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A Hierarchical Method to Solve One Machine Multicriteria Sequencing Problem

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Abstract

The problem of minimizing a function of three criteria maximum earliness, total of square completion times and total lateness in a hierarchical (lexicographical) method is proposed in this article. On one machine, n independently tasks (jobs) must planned. It is always available starting at time zero and can only do mono task (job) at time period. Processing for task (job) $j(j = 1, 2, ..., nj)$ is necessary meantime the allotted positively implementation time p_{tj}. For the problem of three criteria maximization earliness, total of square completion times, and total lateness in a hierarchy instance, the access of limitation that which is desired sequence is hold out. The Generalized Least Deviation Method (GLDM), a robust technique for analyzing historical data to project future trends is analyzed.

Keywords. Sequencing with one machine, Hierarchical, Lexicographic format, Square completion times, Multicriteria.
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1. Introduction

In real-world situations, making decisions is often complicated by competing standards. Making decisions grows increasingly challenging as the number of constraints rises. Modeling and developing sequence techniques has always been a challenge for operations researchers. Several techniques and formulations have been developed for various kinds of problems [1]. Each duty, sometimes referred to as a task, consists of a basic sequencing challenge, an execution time on one of the machines capable of carrying it out, Of course, it should be done in a way that guarantees the result at the end. referred to as a sequencing, is ideal, satisfies all side constraints, or minimizes the given objective function [2]. Sequencing theory was developed to overcome problems with, for instance, nurse sequencing [3].
The one machine example is taken into consideration in this work because it provides a useful laboratory for the development of concepts for heuristics and interactive methods that may be useful in more broader models.
There are two methods for handling multi-criteria problems: the hierarchical approach and the simultaneous approach. the method based on hierarchy. One of the two criteria is the major criterion, while the other is the secondary criterion. The secret must be to minimize the first performance measurement while using the lowest second performance measurement value to defeat similarity in preference sequencing. The simultaneous approach considers two standards at the same time. This method usually generates all possible sequencings and selects the optimal one according to the values of the assembly goal function for both criteria. Most problems using multiple criteria sequencing are NP-hard [4]. Evolutionary algorithms (EAs) have emerged as a strength optimizing toolset to tackle sequencing issues [5, 6]. Erne [7] offered an integer programming model heuristic method for minimizing the weighted sum of total completion time, maximum tardiness, and maximum earliness for a sequencing problem with many criteria and sequencing-dependent setup time. Nelson et al. [8] provided many sequences for the three-criteria problems, flow time g, maximum tardiness Tmax, and number of tardy jobs nT, using mean algorithms. Hoogeveen [9] offered a technique for reducing the growing measure of R regularity functions. For the multi-criteria problem $1/F(C_j, \sum T_j, L_{max})$ [10] provided an

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efficient approach for discovering the set of all efficient sequences. Research on multi-criteria decision-making problems is extensively covered in [11]. Approximate techniques and mathematical programming are employed to handle multi-criteria decision making problems [12]. Using a hierarchical approach, [13] presented a multi-criteria problem. The multi-criteria problem in is solved using a modified branch and bound technique in [14].

A sequencing σ establishes the completion time $C_{tj}(\sigma)$ for each job j so that the jobs do not execute concurrently. The penalty function g_j calculates the freckle set back accomplish j at period C_{tj}. The totally square completing period $\sum C_{tj}^2$ and maximization costing g_{max}, where maximization costing means $g_j(C_{tj})$, is minimized hierarchically (C_{tj}), is the multi criteria problem in this article , where g_j stands for a cost function, either regular or irregular, routine implies that $g_j(C_{tj})$ does not disappear as C_{tj} rises, adapt Tt_{max}, $\sum L_{tj}, \sum C_{tj}^2$. If not, a measure said to as not regulator, like Et_{max}.

The fundamental planning issue can be portrayed as finding for each of the assignments, which are too called occupations, an execution interim on one of the machines that are able to execute it, such that all side limitations are met; obviously, this ought to be wiped out such a way that the resulting solution, which is called a plan, is best conceivable, that's , it minimizes the given objective work. Planning hypothesis has been created to illuminate issues happening in for occurrence nurture planning.

