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Modelling the transmission of Mpox with case study in Nigeria and Democratic Republic of Congo (DRC)

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Abstract

This paper focuses on the dynamics of Mpox, a viral disease, in Nigeria and the Democratic Republic of Congo (DRC), employing mathematical modeling and parameter estimation techniques. Utilizing optimization methods, the model parameters were calibrated to match the observed Mpox cases and deaths. The basic reproduction number ($\mathcal{R}_0$) was calculated for each region, indicating the disease's transmission potential, and a sensitivity analysis was conducted to identify key parameters influencing disease outcomes. Subsequently, numerical simulations were performed to assess the impact of intervention scenarios on Mpox cases and deaths. The primary goal is to create mathematical methods that can evaluate the risk of Mpox transmission and implement control measures in Nigeria and DRC, potentially extending the findings to other countries. Results show that reducing parameters related to transmission and progression significantly decreases disease burden, highlighting the importance of preventive measures. These findings provide valuable insights for policymakers and public health officials in designing effective strategies to mitigate Mpox's impact on human populations..

Keywords. Non-linear mathematical Mpox model, Control measure, Effective reproduction number, Sensitivity analysis, Case and death averted, Immunization.

2010 Mathematics Subject Classification. 91A40, 34D23.

1. Introduction

Infectious diseases remain one of the major challenges facing the landscape of global public health and this continuous calls for the need for innovative strategies that could comprehensively address their complex transmission dynamics. However, within the spectrum of the diseases mitigating the human population, mpox takes centre stage due to the goal of having an in-depth understanding of the transmission dynamics and providing the mitigating measure to combat the disease. Mpox is a viral disease caused by the mpox virus (MPXV), which belongs to Orthopoxvirus genus and this virus is understood to primarily target animals such as monkeys, squirrels, and rodents [19]. Moreover, the mpox belongs to the class of zoonotic disease and as a result, the disease is transmitted to the human population through animals while it is also discovered that the transmission is not limited to animals to humans. According to several researchers, it is understood that there could also be human-to-human transmission through close contact with infected individuals, particularly via respiratory droplets or contact with skin lesions or other body fluids. Besides these, other possible modes of transmission include contact with contaminated objects or surfaces known as environmental transmission. Some of the known symptoms of the disease infection are fever, headache, muscle pain, tiredness, etc. Similarly, according to the Centers for Disease Control and Prevention, the incubation period of mpox spans from $7-14$ days, while in some cases, the infection could vary between 5 and 21 days. Some of the symptoms

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of diseases are usually mild, and the larger proportion of the patients are known to naturally recover within a few weeks while those with weakened immune systems may exhibit severe symptoms and a longer time to recover [16]. As the incidence of mpox continues to rise, this further raises concern in public health and several approaches are into consideration as a means of controlling the outbreak. However, it is worth noting there is no safe approved treatment mechanism specifically for mpox virus infection at the moment as there is still continuous ongoing research on identifying a control measure specific for mpox. Nevertheless, research discovery suggests that smallpox control measures can also be adopted to combat the disease. However, the smallpox vaccine is currently unavailable as smallpox has been eradicated worldwide [21].

Several mathematical models have been formulated to understand the dynamics of mpox before, during, and after the 2022-2023 mpox outbreak. Some have focused on the transmission of mpox from animals to humans [23], [24], [25], [3], [17], [18], [26]. Some other studies focused on the transmission of mpox via sexual transmission or among the high-risk population [22], [27], [36], [37]. In addition, some models have also explored the impact of smallpox or mpox vaccination in curtailing the spread of mpox [9], [10]. Models to understand the co-interaction between mpox and other diseases have also been investigated [7], [22]

Motivated by the above modelling studies, there is a need to further understand the intrinsic dynamic of the mpox disease through the development of an appropriate mathematical model which would help to foster the understanding of the transmission dynamic in aiding the global population to reduce the disease burdens through enhanced access to vaccines and immunization programs. Consequently, our focus will be on providing recommendations to curtail the transmission of infectious diseases, particularly mpox disease, and to alleviate the collective disease burden within the populations of Nigeria and the Democratic Republic of Congo (DRC). To achieve this goal, we propose the implementation of a mathematical model that delineates the dynamics of Mpox. This model will provide the requisite scientific evidence to facilitate the control of Mpox disease, along with the necessary interventions to mitigate the impact of diseases on communities. The study's main goals will be addressed through the construction, analysis, and parameterization of the model using real data from Nigeria and the Democratic Republic of the Congo, as well as simulations using various scenarios. The rest of the paper is organized as follows: the model formulation and justification are presented in section 2 while our fitting to both the Nigeria and Democratic Republic of Congo (DRC) data together with the estimation of model parameters are presented in section 3. In addition, the section further discusses the spread of disease through the derivation of the reproduction number $\mathcal{R}_0$, and the sensitivity of the model parameters with respect to the rate of disease spread is further discussed. In section 4, we present the model simulations and subsequent research findings, encompassing the behavior of Mpox dynamics under various intervention scenarios, along with the cases and deaths averted due to Mpox under these scenarios. The conclusions and recommendations are outlined in section 5, offering a summary of our scientific findings and utilizing these insights to provide evidence-based recommendations for disease control strategies.

2. Methods

According to the World Health Organization (WHO) [35], Mpox transmission occurs from human to human, and also from animal to human. As a result, we develop a mathematical model that describes the dynamics of Mpox between the human and animal populations. The two-host population (humans and animals) are further sub-grouped into different compartments based on their epidemiological status as described in Table 1. The susceptible human population which is denoted by S_h contains the individuals who are Mpox-free but are prone to be infected by the virus after an effective contact with an infected host. This population is derived through immigration processes or by birth at a recruitment rate θ_h. As reported by WHO [35] there have been some cases of second Mpox infections reported, as a result of this, the susceptible population is modeled to also increase by the loss of immunity of the recovered individuals at the rate ε. After effective contact with an infected host, the susceptible population is reduced by the force of infection rate ϕ_1 which is defined as

$$\phi_1 = \frac{\alpha_h I_h + \alpha_r I_r}{N_h}, \tag{2.1}$$

The parameters α_h and α_r, represent the effective transmission probability per contact with infected humans and animals respectively. Lastly, all the human populations are reduced through natural death at the rate μ_h. The

exposed human population which we denoted by E_h contains the individuals who have been exposed to Mpox without any symptoms. This population is increased by the number of susceptible individuals who had successful contact with the infected host and further depopulated by the progression rate from exposed state to infectious state (denoted by τ) because of the incubation period reached. The infectious human population denoted as I_h contains the individuals who are infected with Mpox with an exhibition of symptoms and can transmit the virus. This population is increased by the progression rate of exposed individuals because of the incubation period reached. According to WHO [**?**], the incubation period of Mpox is between 1-21 days after which an individual is expected to be showing symptoms of the disease. The infectious human population is reduced due to a recovery from the infection at a rate σ and reduced by the disease-induced mortality rate δ_h. The individuals who died due to Mpox progress to increase the dead human population which is denoted as D_h. The recovered human population denoted as R_h contains the individuals who have been cleared of the disease after a period of time. This population is increased by the number of infectious humans that recover from infection at the rate σ, and further reduced by the loss of Mpox immunity at the rate ε. The animal population is classified into susceptible and infectious populations. The susceptible animal population is increased due to recruitment into the population by birth at the rate θ_r. This population is reduced following effective contact with infected animals at a rate ϕ_2 defined by

$$\phi_2 = \frac{\alpha_r I_r}{N_r}, \tag{2.2}$$

where the parameter α_r represents the effective transmission probability per contact with infected animals. The infectious animal population is increased by the number of susceptible animals who had successful contact with infected animals at the rate ϕ_2. Both the susceptible and infectious animal populations are reduced by a natural death at the rate μ_r. The total human and animal population denoted as N_h and N_r respectively are defined as $N_h = S_h + E_h + I_h + R_h + D_h$ and $N_r = S_r + I_r$.

Following the description above, the system of nonlinear ordinary differential equations (ODEs) utilized to investigate the dynamics of Mpox transmission in this study is defined by (2.3). The model's state variables and their description are given in Table 1, while the parameter description and values for the two countries are presented in Table 2 and Table 3 respectively.

TABLE 1. Description of the Mpox model variables.

