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Flow of gas-condensate system in a time-dependent deformable reservoir with rock creep effect in the well Bottomhole zone

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Abstract

The problem of flow of a gas-condensate system to a well-draining a time-dependent deformable formation is considered in the case when the laws of compressibility of reservoir rocks in the bottomhole zone and in the part of the reservoir remote from the well are different. Using the idea of a binary representation of a gas-condensate system, a semi-analytical solution to the considered problem is obtained, an algorithm is proposed for calculating the main indicators of depletion of a gas-condensate reservoir, for the case when near the well (inner zone) the formation undergoes creep, and in the distant part of the reservoir (outer zone) elastic deformation occurs. Based on this algorithm, a computer simulator of the considered process created.

The results of a study of the influence of the noted factor on the main indicators of the depletion process of gas-condensate deposits represented by time-dependent (relaxing) reservoirs showed that taking into account the rheological characteristics of the reservoir in the well bottom-hole zone significantly refines the forecasting of the main development indicators. It has been established that when the creep effect of formation rocks in the near-wellbore zone is taken into account, the maximum difference in the current values of formation pressures reaches up to 12.53%. It corresponds to a gas recovery factor value of 0.55.

Keywords. Gas-condensate system, Relaxation, Rocks creeping, Binary model, Depletion, Compressibility, Time-dependent deformation.

2010 Mathematics Subject Classification. 65L05, 34K06, 34K28.

1. Introduction

It is known that the development of deep gas-condensate and oil fields [1], [2], [5], [10], [17] is accompanied by deformation of reservoir rocks [6], as a result of which their reservoir characteristics change. It has been established that with a wider range of changes in reservoir pressure, the deformation of rocks can have a significantly nonlinear character [3], [16], [19]. In addition, creep of rocks may occur in this case [9]. Moreover, in the same formation, depending on the pore pressure value, deformations of the rocks skeleton can manifest themselves differently [8]. So, if near the well bottomhole zone, where the reservoir pressure is much lower than its initial value, deformations of the reservoir formation occur according to the same law, and on the well-drainage areas edge (or far from the well bottomholle zone), where the pressure is relatively high (or pressure above a certain limit), the reservoir skeleton is compressed according to a different law. In this case, we are talking about taking into account changes in rheology along the formation.

In [13], solution to the problem of flow of gas-condensate mixtures in time-dependent, including creeping collectors, was obtained. It was assumed that the formation was rheologically homogeneous.

But in this work, a study of the development of gas-condensate deposits with time-dependent deformable reservoirs is developed to take into account changes in the rheology of the formation along its strike.

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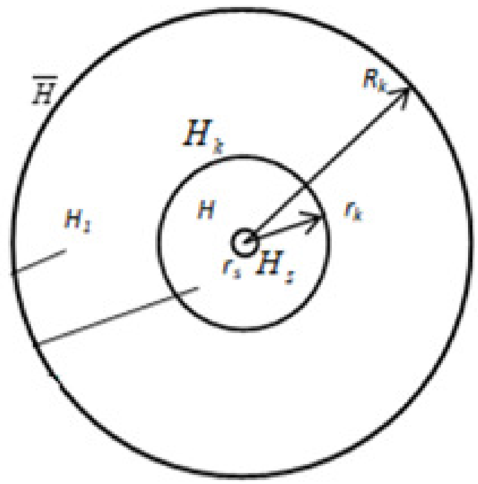

FIGURE 1. Schematic representation of the well and drainage area, which consisted of two zones with different rheological properties.

For this purpose, the gas condensate reservoir appears to consist of two zones (Figure1). In the inner zone of the formation, creep manifests itself when in the outer zone the rocks are still deformed according to the time-dependent elastic law.

2. Flow of gas condensate mixture to the well

It is known that the movement of a gas-condensate mixture in deformable reservoirs is represented by complex nonlinear partial differential equations [14], [15], [18], the solution of which uses various methods. In [12], a technique was proposed for approximate solution of the equations of motion of a gas-condensate mixture using the averaging method and applying the Khristianovich function. Based on the results of [4], [12], [15], the equations of motion of gas and condensate dissolved in it when flowing to the well in the internal and external zones (Figure1), taking the averaging method into account, can be written in the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H}{\partial r}\right) = -\Phi\left(t\right), \tag{2.1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_1}{\partial r}\right) = -\Phi_1\left(t\right), \tag{2.2}$$

where r and t are radial coordinates and time, respectively, $H,\ H_1$ are Khristianovich functions, $H = \int \varphi(p,s)dp + const$, $H_1 = \int \varphi(p_1,s_1)dp_1 + const$,

$$\varphi = \left[\frac{k_{rg}(s)p\beta[1-c(p)\bar{\gamma}(p)]}{\mu_g(p)z(p)p_{at}} + \frac{k_{ro}(s)S(p)}{\mu_o(p)B_o(p)}\right]k(p,t),$$

$$\varphi_1 = \left[\frac{k_{rg}(s_1)p_1\beta[1-c(p_1)\bar{\gamma}(p_1)]}{\mu_g(p_1)z(p_1)p_{at}} + \frac{k_{ro}(s_1)S(p_1)}{\mu_o(p_1)B_o(p_1)}\right]k(p_1,t), \tag{2.3}$$