There are two approaches for the multicriteria issues; the various leveled and the concurrent approach. Within the various leveled approach, one of the two criteria is considered as the essential basis and the other one as the auxiliary criterion. The problem is to play down the essential basis whereas breaking ties in favor of the schedule that has the least auxiliary measure esteem. Within the synchronous approach, two criteria are considered at the same time. This approach regularly produces all effective plans and chooses the one that yields the finest composite objective work esteem of the two criteria. Most multicriteria planning issues are NP-hard in nature. In later a long time, as a effective optimization device, developmental calculations (EAs) have been presented to illuminate the arrange planning issues. Within the generation division, planning can be in a more extensive point of view characterized as a prepare of organizing, controlling, and optimizing work or workloads With respect to finding the ideal plan for a particular structure and generation framework conditions, the planning is considered as a complex combinatorial optimization issue, generally demonstrated of NP-hard sort. Correct optimization strategies are primarily utilized as it were for the frameworks which have a particular topology where exceptionally solid disentangling presumptions must be utilized, so they are not as well pertinent in a real-world situation for more complex frameworks. In that case, surmised optimization strategies and metaheuristics based on stochastic nearby look approach, machine learning procedures, particularly manufactured neural systems (ANN), fluffy rationale strategies, and master frameworks, are at the center of investigate intrigued to discover ideal or near-optimal arrangements rather than correct scientific optimization models.

In differentiate to other strategies, dispatching rules (we moreover utilize the term need rules all through the taking after content) speak to the profitable viable and overwhelming approach of the shop floor control within the complex industry environment, such as, e.g., in semiconductor fabricating for fathoming complex planning issues in real-time. Need rules are well known since they are characterized by the effortlessness of usage, palatable execution, and a significantly diminished computational prerequisite. By the by, the choice of appropriate dispatching rules isn't a unimportant assignment and depends on the significant key execution pointers.

The impacts created by the chosen need run the show are for the most part troublesome to clarify by expository strategies, in this way the recreation is utilized exceptionally frequently to assess the plan effectiveness within the complex planning issue. As an outline, within the recreation consider, Vinod and Sridharan [17] assessed the execution measures based on stream time and lateness of occupations for the distinctive combinations of due-date task strategies and seven planning choice rules connected in a dynamic job shop framework. Xanthopoulos et al. [18] compared seventeen dispatching rules within the consider centered on stochastic energetic planning issues with sequence-dependent setups. Execution measures were cruel work-in-progress, cruel cycle time, cruel lateness, and a division of late employments.

Authors of [19] explained the generalized fifth-order KdV like equation with prime number $p = 3$ via a generalized bilinear differential operator. N-lump was invstigated to the variable-coefficient CaudreyDoddGibbonKoteraSawada equation [20]. Applications of $tan(\phi/2)$-expansion method for the BiswasMilovic equation [21], the GerdjikovIvanov model [22], the KunduEckhaus equation [23] and the fifth-order integrable equations [24] were studied. Lump solutions were analyzed to the fractional generalized CBS-BK equation [25] and the (3+1)-D Burger system [26]. The

approximations of one-dimensional hyperbolic equation with non-local integral conditions were constructed by reduced differential transform method [27]. The generalized Hirota bilinear strategy by the number prime was used to the (2+1)-dimensional generalized fifth-order KdV like equation [28]. The traveling wave solutions and analytical treatment of the simplified MCH equation and the combined KdVmKdV equations were studied [29].
The structure of this paper is given as under:This paper is formed because the section 2 contains the exponent taking fundamental ideas and related results which are thoroughly crucial to know the novelty of this paper. In section 3, we investigate $1//Lex(Et_{max}, \sum_{j=1}^{n}(C_{t_j})^2, \sum_{j=1}^{n} L_{tj})$ problem. Generalized Least Deviation Method Description is discussed in section 4. In addition, soft computing results is provided in section 5. Finally, we approach some kind of results and conclusion in sixth section.

2. Exponent taking fundamental ideas

Tasks (jobs) $j(j = 1, 2, ..., n)$ have been performed on a one machine in this study using the notation.

N_j: tasks collection.

n: the number of tasks (jobs) in given sequencing.

P_{t_j}: operationally time for task (job) j.

D_{t_j}: the period where the task (job) j has to perfectly completing.

$\overline{Dt_j}$: baseline for task (job) j.

Ct_j: the completing period of task (job) j.

C^2t_j: the square completing period of task (job) j.

$Ct_1 = pt_1$

$Ct_j = Ct_{(j-1)} + pt_j, j = 2, , n.$

$s_j = Dt_j - pt_j$: the slacked period of task (job) j.

$Lt_j = Ct_j - Dt_j$: the lately of task (job) j.

$Et_j = max0, Dt_j - Ct_j$: the earliness (premature) of task (job) j.

$\sum C^2t_j$: totally completing period.