Variable	Description
S_h	Population of susceptible individuals
E_h	Population of individuals exposed to Mpox
I_h	Population of Mpox infectious individuals
R_h	Population of recovered individuals
D_h	Population of dead individuals
S_r	Population of susceptible animals
I_r	Population of infectious animals

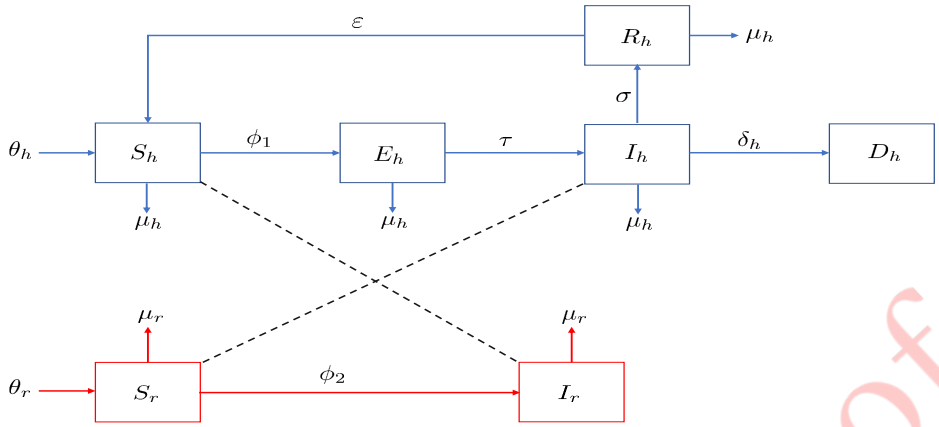


FIGURE 1. Flow chart of the Mpox model.

The mathematical model describing the Mpox transmission dynamic is presented in (2.3) while the visual representation of the model's flow is illustrated in Figure 1.

$$\begin{aligned}
\frac{dS_h}{dt} &= \theta_h - \phi_1 S_h - \mu_h S_h + \varepsilon R_h,\\
\frac{dE_h}{dt} &= \phi_1 S_h - (\mu_h + \tau) E_h,\\
\frac{dI_h}{dt} &= \tau E_h - (\mu_h + \delta_h + \sigma) I_h,\\
\frac{dR_h}{dt} &= \sigma I_h - (\mu_h + \varepsilon) R_h,\\
\frac{dD_h}{dt} &= \delta_h I_h,\\
\frac{dS_r}{dt} &= \theta_r - \phi_2 S_r - \mu_r S_r,\\
\frac{dI_r}{dt} &= \phi_2 S_r - \mu_r I_r.
\end{aligned} \tag{2.3}$$

3. Data fitting and Parameter Estimation

The model (2.3) is now used to study the dynamics of mpox in both Nigeria and the Democratic Republic of Congo (DRC). The model fitting was carried out by applying the *Fmincon* function in the MATLAB Optimization Toolbox [23]. To enhance our approach for data fitting and parameter estimation, we begin by meticulously tailoring our model to suit the specific characteristics of Mpox cases in both Nigeria and the Democratic Republic of Congo (DRC). This entails individually calibrating and fitting the model using the available data.

While certain parameters can be borrowed from existing models and their documented ranges in the literature, we prioritize accuracy by basing these estimates on reliable sources which have been cited accordingly. In order to account for the potential underreporting of Mpox cases, a common issue in disease surveillance, we incorporate a scaling mechanism. This involves adjusting the recorded daily infection figures to compensate for the likelihood of

missed cases. To achieve this, we introduce a detection probability parameter (p), the value of which is also subject to estimation.

The fmincon optimization tool utilizes the least squares method, which is very efficient and reliable. The method seeks to fit the observed data sets, Y_i, with the estimated values, X_i, such that; the sum of squares of errors between the observed and fitted curve is minimal. The sum of squares error, SSE, is illustrated mathematically as:

$$SSE = \sum_{i=1}^{k} (Y_i - X_i)^2.$$

Available data sets on the Mpox outbreak from the Nigeria Center for Disease Control (NCDC) [21] shall be used for the fitting for Nigeria. In particular, we shall use the cumulative number of confirmed Mpox cases and deaths from September 2017 to January 2023 for the model fitting for Nigeria.

The following parameters in model (2.3) are estimated from the fitting of the model to the annual cumulative number of confirmed cases and deaths for Nigeria: Effective transmission probability per contact with infected humans (α_h), Effective transmission probability per contact with infected animals (α_r), immunity waning rate of recovered humans with Mpox (ε), the Progression rate of exposed humans to infected humans population (τ), disease-induced death rate of humans (δ_h), the recovery rate of Mpox humans (σ).

The total population of Nigeria, based on World Bank estimate is 218,541,212 [29]. Also, the life expectancy for Nigeria is 62.6 years [29]. Therefore, the human natural death rate is set as, $\mu_h = \frac{1}{62.6}$ per year. Since the total Nigerian population is, $N(0) = 218,541,212$, we set the human recruitment rate, θ_h to be $\frac{218,541,212}{62.6}$. The initial conditions used for the model fitting with Nigeria data are set as follows $S_h(0) = 218,500,000$, $I_h(0) = 88$, $D_h(0) = 8$, $S_r(0) = 50,000$, $I_r(0) = 2000$. Also, certain information is not available to us at the onset of the data fitting from 2017, such as the initial number of individuals exposed to Mpox and recovered individuals. We have taken into consideration the fact that, at the onset of the simulation, it is expected that there would be some individuals exposed to Mpox and others who have also recovered from infection within the population (since case detection and testing for Mpox might not have begun at that time). Therefore, it is necessary to estimate the potential values of other initial conditions: $E_h(0)$ and $R_h(0)$. For the fittings of the model to the cumulative confirmed Mpox cases and deaths using Nigeria data, depicted by Figures 2 and 3, the obtained R^2 values were 0.83 and 0.85, showing a good fit with our model. Estimated parameters and initial conditions using Nigeria data are presented in Table 2.

TABLE 2. Description of model parameters, initial conditions, and threshold quantity for Nigeria.

Parameter	Description	Value	Source
θ_r	Recruitment rate of animals	0.2000	[24]
μ_r	Mortality rate of animals	0.0020	[24]
θ_h	Recruitment rate of susceptible humans	3.4911×10^6	Estimated from [31]
μ_h	Natural death rate of humans	0.0160	Estimated from [31]
δ_h	Disease-induced death rate of humans	0.1233	Fitted
α_h	Effective transmission probability per contact with infected humans	0.5712	Fitted
α_r	Effective transmission probability per contact with infected animals	0.8489	Fitted
τ	Progression rate of exposed humans to infected humans population	0.0066	Fitted
σ	Recovery rate of Mpox humans	9.2874×10^{-4}	Fitted
ε	Immunity waning rate of recovered humans with Mpox	0.4233	Fitted
Initial condition			
$S_h(0)$	Initial population of susceptible humans	218,500,000	Estimated
$E_h(0)$	Initial population of exposed humans	14	Fitted
$I_h(0)$	Initial population of infectious humans	88	[21]
$R_h(0)$	Initial population of recovered humans	16	Fitted
$D_h(0)$	Initial population of dead humans	8	[21]
$S_r(0)$	Initial population of susceptible animals	50,000	Assumed
$I_r(0)$	Initial population of infectious humans	2,000	Assumed
Threshold Quantity			
$\mathcal{R}_{0N}$	Basic reproduction number of Mpox in Nigeria	1.2027	Estimated

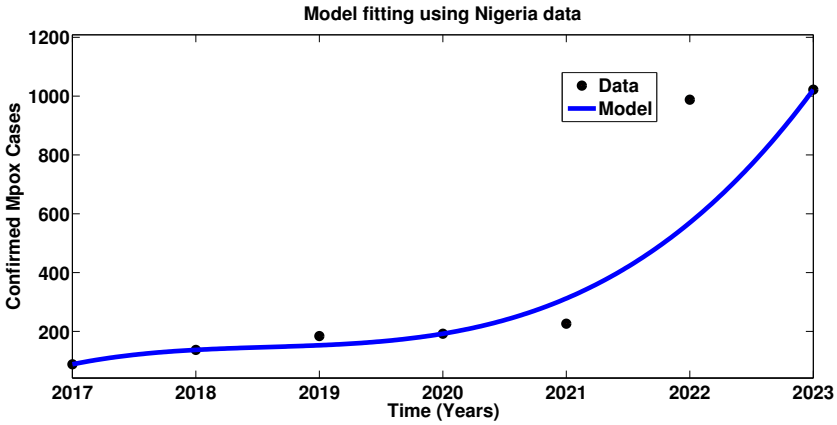


FIGURE 2. Fitting of the model to the Confirmed Mpox cases For Nigeria.

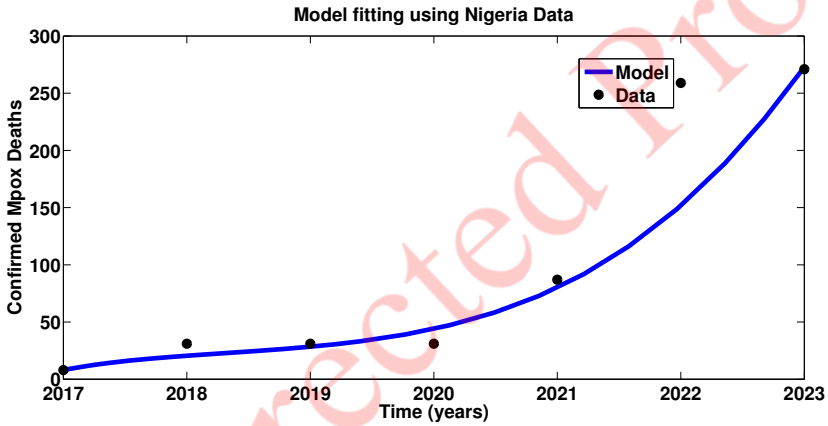


FIGURE 3. Fitting of the model to the Confirmed Mpox Deaths For Nigeria.