$\Phi(t)$ and $\Phi_1(t)$ are unknown functions that depend only on time and are determined for a fixed time using additional conditions, p is pressure, k is the reservoir permeability, $k_{ro}(s)$, $k_{rg}(s)$ are the relative phase permeability of the liquid and gas phases, respectively. s is the condensate saturation. μ_o, μ_g are the dynamic viscosities of condensate and gas, respectively. B_o is the condensate volume factor. S is the gas solubility in liquid phase. z is the gas factor (gas compressibility factor), β is the temperature correction factor. c is the content of potentially liquid hydrocarbons in the gas phase. is the ratio of the specific weight of liquid phase and the specific weight of gas phase at the reservoir pressure p; p_{at} is the atmospheric pressure. Parameters with index "1" correspond to the external zone.

The system of Equations (2.1) and (2.2) is solved under the following boundary conditions:

$$r = r_w, \;\; H = H_w,$$

$$r = r_k, \;\; H = H_k\,, \tag{2.4}$$

$$r = R_e, \;\; H_1 = \bar{H}.$$

Additionally, we have the following conditions and notations:

$$r = r_k, \;\; \frac{\partial H}{\partial r} = \frac{\partial H_1}{\partial r}, \tag{2.5}$$

$$r = R_e, \;\; \frac{\partial H_1}{\partial r} = 0$$

and

$$r = r_k, \;\; H_1 = H_k. \tag{2.6}$$

General solutions of Equations (2.1) and (2.2) under boundary conditions (2.4) can easily be obtained in the form:

$$H = -\frac{1}{4}\Phi(t)\left(r^2 - r_k^2 - \frac{r_k^2 - r_w^2}{\ln\frac{r_k}{r_w}}\ln\frac{r}{r_k}\right) + \frac{H_k - H_w}{\ln\frac{r_k}{r_w}}\ln\frac{r}{r_k} + H_k \tag{2.7}$$

and

$$H_1 = -\frac{1}{4}\Phi_1(t)\left(r^2 - R_e^2 - \frac{R_e^2 - r_k^2}{\ln\frac{R_e}{r_k}}\ln\frac{r}{R_e}\right) + \frac{\bar{H}_k - H_k}{\ln\frac{R_e}{r_k}}\ln\frac{r}{R_e} + H_k. \tag{2.8}$$

From (2.7) and (2.8), taking into account conditions (2.5) and (2.6), it is possible to determine $\Phi(t)$, $\Phi_1(t)$:

$$\Phi(t) = 2\frac{\frac{\bar{H}-H_k}{R_e^2\ln\frac{R_e}{r_k} - \frac{1}{2}(R_e^2 - r_k^2)}\left(r_k^2 - \frac{1}{2}\frac{R_e^2 - r_k^2}{\ln\frac{R_e}{r_k}}\right) - \frac{\bar{H}-H_k}{\ln\frac{R_e}{r_k}} + \frac{H_k - H_w}{\ln\frac{r_k}{r_w}}}{r_k^2 - \frac{1}{2}\frac{r_k^2 - r_w^2}{\ln\frac{r_k}{r_w}}}, \tag{2.9}$$

$$\Phi_1(t) = 2\frac{\bar{H} - H_k}{R_e^2\ln\frac{R_e}{r_k} - \frac{1}{2}\left(R_e^2 - r_k^2\right)}. \tag{2.10}$$

If we take into account that the well flow rate $q = 2\pi r_w h \left.\frac{\partial H}{\partial r}\right|_{r-r_w}$, then the expression for determining the well flow rate will be obtained from (2.7) and (2.9) in the following form:

$$q = 2\pi h \frac{\frac{\bar{H}-H_k}{R_e^2 \ln \frac{R_e}{r_k} - \frac{1}{2}\left(R_e^2 - r_k^2\right)} \left(r_k^2 - \frac{1}{2} \frac{R_e^2 - r_k^2}{\ln \frac{R_e}{r_k}} \right) - \frac{\bar{H}-H_k}{\ln \frac{R_e}{r_k}} + \frac{H_k - H_w}{\ln \frac{r_k}{r_w}}}{r_k^2 - \frac{1}{2} \frac{r_k^2 - r_w^2}{\ln \frac{r_k}{r_w}}} \rightarrow$$
$$\rightarrow \left(r_w^2 - \frac{1}{2} \frac{r_k^2 - r_w^2}{\ln \frac{r_k}{r_w}} \right) + \frac{H_k - H_w}{\ln \frac{r_k}{r_w}}. \tag{2.11}$$