$E_{tmax} = max_j Et_j$: maximization earliness (premature).

$\sum Lt_j$: totally lately tasks.

SPTO= Sequencing the tasks (jobs) in non-decreasing order of processing time, the Rule of the least processing time is used. EDDO= The earliest due date order rule is applied by sequencing the jobs in non-decreasing order of their due dates.

Theorem 2.1. *[15] The following minimizes the $1/g_{max}$ problem: If there are any unassigned jobs, allocate the one with the lowest cost to the final unassigned spot on the timetable.*

Theorem 2.2. *[16] The $1/Et_{max}$ problem is resolved by executing the jobs in a non-decreasing order of $Dt_j - pt_j$ in accordance with the minimum slack time (MSTO) criteria.*

Definition 2.3. [9] Minimization in a hierarchy: The order of relevance for the performance criteria $g_1, g_2, ..., g_k$ is indexed in decreasing order. First, g_1 is reduced, next, g_2 is reduced. provided that the sequencing has a minimum g_1 value, if required, g_1 and g_2 must have values that are equal to those found in the previous stage in order for g_3 to be minimized.

3. The $1/Lex(Et_{max}, \sum_{j=1}^{n}(C_{t_j})^2, \sum_{j=1}^{n} L_{tj})$ problem

This problem (issue) can be defined as follows:

$$\begin{cases} Min \sum_{j=1}^{n} L_{tj}, \\ S.t., \\ Et_{max} = Et^*, Et^* = Et_{max}(MSTO), \\ \sum_{j=1}^{n}(C_{t_j})^2 \leq Ct^*, Ct^* \in [\sum_{j=1}^{n}(C_{t_j})^2(SPTO), \sum_{j=1}^{n}(C_{t_j})^2(MSTO)]. \end{cases} \tag{3.1}$$

Given that Et_{max} is the most crucial function in this problem (3.1) and should be at its best, next way EtCtLt algorithm provides desired solution (outcome).

Algorithm (EtCtLt)

Move 1: Solving Et_{max} problem for finding Et^*.
Move 2: Ascertain $\overline{Dt_j} = Dt_j + Et^* \forall j \in N_j, N_j = 1, 2, , n$.
Move 3: Letting $h = \sum_{j=1}^{n} p_{tj}$.
Move 4: Finding the task (job) $j^* \in N_j$ verifies $\overline{Dt_{j*}} \leq h$ (choose the task with the minimum processing time if there is a tie, and if there is still a tie, select the task with the earliest baseline).
Move 5: Set $h = h - p_{tj^*}, r = r + 1, N_j = N_j - j^*$, sequencing $\sigma = (\sigma, \sigma(r))$, if $N_j = \varphi$ go to move6, otherwise go to move 4.
Move 6: In sequencing σ computing $Et_{max}, \sum_{j=1}^{n}(Ct_j)^2, \sum_{j=1}^{n} L_{tj}$.

Example 3.1. Considering the problem (3.1) with the following inputs.

j	1	2	3	4	5
P_{t_j}	6	3	7	10	10
D_{t_j}	7	15	17	11	10

$E^* = 0, h = 36$.

$\overline{D_{t_1}} = 7, \quad \overline{D_{t_2}} = 15, \quad \overline{D_{t_3}} = 17, \quad \overline{D_{t_4}} = 11, \quad \overline{D_{t_5}} = 10$

r	h	t^*
$r1$	36	$2j$
$r2$	33	$1j$
$r3$	27	$3j$
$r4$	20	$5j$
$r5$	10	$4j$

Sequencing $(2j, 1j, 3j, 5j, 4j)$ getting $(Et_{max}, \sum_{j=1}^{n}(C_{tj})^2, \sum_{j=1}^{n} L_{tj}) = (12, 2318, 31)$ based on an algorithm (EtCtLt).

4. Generalized Least Deviation Method Description

A time series forecasting model employing the Generalized Least Deviation Method (GLDM) is considered. The time series dataset is characterized as follows:

$$\{y_t\}_{t=1}^{T} \subset \mathbb{R}, \tag{4.1}$$

where y_t denotes a real-valued observation at time index t.

The GLDM Estimator is utilized to determine an optimal set of coefficients $\{a_j\}_{j=1}^{n(m)}$, which minimize the objective function $F(\mathbf{a})$, defined as the sum of the arctangents of absolute deviations:

$$F(\mathbf{a}) = \sum_{t=1}^{T} \arctan \left| y_t - \sum_{j=1}^{n(m)} a_j g_j(\{y_{t-k}\}_{k=1}^{m}) \right|, \tag{4.2}$$

with each function g_j representing a unique combination of preceding values up to the m-th order.