Available data sets from the WHO African Region [1] shall be used for the fitting for the Democratic Republic of Congo. Particularly, the cumulative number of confirmed Mpox cases for DRC from July 2022 to May 2023 shall be used for the model fitting for DRC.
The total population of DRC, based on World Bank estimate is 99,010,212 [29]. Also, the life expectancy for DRC is 62.4 years [32, 33]. Therefore, the human natural death rate for DRC is set as, $\mu_h = \frac{1}{62.4}$ per year. Since the total DRC population is, $N(0) = 99,010,212$, we set the human recruitment rate, θ_h to be $\frac{99,010,212}{62.4}$. The initial conditions used for the model fitting with DRC data are set as follows $S_h(0) = 99,000,000$, $I_h(0) = 2$, $S_r(0) = 50,000$, $I_r(0) = 2000$. Also, certain information which are not available to us at the onset of the data fitting from 2017, such as the initial number of individuals exposed to mpox and recovered individuals, and those who have died due to Mpox.

We have equally taken into consideration the fact that, at the onset of the simulation, it is expected that there would be some individuals exposed to Mpox and others who have also recovered from or died due to Mpox infection within the population. Thus, it is important to estimate the likely values of these initial conditions: $E_h(0)$, $D_h(0)$ and $R_h(0)$. For the fitting of the model to the cumulative confirmed Mpox cases using DRC data, depicted by Figure 4, the obtained R^2 value was 0.97, showing a very good fit with our designed model. Estimated parameters and initial conditions using Nigeria data are presented in Table 3.

TABLE 3. Description of model parameters, initial conditions, and threshold quantity for DRC.

Parameter	Description	Value	Source
θ_r	Recruitment rate of animals	0.2000	[24]
μ_r	Mortality rate of animals	0.0020	[24]
θ_h	Recruitment rate of susceptible humans	1.5867×10^6	Estimated from [29]
μ_h	Natural death rate of humans	0.0160	Estimated from [33]
δ_h	Disease-induced death rate of humans	0.0501	Fitted
α_h	Effective transmission probability per contact with infected humans	0.1005	Fitted
α_r	Effective transmission probability per contact with infected animals	0.1284	Fitted
τ	Progression rate of exposed humans to infected humans population	0.0428	Fitted
σ	Recovery rate of Mpox humans	9.2874×10^{-4}	Fitted
ε	Immunity waning rate of recovered humans with Mpox	0.9094	Fitted
Initial condition			
$S_h(0)$	Initial population of susceptible humans	99,000,000	Estimated
$E_h(0)$	Initial population of exposed humans	22	Fitted
$I_h(0)$	Initial population of infectious humans	2	Estimated from [1]
$R_h(0)$	Initial population of recovered humans	30	Fitted
$D_h(0)$	Initial population of dead humans	5	Fitted
$S_r(0)$	Initial population of susceptible animals	50,000	Assumed
$I_r(0)$	Initial population of infectious humans	2,000	Assumed
Threshold Quantity			
$\mathcal{R}_{0D}$	Basic reproduction number of Mpox in DRC	1.1038	Estimated

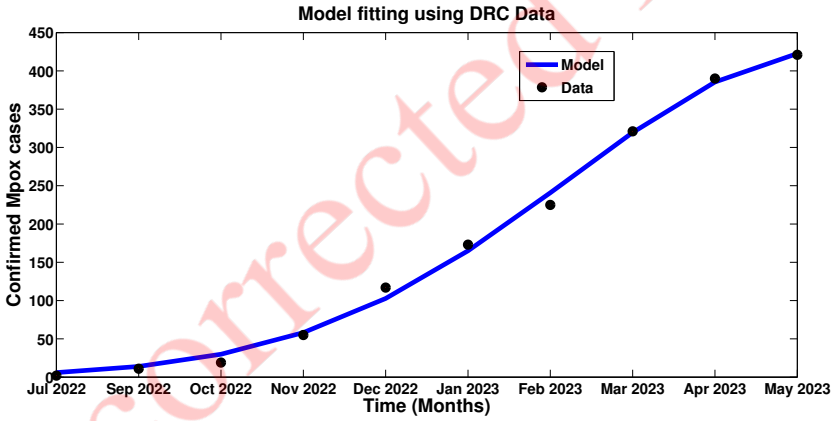


FIGURE 4. Fitting of the model to the Confirmed Mpox cases For Democratic Republic of Congo (DRC).

3.1. **Basic reproduction number.** The basic reproduction number is an epidemiologic metric that is used in describing the spread of infectious agents like viruses, bacteria, and fungi to mention a few [13]. While using mathematical models as an important tool in investigating the dynamics of diseases and possible control strategies that can help facilitate the reduction of the threat they pose to humanity, the basic reproduction number is used as an important threshold quantity in estimating the transmission potential of a disease in a given population. The basic reproduction number $\mathcal{R}_0$ is defined as an average secondary case that a single infective individual can produce in a population that is completely susceptible. According to [13, 15], an outbreak of a disease is expected to continue in a population if the basic reproduction number is above unity ($\mathcal{R}_0 > 1$), and when the reproduction number is below unity ($\mathcal{R}_0 < 1$), the outbreak of the disease is expected to end. In epidemiological models, the potential size of a disease outbreak or epidemic is often based on the abundance of the reproduction number. To determine the basic reproduction number of Mpox in Nigeria and DRC, which are the scope of this study, we employed the next-generation matrix technique

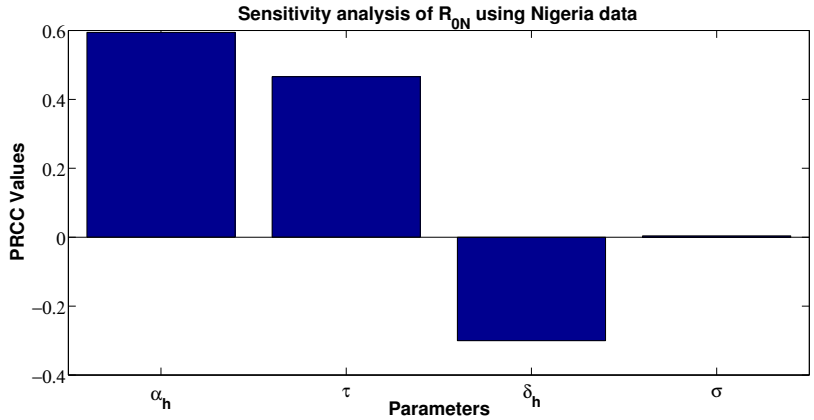
as given in [15], and the expression of the reproduction number is obtained as

$$\mathcal{R}_0 = max\left\{\mathcal{R}_h, \mathcal{R}_r\right\} = max\left\{\frac{\alpha_h \tau}{(\mu_h+\tau)(\mu_h+\delta_h+\sigma)}, \frac{\alpha_r}{\mu_r}\right\}. \tag{3.1}$$

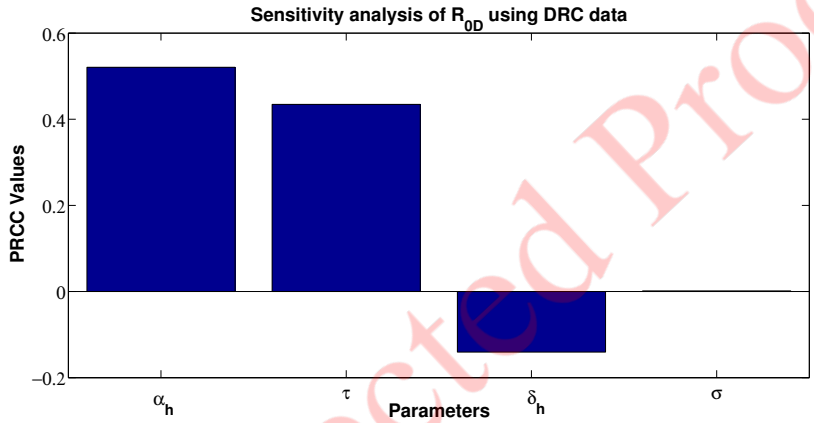
From the above Equation (3.1), $\mathcal{R}_h$ is the human reproduction number, while $\mathcal{R}_r$ is the animal reproduction number. It is important to emphasize that the value of the reproduction number of a disease is reported as a single numeric value or low-high range, and this estimate depends on many variables such as the location or region where the outbreak occurs and the density of the population [28]. Since in this work, we are investigating the dynamics of Mpox in two different regions named Nigeria and DRC, we will estimate their respective basic reproduction number by substituting the parameter values that were obtained from fitting the developed model to real data from each region. By doing this, the basic reproduction number of Mpox in Nigeria (denoted as $\mathcal{R}_{0N}$), and the reproduction number of Mpox in DRC (denoted as $\mathcal{R}_{0D}$) are obtained as $\mathcal{R}_{0N} = 1.2027$, and $\mathcal{R}_{0D} = 1.1038$ respectively.