Now let's consider determining the gas flow rate flowing from the external zone to the internal zone at the r_k boundary. If we take into account that $q_1 = 2\pi r_k h \left.\frac{\partial H_1}{\partial r}\right|_{r-r_k}$, then from (2.8) and (2.10) we can obtain an expression for q_1 in the following form:

$$q_1 = 2\pi h \left[\frac{\bar{H}_k - H_k}{\frac{1}{2}\left(R_k^2 - r_k^2\right) - R_k^2 \ln \frac{R_k}{r_k} -} \left(r_k^2 - \frac{1}{2} \frac{R_k^2 - r_k^2}{\ln \frac{R_k}{r_k}} \right) - \frac{\bar{H}_k - H_k}{\ln \frac{R_k}{r_k}} \right], \tag{2.12}$$

where the value of the H function is determined by the following integral:

$$H = \int \varphi(p,s)dp + const,$$

where the $\varphi(p,s)$ function is determined by expressions (2.3) depending on the zone under consideration.

Taking (2.12) into account, we can reduce (2.11) to the following form:

$$q = q_1 \cdot \varepsilon + 2\pi h \frac{H_k - H_w}{\ln \frac{r_k}{r_w}} (1 - \varepsilon), \tag{2.13}$$

where $\varepsilon = \frac{\frac{R_e^2 - r_k^2}{\ln \frac{R_e}{r_k}} - \frac{r_k^2 - r_w^2}{\ln \frac{r_k}{r_w}}}{R_e^2\left(2 \ln \frac{R_e}{r_k} - 1\right) + r_k^2}$.

The resulting formula (2.13), taking (2.12) into account, makes it possible to determine the gas flow rate of the well under the considered development conditions for the gas condensate deposits. At the same time, the difference between the pseudo pressures $H_k - H_w$ and $\bar{H} - H_k$ can be determined by the following expressions:

$$H_k - H_w = \frac{A}{3}(p_k^3 - p_w^3) + \frac{B}{2}(p_k^2 - p_w^2) + C(p_k - p_w), \tag{2.14}$$

$$\bar{H} - H_k = \frac{A_1}{3}(p_1^3 - p_k^3) + \frac{B_1}{2}(p_1^2 - p_k^2) + C_1(p_1 - p_k), \tag{2.15}$$

where following [12], the approximation coefficients A, B, C and A_1, B_1, C_1 can be analytically calculated from the relations:

$$A = \frac{2(\varphi_k + \varphi_w - 2\varphi_{sr})}{(p_k - p_w)^2}, B = \frac{\varphi_k - \varphi_w}{p_k - p_w} - A(p_k + p_w), C = \varphi_k - A p_k^2 - B p_k, \tag{2.16}$$

$$A_1 = \frac{2(\varphi_k + \varphi_1 - 2\varphi_{1sr})}{(p_1 - p_k)^2}, B_1 = \frac{\varphi_1 - \varphi_k}{p_1 - p_k} - A_1(p_k + p_1), C_1 = \varphi_1 - A_1 p_1^2 - B_1 p_1. \tag{2.17}$$

Here φ_w, φ_k and φ_1 are the values of the integrand at the well bottomhole, at the border of the external and internal zones, i.e. at pressure p_k and at the well supply boundary; $\varphi_{sr} = \varphi(p_{sr}, s_{sr})$, $p_{sr} = \frac{p_k + p_w}{2}$, $\varphi_{1sr} = \varphi(p_{1sr}, \rho_{1sr})$, $p_{1sr} = \frac{p_1 + p_k}{2}$.

However, to apply the algorithm outlined above, it will be necessary to determine reservoir pressures and condensate saturations at the boundary between the zones under consideration and at the supply boundary over time. To do this, we will use the equations of material balances of gas and condensate.

3. Determination of average reservoir parameters over time

We write the material balance equation for gas and condensate in the internal zone in the following form:

$$q_g - q_{g1} = -\frac{d}{dt}\left\{\left[\frac{(1-s)p\beta}{z(p)p_{at}}[1-c(p)\bar{\gamma}(p)] + \frac{sS(p)}{B_o(p)}\right]\Omega(p,t)\right\}, \tag{3.1}$$

$$q_o - q_{o1} = -\frac{d}{dt}\left\{\left[\frac{s}{B_o(p)} + (1-s)\frac{p\beta c(p)}{z(p)p_{at}}\right]\Omega(p,t)\right\}, \tag{3.2}$$

where $q_g,\ q_o$ are the well production rate of gas and condensate, respectively, $q_{g1},\ q_{o1}$ are the flow rate of gas and condensate invading from the external zone in the internal zone across the boundary of r_k. Similar equations for the external zone can be written in the following form:

$$q_{g1} = -\frac{d}{dt}\left[\frac{(1-s_1)p_1\beta}{z(p_1)p_{at}}[1-c(p_1)\bar{\gamma}(p)] + \frac{s_1S(p_1)}{B_o(p_1)}\right]\Omega_1(p_1,t), \tag{3.3}$$

$$q_{o1} = -\frac{d}{dt}\left[\frac{s_1}{B_o(p_1)} + (1-s_1)\frac{p_1\beta c(p_1)}{z(p_1)p_{at}}\right]\Omega_1(p_1,t), \tag{3.4}$$

where $p_1,\ s_1$ are average reservoir pressure and condensate saturation in the outer zone. From (3.1)-(3.2) and (3.3)-(3.4) we can obtain equations describing changes in average reservoir pressures and condensate saturations over time for the internal and external zones, respectively:

$$\frac{dp}{dt} = -\frac{\frac{q_g-q_{g1}}{\Omega_0\bar{\Omega}}(\alpha_4+\frac{\alpha_2}{G}) - (\alpha_2\alpha_3+\alpha_1\alpha_4)\frac{1}{\bar{\Omega}}\frac{d\bar{\Omega}}{dt}}{(\alpha_5+\alpha_6)\alpha_4 + (\alpha_7+\alpha_8)\alpha_2}, \tag{3.5}$$

$$\frac{ds}{dt} = -\frac{\frac{q_g-q_{g1}}{\Omega_0\bar{\Omega}G} + (\alpha_7+\alpha_8)\frac{dp}{dt} + \alpha_3\frac{1}{\bar{\Omega}}\frac{d\bar{\Omega}}{dt}}{\alpha_4}, \tag{3.6}$$

$$\frac{dp_1}{dt} = -\frac{\frac{q_{g1}}{\Omega_{01}\bar{\Omega}_1}(\alpha_4+\frac{\alpha_2}{G_1}) - (\alpha_2\alpha_3+\alpha_1\alpha_4)\frac{1}{\bar{\Omega}_1}\frac{d\bar{\Omega}_1}{dt}}{(\alpha_5+\alpha_6)\alpha_4 + (\alpha_7+\alpha_8)\alpha_2}, \tag{3.7}$$

$$\frac{ds_1}{dt} = -\frac{\frac{q_{g1}}{\Omega_{01}\bar{\Omega}_1G_1} + (\alpha_7+\alpha_8)\frac{dp_1}{dt} + \alpha_3\frac{1}{\bar{\Omega}}_1\frac{d\bar{\Omega}_1}{dt}}{\alpha_4}, \tag{3.8}$$

where the gas condensate factor for the internal and external zones at the corresponding pressures $(p,\ p_1)$ and condensate saturations $(s,\ s_1)$ is determined by the following expression:

$$G = \frac{\frac{\bar{\mu}(p)B_o(p)p\beta}{z(p)p_{at}}[1-c(p)\bar{\gamma}(p)] + \frac{S(p)}{\psi(s)}}{\frac{1}{\psi(s)} + \frac{\bar{\mu}(p)B_o(p)p\beta c(p)}{z(p)p_{at}}}, \tag{3.9}$$

where $\bar{\Omega} = \frac{\Omega}{\Omega_0}$ is the ratio of the current volume of gas-saturated pores in the internal zone to its initial value, $\bar{\Omega}_1 = \frac{\Omega_1}{\Omega_{10}}$ is the ratio of the current pore volume of the external zone to its initial value;

$$\begin{aligned}
\alpha_1 &= (1-s)\frac{p\beta}{z(p)p_{at}}[1-c(p)\bar{\gamma}(p)] - s\frac{S(p)}{B_o(p)},\\
\alpha_2 &= \frac{p\beta}{z(p)p_{at}}[1-c(p)\bar{\gamma}(p)] - \frac{S(p)}{B_o(p)},\\
\alpha_3 &= s\frac{1}{B_o(p)} - (1-B_o)\frac{p\beta c(p)}{z(p)p_{at}},\\
\alpha_4 &= \frac{1}{B_o(p)} - \frac{p\beta c(p)}{z(p)p_{at}},\\
\alpha_5 &= (1-s)\left\{\frac{p\beta}{z(p)p_{at}}[1-c(p)\bar{\gamma}(p)]\right\}',\\
\alpha_6 &= s\left[\frac{S(p)}{B_o(p)}\right]',\\
\alpha_7 &= s\left[\frac{1}{B_o(p)}\right]',\\
\alpha_8 &= (1-s)\left[\frac{p\beta c(p)}{z(p)p_{at}}\right]',
\end{aligned}$$

"$'$" means the derivative of p. Remaining notations have the same meaning as in [1, 2].

To solve systems of Equations (3.5), (3.6), and (3.7), (3.8), time dependences of pore volume for the corresponding zones of the formation are required. As such equations, we can use the equations obtained in [13]. Below we present these equations.

