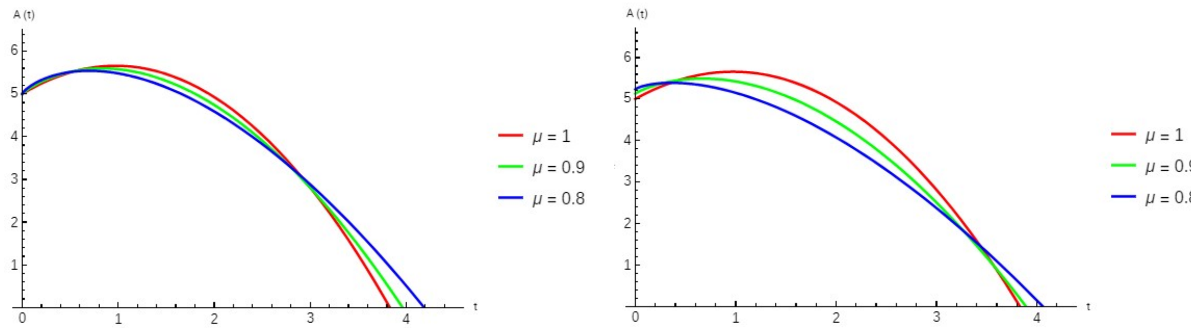


(b) Variation of S(t) for MABC Operator.

Figure 7. Approximate solution for temporarily quit smokers S(t) for $\mu = 1$, 0.9, 0.8.

7. Conclusion

The main objective of this study is the successful implementation of the Yang transform decomposition technique to obtain an approximate analytical solution for the fractional smoking model. The addition of a parameter prevented dimensional inconsistencies. The second objective of our research is to demonstrate the impact of the order of the fractional derivative and the singular and non-singular operator via an efficient graphical depiction of an approximate solution. This study provides a comprehensive framework for analyzing and predicting smoking behavior over time, facilitating more effective public health interventions. In comparison to integer-order differential calculus, fractional operators are more favourable due to their non-local characteristic and degree of freedom in the modeling process. The fractional model shows accuracy, leads to better outcomes, and provides a more accurate depiction. The figures clearly show the distinction between the modified Atangana-Baleanu Caputo derivative and Caputo derivative behaviours, and when $\mu \to 1$, both the fractional solutions converge to the classical integer solution. It is clear that the Adomian



(a) Variation of A(t) for Caputo Operator. (b) Variation of A(t) for MABC Operator.

Figure 8. Approximate solution for permanently quit smokers A(t) for μ = 1, 0.9, 0.8.

Table 2. Approximate solution for potential smokers L(t).

L(t)	μ = 1						μ = 0.9					
t	0	0.1	0.2	0.3	0.4	0.5	0	0.1	0.2	0.3	0.4	0.5
YTDM$_{C}$	40	30.8717	25.8868	25.0452	28.347	35.7923	40	29.2634	26.2826	28.8764	36.4019	48.4892
YTDM$_{MABC}$	40	30.8717	25.8868	25.0452	28.347	35.7923	32.7828	33.3347	39.577	50.448	65.5342	84.5777
q-HATM[32]	40	30.7667	25.6668	24.7002	27.867	35.1672	40	29.1162	26.0052	28.4733	35.8752	47.8402
LADM [1]	40	30.7738	25.7037	24.8025	28.0829	35.5576	40	29.132	26.0772	28.6572	36.24	48.4674
NTDM$_{C}$[22]	40	30.8717	25.8868	25.0452	28.347	35.7923	40	29.2565	26.28233	28.8935	36.4457	48.56872
NTDM$_{CF}$[22]	40	30.8717	25.8868	25.0452	28.347	35.7923	32.9434	31.9995	34.4118	40.1803	49.3048	61.7855
NTDM$_{ABC}$[22]	40	30.8717	25.8868	25.0452	28.347	35.7923	32.9434	32.6841	37.5872	46.6408	59.4646	75.8233

Table 3. Approximate solution for occasional Smokers R(t).

R(t)	μ = 1						μ = 0.9					
t	0	0.05	0.1	0.15	0.2	0.25	0	0.05	0.1	0.15	0.2	0.25
YTDM$_{C}$	10	15.0559	19.0756	22.0591	24.0063	24.9174	10	16.689	20.6672	22.8844	23.5858	22.9052
YTDM$_{MABC}$	10	15.0559	19.0756	22.0591	24.0063	24.9174	17.1604	17.7145	16.5384	14.0011	10.2333	5.3145
q-HATM[32]	10	15.0559	19.0756	22.0590	24.0063	24.9173	10	16.6946	20.6741	22.8892	23.5858	22.8977
NTDM$_{C}$[22]	10	15.0559	19.0756	22.0590	24.0063	24.9173	10	16.6946	20.674	22.8892	23.5858	22.8977
NTDM$_{ABC}$[22]	10	15.0559	19.0756	22.0590	24.0063	24.9173	17.0031	17.8962	17.1965	15.2684	12.2346	8.16908

decomposition technique works effectively when operating the non-linear terms, whereas the Yang transform is a quicker and more straightforward technique. The solutions obtained via YTDM are in good correspondence with the previous research conducted using q-HATM, LADM, and NTDM. Thus, it can be concluded that the YTDM technique for smoking model effectively reduces the negative effects of smoking over a range of time periods. We anticipate that our method will be useful in addressing more fractional order problems, particularly when studying the modeling of real-world phenomena.