3.2. **Sensitivity analysis.** Sensitivity analysis is an effective mathematical tool that is used by experts in all various kind of fields to determine how different values of independent variables affect a specific dependent variable. In the mathematical modeling of infectious diseases, the use of different parameters increases the uncertainty of the model result, as a result, the selection of parameter values is a critical part of mathematical modeling since the uncertainty of the input parameter values determines disease dynamics in the population. Following the parameterization of the model with the real data from our two case studies Nigeria and DRC, we perform a sensitivity analysis to help measure the suitability of the Mpox model output by investigating the impact of each parameter on the respective threshold quantity, infected human population, and dead human population. This task was done to determine which parameter(s) significantly impact the model output.

In this work, we employed the partial rank correlation coefficient (PRCC) to carry out the global sensitivity analysis based on each country considered in this study. The PRCC values obtained from the sensitivity analysis of the reproduction number, infected human population, and dead human population for the Nigeria and DRC region are presented through a bar graph in Figure 5a through Figure 7b. The sensitivity analysis of the Mpox reproduction number in Nigeria and DRC are depicted in Figure 5a and Figure 5b respectively. It is important to note that we investigate the sensitive nature of all the parameters that are included in the reproduction number except for the natural death rate of humans. This is because realistically, the natural death of humans cannot be controlled since humans can die due to unrelated Mpox death. As a result of this, epidemiologically, the natural death rate of humans cannot be considered as a contributing factor in increasing or decreasing disease burden in the human population. In Figure 5a and 5b, the result shows that the effective transmission probability per contact with infected humans (α_h), and the progression rate of exposed humans to infected humans population (τ) has the highest positive PRCC values, while the Mpox induced death rate of humans (δ_h) has the highest negative PRCC value.
A simple interpretation of the sensitivity analysis results is that an increase in the parameter value of any of the parameters with a positive PRCC value will directly increase the value of the dependent variable, while a decrease in the parameter value of any of the parameters with a positive PRCC value will reduce the value of the dependent variable. Similarly, an increase in the parameter value of any of the parameters with a negative PRCC value will decrease the value of the dependent variable, and vice versa. Based on this interpretation, it can be deduced from Figure 5a and 5b that an increase in the dependent variable (Mpox reproduction number in Nigeria $\mathcal{R}_{0N}$ and Mpox reproduction number in DRC $\mathcal{R}_{0D}$) is associated with an increase in the effective transmission probability per contact with infected humans (α_h), and the progression rate of exposed humans to infected humans population (τ). This implies that to reduce the numeric value of the reproduction number of Mpox in either of the countries, the transmission probability per contact with infected humans and the progression rate of exposed humans to an infectious state need to be reduced. In addition, an increase in the reproduction number of Mpox in Nigeria and DRC is associated with a decrease in the Mpox-induced death rate. These results inform policymakers that implementing preventive measures that will reduce disease transmission between humans and implementing effective care facilities that can help reduce the progression of exposed humans into infectious states would be effective in reducing the Mpox burden in the human population.



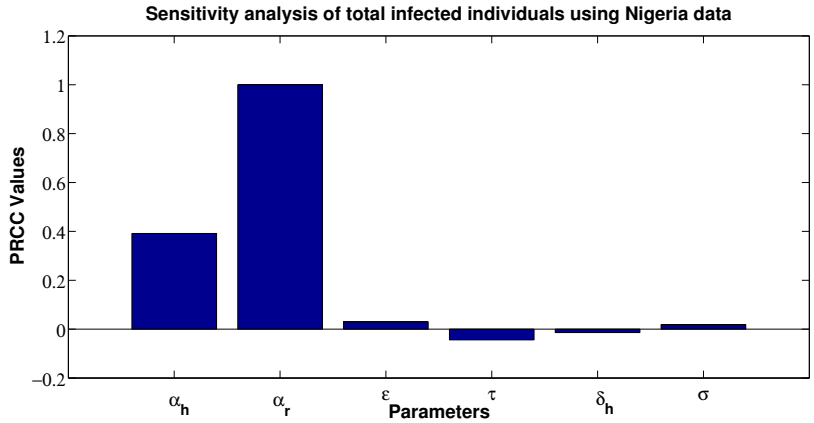
(A) Sensitivity analysis of $\mathcal{R}_{0N}$ for Nigeria.



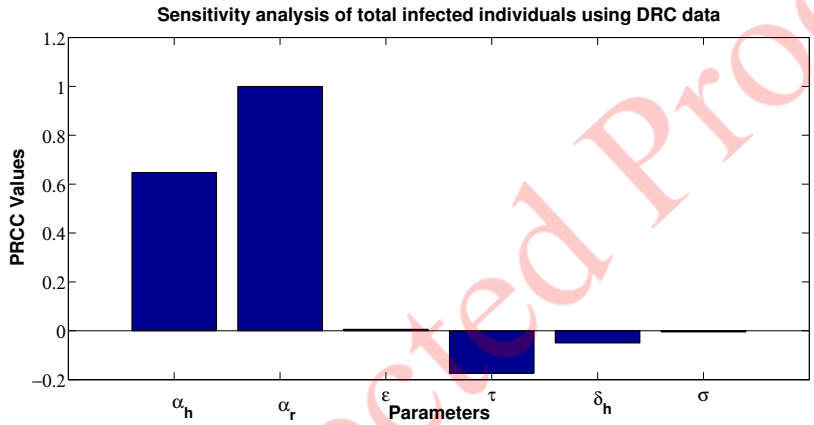
(B) Sensitivity analysis of $\mathcal{R}_{0D}$ for DRC.

The sensitivity analysis of the total infected human population in Nigeria and DRC is depicted in Figure 6a and 6b. The result shows that an increase in the Mpox burden due to the abundance of the infected human population, is associated with an increase in the effective transmission probability per contact with infected humans (α_h), and effective transmission probability per contact with infected animals (α_r). In Figure 7a and 7b, the sensitivity analysis of the dead human population in Nigeria and DRC is shown. The result shows that an increase in the Mpox-induced death rate of humans (δ_h), and the progression rate of exposed humans to infected human population (τ) is associated with an increase in the abundance of the dead humans' populace in both countries.

Based on the significance of the effective transmission probability per contact with infected humans (α_h), and the progression rate of exposed humans to infected humans population (τ) on the abundance of the reproduction number of Mpox in both countries, we investigate the impact of the two parameters simultaneously on the reproduction number of Mpox in Nigeria and DRC as shown in Figure 8a and 8b. It is noted that the parameter (α_h) is more influential in contributing to the increase in the reproduction number of Mpox in Nigeria such that for the value of the threshold quantity $\mathcal{R}_{0N}$ to be below unity, the value of (α_h) has to be at least below 0.17. Similarly, the effective transmission probability per contact with infected humans (α_h) is a contributing factor to the abundance of the reproduction number of Mpox in DRC such that for the value of the threshold quantity $\mathcal{R}_{0D}$ to be reduced below one, the value of (α_h) has to be at least below 0.10. In general, reducing the effective transmission probability per contact with infected humans (α_h), and the progression rate of exposed humans to infected human population (τ) simultaneously will reduce the reproduction number of Mpox in both countries. Thus, preventive and control strategies that will



(A) Sensitivity analysis of total infected individuals for Nigeria.

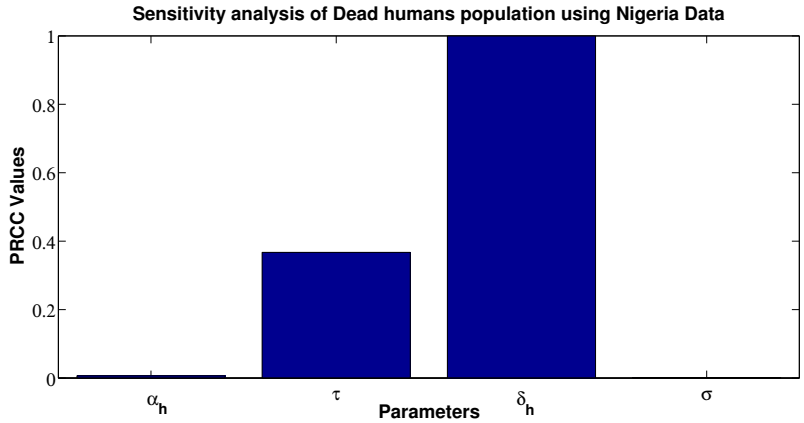


(B) Sensitivity analysis of total infected individuals for DRC.