In the case of time-dependent elastic deformation

$$\frac{d\bar{\Omega}}{dt} = \frac{\bar{\phi}(p) - \bar{\phi}}{\tau_m}, \tag{3.10}$$

where $\bar{\phi}(p) = \frac{\phi(p)}{\phi_0}$ and $\bar{\phi}(p) = \exp[c_m(p-p_0)]$; $\phi_0,\ \tau_m$ are the initial value of the porosity coefficient and relaxation time, respectively; $\bar{\phi} = \frac{\phi}{\phi_0}$.

And in the case of time-dependent creeping deformation

$$\frac{d\bar{\Omega}}{dt} = c_m\frac{dp}{dt} + (p-p_0)(m_1 + c_m\gamma_m) + \gamma_m(1-\bar{\phi}), \tag{3.11}$$

where m_1 is the creep factor, $\gamma_m = \frac{1}{\tau_m}$.

Let us assume that in the external zone the reservoir is deformed according to the time-dependent elastic law, and the internal zone is subject to time-dependent creeping deformation. Then, taking (3.10) into account and (3.11), we rewrite Equations (3.5) and (3.7), respectively, in the following form:

$$\frac{dp}{dt} = -\frac{\frac{q_g - q_{g1}}{\Omega_0\bar{\Omega}}(\alpha_4 + \frac{\alpha_2}{G}) + (\alpha_2\alpha_3 + \alpha_1\alpha_4)\frac{1}{\bar{\Omega}}\left[(p-p_0)m_1 e^{-\gamma_m t}\right]}{(\alpha_5+\alpha_6)\alpha_4 + (\alpha_7+\alpha_8)\alpha_2 + (\alpha_2\alpha_3+\alpha_1\alpha_4)\frac{1}{\bar{\Omega}}(c_m + \frac{m_1}{\gamma_m} - \frac{1}{\gamma_m}m_1 e^{-\gamma_m t})}, \tag{3.12}$$

$$\frac{dp_1}{dt} = -\frac{\frac{q_{g1}}{\Omega_{01}\bar{\Omega}_1}(\alpha_4 + \frac{\alpha_2}{G_1}) - (\alpha_2\alpha_3 + \alpha_1\alpha_4)\frac{1}{\bar{\Omega}_1}\frac{\bar{\phi}(p_1) - \bar{\phi}}{\tau_m}}{(\alpha_5+\alpha_6)\alpha_4 + (\alpha_7+\alpha_8)\alpha_2}, \tag{3.13}$$

where the flow rate of gas filtering from the external zone to the internal zone (q_{g1}) is determined by (2.12).

It should be noted that other models of rock deformation can be taken into account in Equations (3.5) and (3.7) in a similar way [6], [7]. For example, when in the internal zone the rocks are time-dependent elastic, and in the external zone they are nonlinearly elastic (i.e. when $\frac{d\bar{\Omega}}{dt} = c_m \exp[c_m(p - p_0)]$), expressions for determining the pressures in the internal and external zones, respectively, will be written out from (3.5) and (3.7) taking (3.10) into account as follows:

$$\frac{dp}{dt} = -\frac{\frac{q_g - q_{g1}}{\Omega_0 \bar{\Omega}}(\alpha_4 + \frac{\alpha_2}{G}) + (\alpha_2\alpha_3 + \alpha_1\alpha_4)\frac{1}{\bar{\Omega}}\frac{\bar{\phi}(p) - \bar{\phi}}{\tau_m}}{(\alpha_5 + \alpha_6)\alpha_4 + (\alpha_7 + \alpha_8)\alpha_2}, \tag{3.14}$$

$$\frac{dp_1}{dt} = -\frac{\frac{q_{g1}}{\Omega_{01}\bar{\Omega}_1}(\alpha_4 + \frac{\alpha_2}{G_1})}{(\alpha_5 + \alpha_6)\alpha_4 + (\alpha_7 + \alpha_8)\alpha_2 - (\alpha_2\alpha_3 + \alpha_1\alpha_4)\frac{c_m}{\bar{\Omega}_1}e^{c_m(p_1 - p_0)}}. \tag{3.15}$$

Thus, systems of differential equations were obtained, consisting of Equations (3.6), (3.12), and (3.8), (3.13) (or (3.6), (3.14), and (3.8), (3.15)), the solutions of which, with a known well flow rate q_g, make it possible to determine the average reservoir pressure and condensate saturation in both parts of the reservoir at any time in the case when in the internal and external zones the rocks are subjected, respectively, to time-dependent creeping and time-dependent elastic deformations (or time-dependent elastic and nonlinear elastic deformations etc.).

4. Algorithm for simulation of development

The above approach allows us to determine the main indicators of the development of gas condensate deposits under various technological conditions, taking into account the difference in the nature of deformations of the near-wellbore zone and those remote from the well bottomhole. In this case, you can use the algorithm below.