Table 4. Approximate solution for smokers M(t).

M(t)	μ = 1						μ = 0.9					
t	0	0.1	0.2	0.3	0.4	0.5	0	0.1	0.2	0.3	0.4	0.5
$YTDM_C$	20	18.4244	16.9938	15.708	14.5671	13.5711	20	17.9817	16.4546	15.1952	14.1534	13.3036
$YTDM_{MABC}$	20	18.4244	16.9938	15.708	14.5671	13.5711	18.4155	16.8655	15.7247	14.8152	14.0962	13.5467
q-HATM[32]	20	18.4244	16.9938	15.708	14.5671	13.5711	20	17.9798	16.4515	15.1913	14.1491	13.2994
LADM [1]	20	18.1142	16.9805	15.6782	14.5141	13.4883	20	17.9736	16.4298	15.1463	14.0735	13.1863
$NTDM_C$[22]	20	18.4244	16.9938	15.708	14.5671	13.5711	20	17.9798	16.4515	15.1913	14.1491	13.2994
$NTDM_{CF}$[22]	20	18.4244	16.9938	15.708	14.5671	13.5711	18.4969	17.3332	16.2868	15.3578	14.5462	13.8519
$NTDM_{ABC}$[22]	20	18.4244	16.9938	15.708	14.5671	13.5711	18.4969	17.0078	15.8974	14.9976	14.2704	13.6961

Table 5. Approximate solution for temporarily quit smokers S(t).

S(t)	μ = 1						μ = 0.9					
t	0	0.1	0.2	0.3	0.4	0.5	0	0.1	0.2	0.3	0.4	0.5
$YTDM_C$	10	11.276	12.4241	13.4443	14.3365	15.1008	10	11.6316	12.8495	13.8378	14.6381	15.2718
$YTDM_{MABC}$	10	11.276	12.4241	13.4443	14.3365	15.1008	11.2769	12.5017	13.3844	14.0693	14.5899	14.9636
q-HATM[32]	10	11.276	12.4241	13.4443	14.3365	15.1008	10	11.6331	12.8520	13.8408	14.6413	15.2749
LADM [1]	10	11.2543	12.3374	13.2429	13.9896	14.5587	10	11.592	12.7091	13.5444	14.1439	14.5317
$NTDM_C$[22]	10	11.276	12.4241	13.4443	14.3365	15.1008	10	11.6331	12.852	13.8408	14.6413	15.2749
$NTDM_{CF}$[22]	10	11.276	12.4241	13.4443	14.3365	15.1008	11.2121	12.1359	12.9562	13.6729	14.2859	14.7953
$NTDM_{ABC}$[22]	10	11.276	12.4241	13.4443	14.3365	15.1008	11.2121	12.3913	13.2542	13.9372	14.4716	14.8736

Table 6. Approximate solution for permanently quit smokers A(t).

A(t)	μ = 1						μ = 0.9					
t	0	0.1	0.2	0.3	0.4	0.5	0	0.1	0.2	0.3	0.4	0.5
$YTDM_C$	5	5.1281	5.2423	5.3426	5.4291	5.5018	5	5.1635	5.2839	5.3800	5.4562	5.5145
$YTDM_{MABC}$	5	5.1281	5.2423	5.3426	5.4291	5.5018	5.1276	5.2475	5.3322	5.3959	5.4423	5.4729
q-HATM[32]	5	5.1281	5.2423	5.3426	5.4291	5.5018	5	5.1636	5.2841	5.3803	5.4565	5.5148
LADM [1]	5	6.6449	8.4294	10.3537	12.4177	14.6215	5	6.9295	8.3871	9.587	10.5772	11.3822
$NTDM_C$[22]	5	5.1281	5.2423	5.3426	5.4291	5.5018	5	5.1636	5.2841	5.3803	5.4565	5.5148
$NTDM_{CF}$[22]	5	5.1281	5.2423	5.3426	5.4291	5.5018	5.1211	5.2121	5.2918	5.3603	5.3175	5.4636
$NTDM_{ABC}$[22]	5	5.1281	5.2423	5.3426	5.4291	5.5018	5.1211	5.2369	5.32	5.3842	5.4326	5.4668

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Declaration of Competing Interest

The authors declare that there is no conflict of interest.

Data Availability and Access

Not applicable.

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