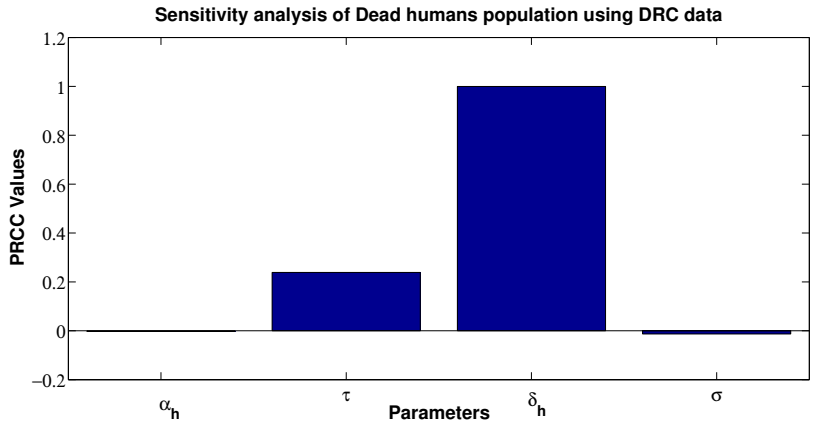
reduce the effective transmission probability per contact with infected humans, and the progression rate of exposed humans to infected humans population will reduce the Mpox burden in the human population.

4. Numerical Simulation

We perform a numerical simulation to investigate different scenarios of interventions that can impact disease burden, to estimate the Mpox cases and deaths averted in each country. We perform a numerical simulation to investigate the impact of implementing different scenarios of interventions on Mpox cases and deaths averted in each country. The potential threat Mpox poses to the human population cannot be overstated; as a result, it is critical to examine how the Mpox burden can be mitigated in each country. Because of the risk that Mpox poses to human health, intervention techniques are essential to reduce the disease's burden on the human population. These can be achieved by implementing preventive or control mechanisms like vaccination to reduce the potential of disease spread in the human populace. In this section, we investigate the impact of the most influential parameters on the respective threshold quantities as determined by the sensitivity analysis. These parameters include the effective transmission probability per contact with infected humans (α_h), the effective transmission probability per contact with infected animals (α_r), and the progression rate of exposed humans into their infectious state (τ). Based on the result from the sensitivity analysis, it was determined that an increase in any of these parameter values will increase the respective reproduction number, and as a result, will upsurge disease burden.



(A) Sensitivity analysis of dead human population for Nigeria.

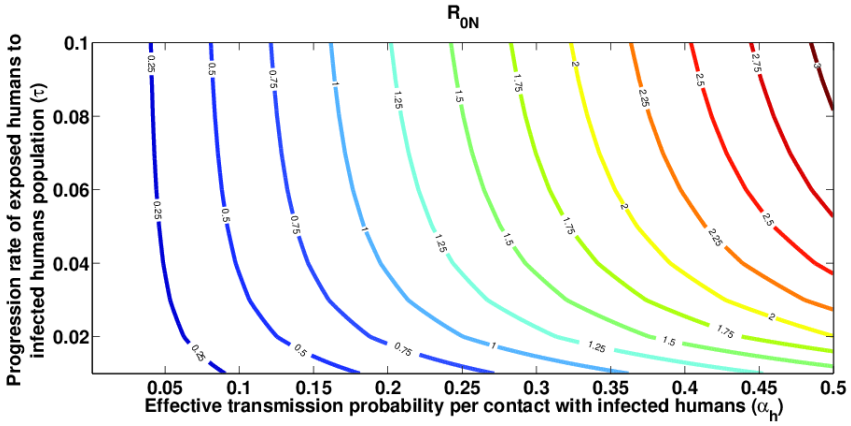


(B) Sensitivity analysis of dead human population for DRC.

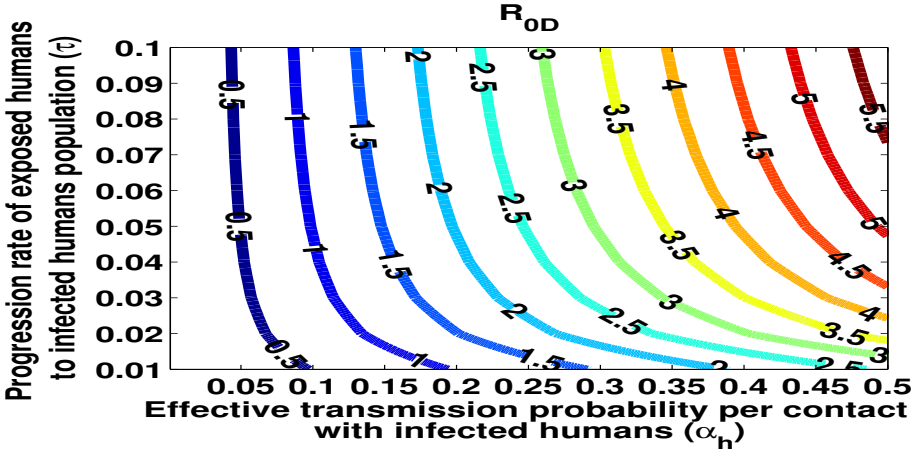
Based on this fact, we categorize three different intervention scenarios that were targeted at reducing each parameter value (as a result of preventive or control measures) based on percentage, to investigate both Mpox cases and death projection and also Mpox cases and deaths averted in both countries. Simulations were computed with the baseline parameter values obtained from data fitting and estimation for each country, and three intervention scenarios were also simulated to estimate Mpox cases and deaths averted as a result of an intervention.

4.1. **Mpox cases and death averted in Nigeria.** Since the Mpox cases and deaths data from Nigeria are reported in years, we simulate all scenarios up to 20 years to forecast the number of expected projections and averted cases and deaths. The details of each intervention scenario, Mpox cases projection and averted, and Mpox deaths projection and averted by using Nigeria data are presented in Table 4. The associated graphical representation is presented in Figure 9 through Figure 12.

Overall, the result shows that the higher the reduction in the percentage of the effective transmission probability per contact with infected humans (α_h), the effective transmission probability per contact with infected animals (α_r), and the progression rate of exposed humans into their infectious state (τ), the higher the number of cases and deaths averted. For instance, by implementing the Intervention C scenario, which entails a reduction of (α_h) by 91.3%, (α_r) by 91.3%, and (τ) by 92.4%, the projection of the number of Mpox cases is estimated as 49 in 20 years, leading to a total of 18,611 averted cases. Similarly, the number of Mpox deaths projection with the implementation of Intervention C is estimated as 110 in the next 20 years, thus leading to a total of 14,940 averted deaths.



(A) Contour plot of $\mathcal{R}_{ON}$ as a function of α_h and τ for Nigeria.



(B) Contour plot of $\mathcal{R}_{OD}$ as a function of α_h and τ for DRC.

TABLE 4. Estimated Mpox cases and deaths projection, and Mpox cases and deaths averted in Nigeria in 20 years under different Interventions.

Scenario	% of Reduction			Cases Projection	Cases Averted	Deaths Projection	Deaths Averted
	α_h	α_r	τ				
No Intervention	0	0	0	18,660	0	15,050	0
Intervention A	47.5	41.1	54.6	3,778	14,882	2,781	12,269
Intervention B	82.5	70.6	84.9	316	18,344	252	14,798
Intervention C	91.3	91.3	92.4	49	18,611	110	14,940

This result suggests to policymakers that efforts should be made to implement preventive measures that will help reduce the effective transmission of Mpox in the population. This includes but is not limited to the implementation of vaccine usage against the disease. Additionally, control measures that will reduce the progression rate of exposed individuals into their infectious condition should be implemented to reduce the disease burden over a period of time. This includes but is not limited to the creation of medical facilities that can support the diagnosis and treatment of infected individuals.

4.2. **Mpox cases and death averted in DRC.** We follow the same approach that was used earlier in estimating Mpox cases and deaths in Nigeria. However, it is important to mention that we simulate all scenarios up to 40 months

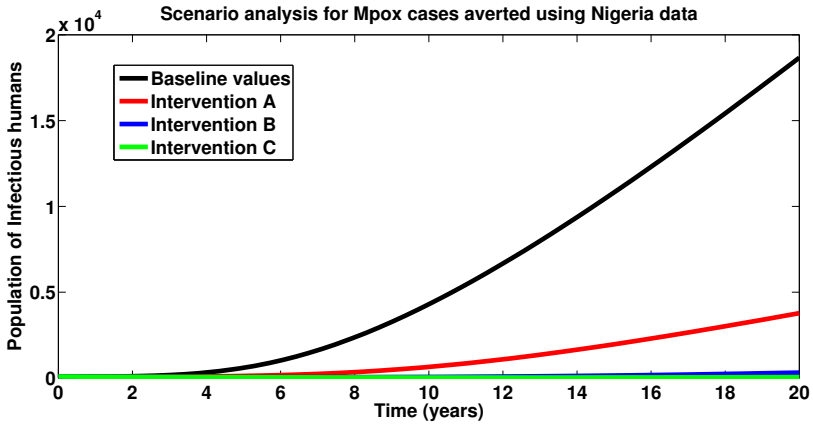


FIGURE 9. Scenario analysis for Mpox cases averted using Nigeria data. Here, Baseline values: $\alpha_h = 0.5712, \alpha_r = 0.8489, \tau = 0.0066$; Intervention A: $\alpha_h = 0.3, \alpha_r = 0.5, \tau = 0.003$; Intervention B: $\alpha_h = 0.1, \alpha_r = 0.25, \tau = 0.001$; Intervention C: $\alpha_h = 0.05, \alpha_r = 0.15, \tau = 0.0005$.