(1) Input initial data

$$t = 0,\ p_w = p_0,\ p_k = p_0,\ p_1 = p_0,\ s = s_0,\ s_1 = s_0,\ \phi = \phi_0,\ k = k_0,\ q_1 = 0;$$

(2) Initial values are calculated:
gas condensate ratio $G_0 = \frac{1 - c(p_0)\bar{\gamma}(p_0)}{c(p_0)}$,
gas and condensate reserves (at $s_0 = 0$) $V_g = \pi R_e^2 h \phi_0 \left[\frac{1 - c(p_0)\bar{\gamma}(p_0)}{z(p_0)p_{atm}}\right]$, $V_o = \frac{V_g}{G_0}$;

(3) If we consider the case of a given gas production rate (percent per year of the initial balance reserves), the flow rate is determined by the following expression: $q_g = \frac{V_g n}{100}$ and goes to step "6.4", otherwise go to step "4";

(4) If the case of a given depression (Δp) is considered, the depression value is set, otherwise go to step "6";

(5) Bottomhole pressure is calculated by the expression $p_w = p_1 - \Delta p$;

(6) Calculation of well flow rate for gas (q_g) and condensate (q_o):
6.1. The values of $\varphi(p, s)$ and A, B, C are calculated for pressures p_w, p_{sr}, p_k using (2.3) and (2.16), respectively;
6.2. Pseudo depression is determined $H_k - H_w$ using (2.14);
6.3. The current gas flow rate (q_g) is calculated using (2.13).
6.4. The current value of the condensate flow rate is determined : $q_o = \frac{q_g}{G}$.

(7) Calculation of gas flow rate filtered through the boundaries of the internal and external zones (q_{g1}):
7.1. The values of the integrand function φ_1 and the approximation coefficients A_1, B_1, C_1 are calculated for the current pressure values p_1, p_{1sr}, p_k using (2.3) and (2.17), respectively;
7.2. Pseudodepression $\bar{H} - H_k$ is determined by expression (2.15);
7.3. The flow rate of gas filtering through the boundaries of the internal and external zones q_{g1} is calculated by (2.12).

(8) The current value of the gas condensate ratio G is calculated by (3.9).

(9) For time $t + \Delta t$, the current values of condensate saturation (s_1) and pressure (p_1) for the external zone by (3.8) together with (3.13) taking into account (3.10) are calculated.

(10) For time $t+\Delta t$, the current values of condensate saturation (s) and pressure (p) for the external zone by (3.6) together with (3.12) taking into account (3.11) are calculated.
(11) The current values of the accumulated gas and condensate production and their recovery factors are determined: $K_g = \frac{\sum_{t=0}^{t} q_g \Delta t}{V_g}$ and $K_k = \frac{\sum_{t=0}^{t} q_o \Delta t}{V_o}$.
(12) Checking the value of the reservoir pressure, if it is greater than its certain value as the final one, go to step "4", otherwise go to step "13";
(13) Output of results and halt.

5. Analysis of the results of computer simulations

Using the above algorithm, a computer simulator was created for the process under consideration and a series of calculations was performed. The research was carried out in two stages. At the first stage, the features of the influence of heterogeneity on the dynamics of the drop in reservoir pressure during development are studied. For this purpose, calculations were performed in two versions: in the first version, it is assumed that the drainage zone of the well consists of two parts. In this case, in the internal zone the rocks are creeping, while in the external zone they deform in a time-dependent elastic manner. In the second version, for comparison, calculations were performed for the case when the rocks are creeping throughout the entire formation.

In the second stage, the aim is to study the influence of the ratio of the dimensions of the outer and inner zones. In the second stage of the investigation, the goal is to study the influence of the ratio of the dimensions of the external and internal zones. Three variants for the R_e/r_k ratio are considered: 1.5, 5 and 10.

In all variants, calculations were performed based on the following initial data:
Initial reservoir pressure p_0=40.0 MPa;
Reservoir thickness $h = 20$ m.
Reservoir external boundary radius R_e=1000 m;
External zone radius $r_k = 500\,\text{m}$;
Well radius $r_s = 0.10$ m;
Initial reservoir permeability $k_0 = 0.1 * 10^{-12} m^2$;
Initial reservoir porosity coefficient ϕ_0= 0.2;
Porosity creep coefficient $m_1 = 3.4 * 10^{-7}$ MPa$^{-1} s^{-1}$.
Permeability creep coefficient $k_1 = \frac{m_1}{4}$.
Relaxation time $\tau_m = 0.23 \cdot 10^6$ s;
Elasticity coefficient of porosity $c_m = 0.001\,1/\text{MPa}$
Elasticity coefficient of permeability $\beta_k = 0.01\,1/\text{MPa}$.