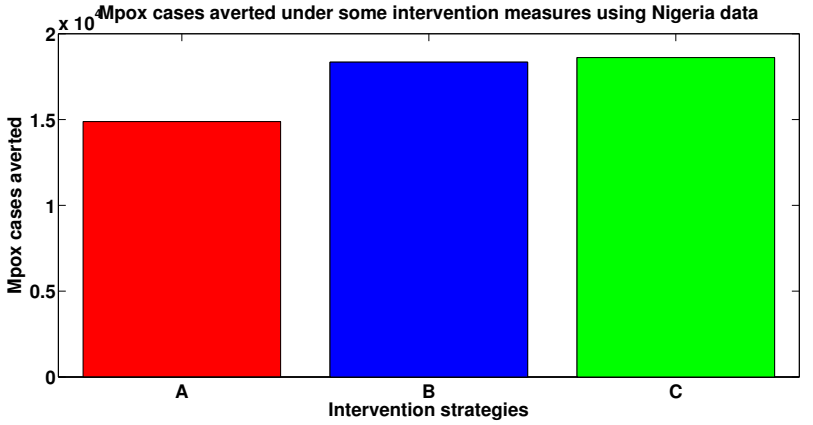


FIGURE 10. Mpox cases averted under different intervention measures using Nigeria data.

to forecast the number of expected projections and averted cases and deaths in DRC since the Mpox cases and deaths data from DRC are reported in months. The details of each intervention scenario, Mpox cases projection and averted, and Mpox deaths projection and averted by using DRC data are presented in Table 5. The associated graphical representation is presented in Figure 13 through Figure 16

TABLE 5. Estimated Mpox cases and deaths projection, and Mpox cases and deaths averted in DRC in 40 months under different Interventions.

Scenario	% of Reduction			Cases Projection	Cases Averted	Deaths Projection	Deaths Averted
	α_h	α_r	τ				
No Intervention	0	0	0	22,510	0	11,950	0
Intervention D	50.3	37.7	30.0	4,873	17,637	2,864	9,086
Intervention E	75.1	68.9	76.6	437	22,073	314	11,636
Intervention F	90.1	84.4	98.8	80	22,430	69	11,881

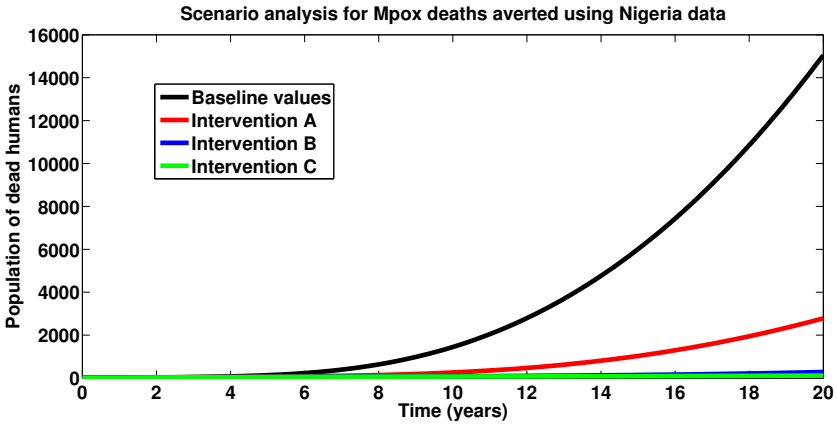


FIGURE 11. Scenario analysis for Mpox deaths averted using Nigeria data. Here, Baseline values: $\alpha_h = 0.5712, \alpha_r = 0.8489, \tau = 0.0066$; Intervention A: $\alpha_h = 0.3, \alpha_r = 0.5, \tau = 0.003$; Intervention B: $\alpha_h = 0.1, \alpha_r = 0.25, \tau = 0.001$; Intervention C: $\alpha_h = 0.05, \alpha_r = 0.15, \tau = 0.0005$.

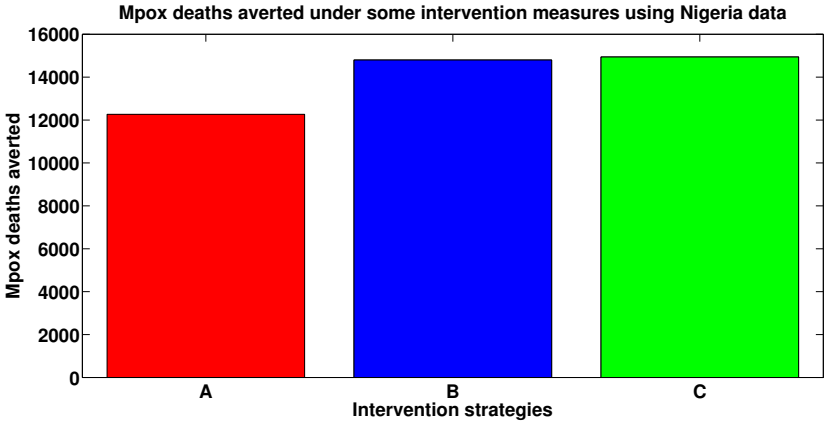


FIGURE 12. Mpox deaths averted under different intervention measures using Nigeria data.

Similar to the results obtained from simulating the impact of interventions on Mpox burden in Nigeria, our results here also show that the higher the reduction in the percentage of the effective transmission probability per contact with infected humans (α_h), the effective transmission probability per contact with infected animals (α_r), and the progression rate of exposed humans into their infectious state (τ), the higher the number of Mpox cases and deaths averted. For example, by implementing the Intervention F scenario, which entails a reduction of α_h by 90.1%, α_r by 84.4%, and τ by 98.8%, the projection of the number of Mpox cases in DRC is estimated as 80 in 40 months, leading to a total of 22,430 averted cases. Similarly, the number of Mpox deaths projection with the implementation of Intervention F in DRC is estimated as 69 in the next 40 months, thus leading to a total of 11,881 averted deaths. This result recommends to the health policymakers that efforts should be made towards the implementation of preventive measures that will help reduce the effective transmission of Mpox in the population.

5. Conclusion and recommendations

Since early May 2022, many instances of Mpox have been reported from non-endemic locations, with several reported cases continuing in endemic countries. As a result of this, the Mpox burden has become a global threat to the human

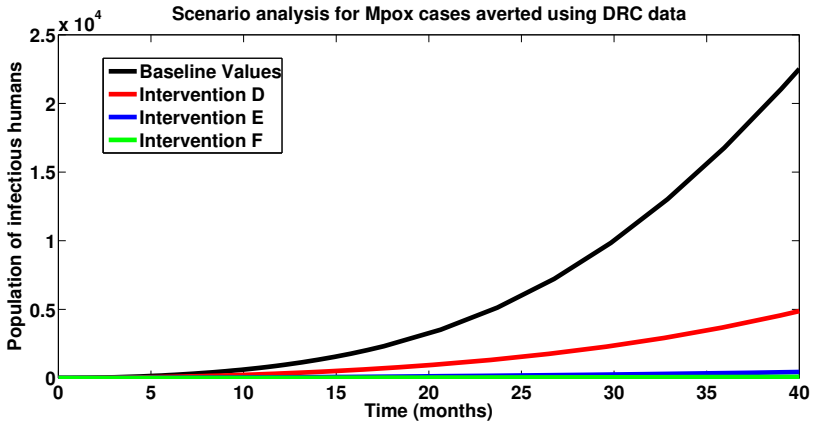


FIGURE 13. Scenario analysis for Mpox cases averted using DRC data. Here, Baseline values: $\alpha_h = 0.1005, \alpha_r = 0.1284, \tau = 0.0428$; Intervention D: $\alpha_h = 0.05, \alpha_r = 0.08, \tau = 0.03$; Intervention E: $\alpha_h = 0.025, \alpha_r = 0.04, \tau = 0.01$; Intervention F: $\alpha_h = 0.010, \alpha_r = 0.02, \tau = 0.0005$.

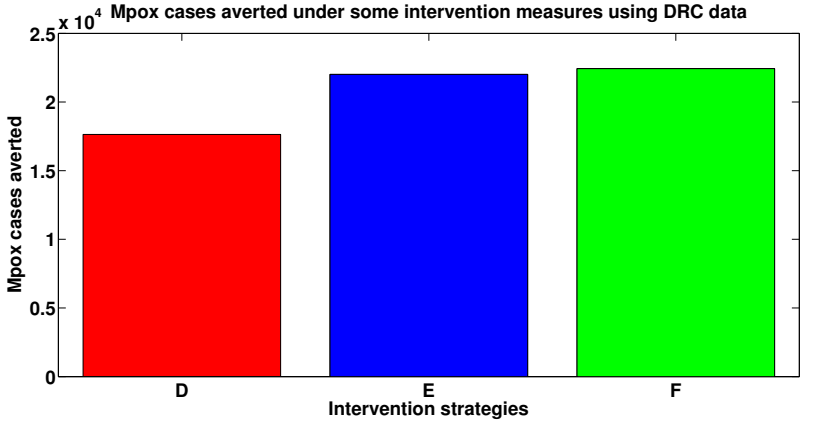


FIGURE 14. Mpox cases averted under different intervention measures using DRC data.

population. According to the director of Africa Centres for Disease Control and Prevention, in the first five months of 2022, one thousand four hundred and five Mpox cases and sixty-two deaths have been reported in four of the endemic African regions which include Nigeria and the Democratic Republic of Congo [1]. In order to support the global population in reducing the Mpox burden, particularly in Nigeria and DRC, this study is presented to govern the decision-making of policymakers in enhancing access to vaccines and immunization programs that focus on disease reduction in the human population.

In this work, we develop a deterministic mathematical model to study the dynamic of Mpox disease in Nigeria and DRC. Particularly, we investigated the burden of the disease in these regions by forecasting the expected number of cases in the absence of preventive or control interventions and estimating the number of averted cases and deaths due to the implementation of some interventions. The formulated model was used to obtain the respective parameter values in each country by fitting the model using the reported data from each region. These parameter values were used to estimate the basic reproduction number of each country and further used in simulating different intervention scenarios to estimate cases and deaths projections and averted. The core results from this study are listed below:

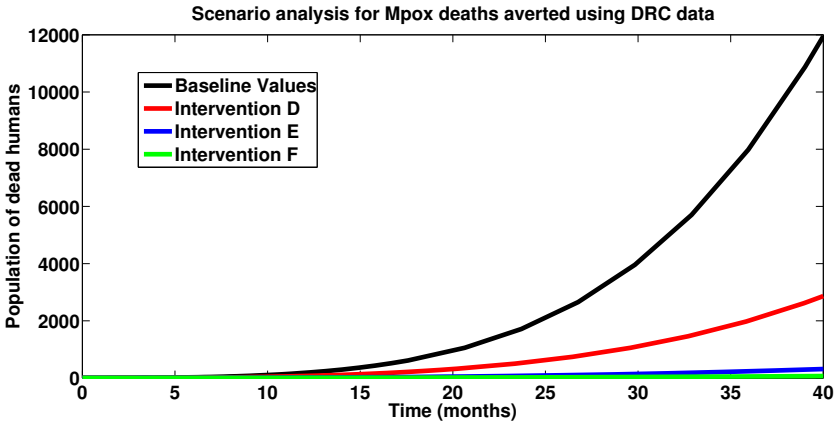


FIGURE 15. Scenario analysis for Mpox deaths averted using DRC data. Here, Baseline values: $\alpha_h = 0.1005, \alpha_r = 0.1284, \tau = 0.0428$; Intervention D: $\alpha_h = 0.05, \alpha_r = 0.08, \tau = 0.03$; Intervention E: $\alpha_h = 0.025, \alpha_r = 0.04, \tau = 0.01$; Intervention F: $\alpha_h = 0.010, \alpha_r = 0.02, \tau = 0.0005$.

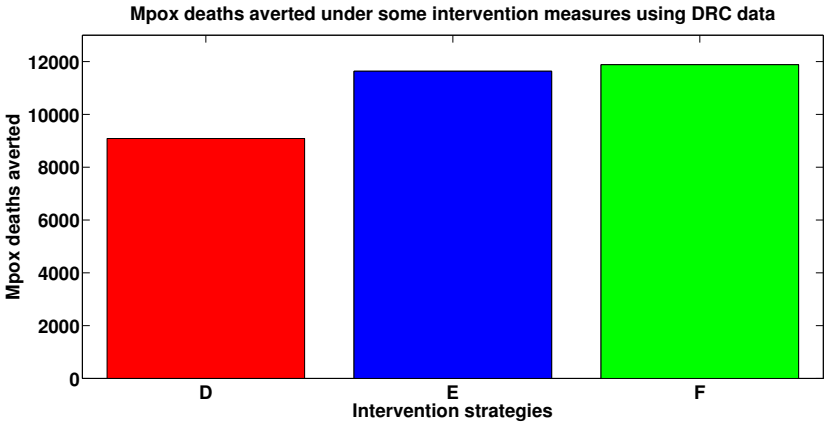


FIGURE 16. Mpox deaths averted under different intervention measures using DRC data.

1. The basic reproduction number of Mpox in Nigeria is estimated to be $\mathcal{R}_{0N} = 1.2027$, while the basic reproduction number of Mpox in DRC is estimated to be $\mathcal{R}_{0D} = 1.1038$.
2. The sensitivity analysis reveals that an increase in the effective transmission probability per contact with infected humans (α_h), and the progression rate of exposed humans to infected human population (τ) will lead to an increase in the respective reproduction number of each country, as a result leading to an upsurge in Mpox burden. Similarly, an increase in the Mpox burden due to the abundance of the infected human population, is associated with an increase in the effective transmission probability per contact with infected humans (α_h), and effective transmission probability per contact with infected animals (α_r).
3. The sensitivity analysis also shows that an increase in the progression rate of exposed humans to their infectious state (τ) and Mpox-induced death rate of human (δ_h) will increase the dead human population.
4. The numerical simulation results reveal that the higher the reduction in the percentage of the effective transmission probability per contact with infected humans (α_h), the effective transmission probability per contact with infected animals (α_r), and the progression rate of exposed humans into their infectious state (τ), the higher the number of cases and deaths averted in both country.

5. The number of cases and deaths projected, and the number of cases and deaths averted in Nigeria and DRC based on different intervention scenarios are presented in Table 4 and Table 5 respectively.

Based on the results from above, this study suggests to policymakers that efforts should be made to implement preventive measures that will help reduce the effective transmission of Mpox in the population. This includes but is not limited to the implementation of vaccine usage against the disease. Additionally, control measures that will reduce the progression rate of exposed individuals into their infectious condition should be implemented to reduce the disease burden over a given time. This includes but is not limited to the creation of medical facilities that can support the diagnosis and treatment of infected individuals.

It is very important to state that the results and recommendations from this work apply to other countries not considered in this study. However, there would be variation in the projection and averted cases and deaths as seen between the two countries due to the state of Mpox in each country (i.e., the estimate of each parameter value). The current study has some limitations. As it has been stated earlier in the introduction section, this study focuses on the dynamics of Mpox in Nigeria and DRC in the absence of the implementation of vaccination. As a result of this, the developed model does not include the vaccinated human population against Mpox. Thus, in the future, we will extend the current model by including the vaccinated human population. The extended model would be used to investigate the impact of vaccine characteristics on the Mpox burden in the two countries. This includes but is not limited to the investigation of the impact of vaccine coverage and its efficacy on the Mpox burden. In addition, we will focus on estimating the cases and deaths averted in each country in the presence of vaccination. Also, this model does not consider the impact of other infectious diseases (such as sexually transmitted infections, bacterial or airborne diseases) on the transmission dynamics of mpox. This is much anticipated for a future study. The 2022-2023 mpox outbreak disproportionately affected homosexual men and individuals with high-risk sexual activities. Thus, further research shall also incorporate the effect of these assumptions. We also hope to consider the impact of behavioral, social, and cultural factors and their impact on the control of mpox among Nigeria and DRC.

Ethics approval and consent to participate

I hereby certify that, to the best of my knowledge and belief, the information provided above are true

Consent for publication

None

Availability of data and materials

The data utilized in this study is accessible upon request from the corresponding author

Competing interests

There are no conflicts of interest to declare.

Authors' contributions

Olumuyiwa James Peter conceptualized the study, developed the methodology, collected and analyzed the data, formulated the model, developed the software, supervised the project, drafted the original manuscript, significantly contributed to the review and editing of the manuscript, and created the visualizations. Oluwatosin Babasola assisted in developing the methodology, contributed to data collection, participated in the review and editing of the manuscript, and helped with visualizations. Mayowa M. Ojo supported data collection, contributed to the review and editing of the manuscript, and assisted in creating visualizations. Andrew Omame also contributed to data collection, participated in the review and editing of the manuscript, and helped with visualizations.

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References

[1] Africa Centres for Disease Control and Prevention (Africa CDC), *20222023 Mpox outbreak*, https://en.wikipedia.org/wiki/2022%E2%80%932023_mpox_outbreak#cite_note-38, (2023). [Online; Accessed on August 24, 2023].

[2] A. Aldurayhim, A. Elsonbaty, W. Adel, and A. El-Mesady, *Mathematical modeling and analysis of a novel monkeypox virus spread integrating imperfect vaccination and nonlinear incidence rates*, Ain Shams Engineering Journal, *15*(3) (2024), 102451.

[3] N. Akinwande, F. Oguntolu, and N. Lasisi, *Development and exploration of a mathematical model for transmission of monkeypox disease in humans*, Mathematical Models in Engineering, *6*(1) (2020), 2333.