First, let's look at the results of the first stage calculations. They are shown in Figure 2 and Figure 3 , where the curves plotted with a solid line correspond to the case when heterogeneity is taken into account, and the dotted curves illustrate the case when the well drainage area is considered homogeneous.

Figure 2 shows the change in reservoir pressure over time under the consideration conditions. As can be seen from a comparison of the curves, neglecting the rheological heterogeneity of the reservoir leads to some overvaluation of the current values of reservoir pressure. At the same time, at the beginning of development the difference between these variants increases, and at the end of the process the curves intersect. In the case under consideration, the maximum difference reaches up to 12.53%. This phenomenon is associated with the influence of reservoir pore compressibility, which is confirmed by the curves of formation pressure versus gas recovery factor in Figure 3. From the $p(K_g)$ curves it can be seen that the moment of maximum deviation of the curves corresponds to a value of η equal to 0.55.

As noted above, to identify the peculiarities of the influence of the size of the internal zone on the development process, calculations were performed for three different R_e/r_k ratios. The calculation results are presented in Figures 4-9, in which the curves labeled "1", "2" and "3" correspond to R_e/r_k values of 1.5, 5 and 10, respectively.

The curves in Figure 4 illustrate the dynamics of changes in reservoir pressure in a rheologically heterogeneous formation under the considered variants of R_e/r_k. As we can see, an increase in the radius of the internal zone leads to an increase in current reservoir pressures. If you look at the curves of well production versus formation pressure ($q_g(p)$) in Figure 5, it can be seen that as the radius of the internal zone increases, the current flow rates become lower.

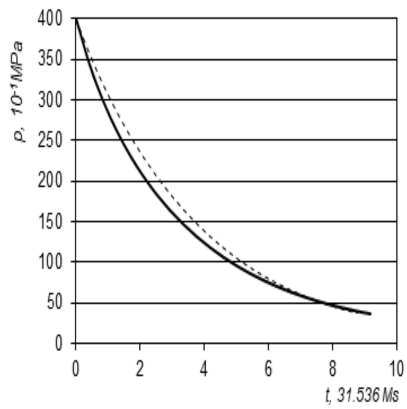

Figure 2. Dynamics of reservoir pressures in cases where heterogeneity is taken into account (solid line) and neglected (dashed line).

Moreover, at the same time, the flow rate of gas flowing from the external zone to the internal zone also decreases in Figure 6. The reason for the described phenomenon is the negative effect of increasing the radius of the internal zone, which has poorer permeability, on the flow process, similar to the skin effect.

It is also clear from the $q_{g1}(t)$ curves in Figure 6 that at the beginning of the process, when the redistribution is not yet completed, there is an intensive increase in the gas flow rate flowing from the external zone to the internal one. And this happens in all variants. As soon as the marked period ends, a decrease in q_{g1} begins in all variants. Moreover, the smaller the internal zone, the greater the current values of q_{g1}.

The nature of the $p(t)$ and $q_g(t)$ dependences illustrated in Figure 4 and Figure 7, respectively, led to changes in time of condensate saturation $\rho(t)$ and gas condensate factor $G(t)$. They are presented in Figure 8 and Figure 9, respectively. Thus, the intense drop in reservoir pressure in all variants caused intense condensation formation in Figure 8. In this case, at the smallest radius of the internal zone (curves 3), the lowest reservoir pressures and, consequently, high condensate saturations are observed. By the end of development, the stabilization of condensate saturation and its slight decrease are explained by partial evaporation. This change in condensate saturation led to the nature of the change in the gas condensate factor shown in Figure 9.

6. Conclusions

The paper considers the problem of flow of a gas-condensate system in the case when around a well, where the pressure is significantly lower than the average reservoir pressure, the reservoir rocks are subject to creeping deformations, and in a remote part of the reservoir the rocks are still compressed within the elastic limits. A solution to the problem of flow of a gas-condensate mixture to a well was obtained, taking into account the time-dependent deformation of reservoir rocks and the PVT properties of the gas-condensate system. An algorithm is proposed for calculating the main development indicators for the depletion of gas condensate deposits under the considered conditions.

A number of computer studies were carried out using the described algorithm. The results of these studies showed that taking into account changes in the rheological characteristics of the reservoir in the near-wellbore zone significantly improves the forecasting of the main development indicators. Thus, in the case when the creep of the near-wellbore zone is taken into account, the maximum difference in the current values of formation pressures reaches up to 12.53%. It corresponds to a gas recovery factor value of 0.55.

The obtained solution also takes into account the differences in the absolute permeability values of the well bottomhole zone and the remote part of the well drainage zone, which makes it possible to use it in studies of the skin effect.

The approach used in this work can also be used to solve problems of interpreting well-test data.