[4] A. Augustine, O. I. Marcus, and T. Jonathan, *A co-infection model for monkeypox and HIV/AIDS: Sensitivity and bifurcation analyses*, Journal of Scientific Research and Reports, *30*(5) (2024), 351368.

[5] O. Babasola, E. O. Omondi, K. Oshinubi, and N. M. Imbusi, *Stochastic delay differential equations: a comprehensive approach for understanding biosystems with application to disease modelling*, Applied-Math, *3*(4) (2023), 702721.

[6] B. Bolaji, O. A. Godwin, and O. O. Peace, *A compartmental deterministic epidemiological model with non-linear differential equations for analyzing the co-infection dynamics between COVID-19, HIV, and monkeypox diseases*, Healthcare Analytics, (2024), 100311.

[7] C. P. Bhunu, J. Hyman, and S. Mushayabasa, *Modelling HIV/AIDS and monkeypox co-infection*, Applied Mathematics and Computation, *218*(18) (2012), 95049518.

[8] X. Cai, T. Zhou, W. Shi, Y. Cai, and J. Zhou, *Monkeypox virus crosstalk with HIV: An integrated skin transcriptome and machine learning study*, ACS Omega, *8*(49) (2023), 4728347294.

[9] M. Cavallaro, S. P. Brand, F. Cumming, C. Turner, J. Hilton, I. Florence, L. M. Guzman-Rincon, P. Blomquist, D. J. Nokes, and T. House, *The role of vaccination and public awareness in forecasts of mpox incidence in the United Kingdom*, Nature Communications, *14*(1) (2023), 4100.

[10] P. A. Clay, E. D. Pollock, E. M. Saldarriaga, P. Pathela, M. Macaraig, J. R. Zucker, B. Crouch, I. Kracalik, S. O. Aral, and I. H. Spicknall, *Modeling the impact of prioritizing first or second vaccine doses during the 2022 mpox outbreak*, medRxiv, (2023), 202310.

[11] C. E. Copen, E. D. Pollock, J. M. Asher, K. P. Delaney, P. A. Clay, D. C. Payne, E. M. Saldarriaga, P. Pathela, B. Crouch, and I. H. Spicknall, *Modeling the impact of prioritizing first or second vaccine doses during the 2022 mpox outbreak*, medRxiv, (2023), 202310.

[12] P. A. Clay, J. M. Asher, N. Carnes, C. E. Copen, K. P. Delaney, D. C. Payne, E. D. Pollock, J. Mermin, Y. Nakazawa, W. Still, et al., *Modelling the impact of vaccination and sexual behaviour adaptations on mpox cases in the USA during the 2022 outbreak*, Sexually Transmitted Infections, *100*(2) (2024), 7076.

[13] P. L. Delamater, E. J. Street, T. F. Leslie, Y. T. Yang, and K. H. Jacobsen, *Complexity of the basic reproduction number (R0)*, Emerging Infectious Diseases, *25*(1) (2019), 1.

[14] O. Diekmann, J. A. P. Heesterbeek, and J. A. Metz, *On the definition and the computation of the basic reproduction ratio R0 in models for infectious diseases in heterogeneous populations*, Journal of Mathematical Biology, *28* (1990), 365382.

[15] O. Diekmann, J. Heesterbeek, and M. G. Roberts, *The construction of next-generation matrices for compartmental epidemic models*, Journal of the Royal Society Interface, *7*(47) (2010), 873885.

[16] K. N. Durski, A. Khalakdina, M. G. Reynolds, I. K. Damon, S. Briand, Y. Nakazawa, M. H. Djingarey, V. Olson, and A. M. McCollum, *Emergence of monkeypoxwest and central africa, 19702017*, Morbidity and Mortality Weekly Report, *67*(10) (2018), 306.

[17] A. El-Mesady, W. Adel, A. Elsadany, and A. Elsonbaty, *Stability analysis and optimal control strategies of a fractional-order monkeypox virus infection model*, Physica Scripta, *98*(9) (2023), 095256.

[18] A. Elsonbaty, A. Aldurayhim, W. Adel, and A. El-Mesady, *Investigating the dynamics of a novel fractional-order monkeypox epidemic model with optimal control*, Alexandria Engineering Journal, *73* (2023), 519542.

[19] B. Liu, M. Altanji, R. Nawaz, S. Farid, S. Ullah, and S. Wondimagegnhu Teklu, *Mathematical assessment of monkeypox disease with the impact of vaccination using a fractional epidemiological modeling approach*, Scientific

Reports, *13*(1) (2023), 13550.
[20] J. Mermin, P. A. Clay, E. D. Pollock, J. M. Asher, N. Carnes, C. E. Copen, and K. P. Delaney, *Modelling the impact of vaccination and sexual behaviour adaptations on mpox cases in the USA during the 2022 outbreak*, Sexually Transmitted Infections, *100*(2) (2024), 7076.
[21] Nigeria Centre for Disease Control, *Mpox Monkeypox outbreak situation report*, https://ncdc.gov.ng/report/, (2023). [Online; Accessed on August 23, 2023].
[22] A. Omame, Q. Han, S. A. Iyaniwura, E. Adeniyi, N. L. Bragazzi, X. Wang, J. D. Kong, and W. A. Woldegerima, *Understanding the impact of HIV on mpox transmission in an MSM population: a mathematical modeling study*, Available at SSRN 4762707 (2024).
[23] O. J. Peter, A. Abidemi, M. M. Ojo, and T. A. Ayoola, *Mathematical model and analysis of monkeypox with control strategies*, The European Physical Journal Plus, *138*(3) (2023), 242.
[24] O. J. Peter, S. Kumar, N. Kumari, F. A. Oguntolu, K. Oshinubi, and R. Musa, *Transmission dynamics of Monkeypox virus: a mathematical modelling approach*, Modeling Earth Systems and Environment, (2022), 1–12.
[25] O. J. Peter, C. E. Madubueze, F. A. Oguntolu, M. M. Ojo, and T. A. Ayoola, *Modeling and optimal control of monkeypox with cost-effective strategies*, Modeling Earth Systems and Environment, *9*(2) (2023), 19892007.
[26] E. M. Tag-Eldin, F. Allehiany, M. A. Khan, M. H. DarAssi, and I. Ahmad, *Mathematical modeling and backward bifurcation in monkeypox disease under real observed data*, Results in Physics, *50* (2023), 106557.
[27] M. Rabiu, E. J. Dansu, O. A. Mogbojuri, I. O. Idisi, M. M. Yahaya, P. Chiwira, R. T. Abah, and A. A. Adeniji, *Modeling the sexual transmission dynamics of mpox in the United States of America*, The European Physical Journal Plus, *139*(3) (2024), 120.
[28] P. Van den Driessche, *Reproduction numbers of infectious disease models*, Infectious Disease Modelling, *2*(3) (2017), 288303.
[29] World Health Organization, *Mpox (monkeypox Newsroom)*, https://www.who.int/news-room/questions-and-answers/item/monkeypox, (2023). [Online; Accessed on August 13, 2023].
[30] World Health Organization, *Mpox (monkeypox Fact sheets)*, https://www.who.int/news-room/fact-sheets/detail/monkeypox, (2023). [Online; Accessed on August 13, 2023].
[31] World Bank, *Population, total - Nigeria*, https://data.worldbank.org/indicator/SP.POP.TOTL?locations=NG, (2023). [Online; Accessed on August 18, 2023].
[32] World Health Organization, *Health data overview for the Federal Republic of Nigeria*, https://data.who.int/countries/566, (2023). [Online; Accessed on August 18, 2023].
[33] World Health Organization, Africa Region, *Emergency Preparedness and Response*, https://www.afro.who.int/, (2023). [Online; Accessed on August 25, 2023].
[34] World Bank, *Population, total - Congo, Dem. Rep.*, https://data.worldbank.org/indicator/SP.POP.TOTL?locations=CD, (2023). [Online; Accessed on August 18, 2023].
[35] World Health Organization, *Health data overview for the Democratic Republic of the Congo*, https://data.who.int/countries/180, (2023). [Online; Accessed on August 18, 2023].
[36] M. Xiridou, F. Miura, P. Adam, E. O. de Coul, J. de Wit, and J. Wallinga, *The fading of the mpox outbreak among men who have sex with men: a mathematical modelling study*, The Journal of Infectious Diseases, (2024), jiad414.
[37] S. Yang, X. Guo, Y. Zhao, Z. Zhao, T. Chen, H. Wei, Y. Wang, J. Rui, and W. Song, *Possibility of mpox viral transmission and control from high-risk to the general population: a modeling study*, BMC Infectious Diseases, *23*(1) (2023), 119.
[38] N. Zhang, E. Addai, L. Zhang, M. Ngungu, E. Marinda, and A. JKK, *Fractional modeling and numerical simulation for unfolding Marburgmonkeypox virus co-infection transmission*, Fractals, *31*(7) (2023), 2350086.