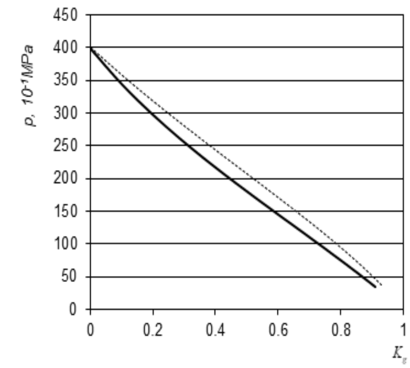

FIGURE 3. Dependence of reservoir pressures on the gas recovery factor in heterogeneous (solid line) and homogeneous reservoirs (dashed line).

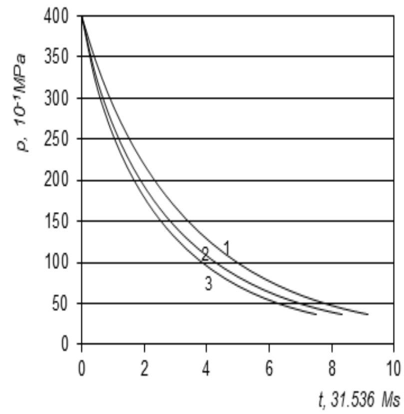

FIGURE 4. Dynamics of the reservoir pressure in a rheologically heterogeneous formation at different values of R_e/r_k: 1-1.5, 2-5 and 3-10.

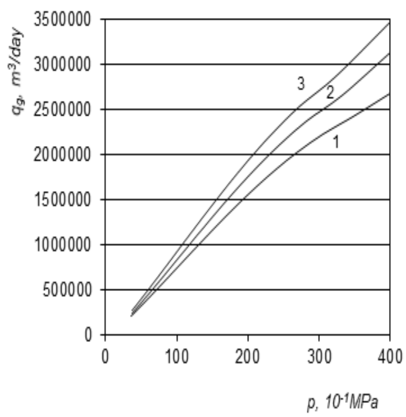

FIGURE 5. Dependency curves of gas flow rate in a rheologically heterogeneous formation at various values of R_e/r_k: 1-1.5, 2-5 and 3-10.

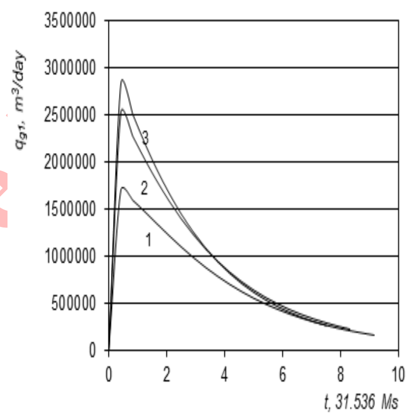

FIGURE 6. Dynamics of the curves of gas flow flowing from the external zone to the internal zone at different values of R_e/r_k: 1-1.5, 2-5 and 3-10.

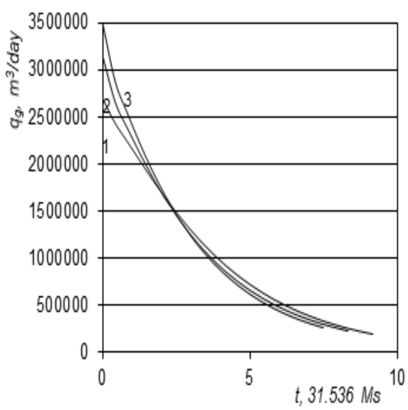

FIGURE 7. Dynamics of well flow rate at different values of R_e/r_k: 1-1.5, 2-5 and 3-10.

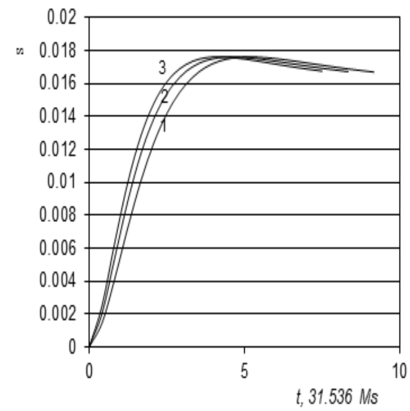

FIGURE 8. Dynamics of the condensate saturation coefficient at different values of R_e/r_k: 1-1.5, 2-5 and 3-10.

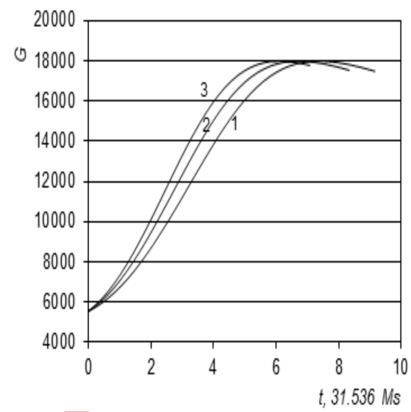

FIGURE 9. Dynamics of the gas condensate factor at various values of R_e/r_k: 1-1.5, 2-5 and 3-10.

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