Within the quasi-linear model framework, the functions g_j capture the influence of historical data. These functions are defined as follows:

$$g_j(\{y_{t-k}\}_{k=1}^{m}) = y_{t-j} + \sum_{\substack{p=1\\p\neq j}}^{m} y_{t-j}\cdot y_{t-p} + \sum_{p=1}^{m} y_{t-p}^2, \tag{4.3}$$

where y_{t-j} signifies the lagged value of the series at time $t-j$. The first summation models the interaction effects between different lagged values, while the second summation encapsulates the non-linear effects through squared terms of the lagged values. These elements allow for the modeling of complex dynamics within time series data.

The total count of coefficients for an m-th order model, which includes linear, interaction, and quadratic components, is described by the following expression:

$$n(m) = m + \binom{m}{2} + m = \frac{m(m+3)}{2}. \tag{4.4}$$

The structure and roles of these coefficients in the modeling process are detailed as follows:

- The term m refers to the linear coefficients, correlating each historical value with the subsequent predicted value.
- The term $\binom{m}{2}$ represents the interaction coefficients, denoting the pairwise combinations between historical values, facilitating the detection of dependencies and interactions at different time lags.
- The final term m denotes the quadratic coefficients, accommodating non-linear trends by reflecting the self-interactions of the historical values.

4.1. Second-Order Time Series Forecasting Model.

A time series forecasting model that incorporates interaction and non-linear terms up to the second order is considered. For a model where $m = 2$, it is determined that the total number of coefficients is five. These coefficients comprise the linear terms, their squares, and the interaction term between them, which are critical for modeling the linear tendencies and capturing the potential synergistic and quadratic effects within the time series data.

For a second-order model ($m = 2$), the coefficients and their corresponding terms are enumerated as follows:

- Linear terms: y_{t-1}, y_{t-2},
- Squared terms: y_{t-1}^2, y_{t-2}^2,
- Interaction term: $y_{t-1}\cdot y_{t-2}$.

The generalized function g_j for this model is explicitly defined in the following manner:

$$g_j(\{y_{t-k}\}_{k=1}^{m}) = \begin{cases} y_{t-j} & \text{for } j = 1, 2,\\ y_{t-1}\cdot y_{t-2} & \text{for } j = 3,\\ y_{t-j+2}^2 & \text{for } j = 4, 5. \end{cases} \tag{4.5}$$

In this configuration, g_1 and g_2 are assigned to the first and second linear terms, respectively, g_3 to the interaction term, and g_4 and g_5 to the squared terms of the first and second variables, respectively. This structural arrangement effectively accounts for both the direct influences and the interactions between the past two values, as well as their individual non-linear influences on the present value.

4.2. **Third-Order Time Series Forecasting Model.** Interactions between past values in a time series are essential for capturing the dynamics and dependencies inherent within the data. In a third-order model, denoted by $m = 3$, linear, squared, and interaction terms are included, facilitating the modeling of complex non-linear behaviors. This comprehensive approach enables the effective capture of intricacies and interdependencies among historical values.

For a third-order model ($m = 3$), the coefficients and their corresponding terms are outlined as follows:

- Linear terms: y_{t-1}, y_{t-2}, y_{t-3},
- Squared terms: y_{t-1}^2, y_{t-2}^2, y_{t-3}^2,
- Interaction terms: $y_{t-1} \cdot y_{t-2}$, $y_{t-1} \cdot y_{t-3}$, $y_{t-2} \cdot y_{t-3}$.

The functions g_j representing these terms in the third-order model are systematically defined as follows:

$$g_j(\{y_{t-k}\}_{k=1}^m) = \begin{cases} y_{t-j} & \text{for } j = 1, 2, 3, \\ y_{t-j+3}^2 & \text{for } j = 4, 5, 6, \\ y_{t-1} \cdot y_{t-2} & \text{for } j = 7, \\ y_{t-1} \cdot y_{t-3} & \text{for } j = 8, \\ y_{t-2} \cdot y_{t-3} & \text{for } j = 9. \end{cases} \tag{4.6}$$

This model structure, incorporating linear, squared, and interaction terms, ensures a robust representation of the time series dynamics. The inclusion of these terms aids in modeling more complex nonlinear relationships that linear terms alone may not capture.

The complete mathematical model of the time series, utilizing the coefficients defined above, is given by the following equation:

$$y_t = \sum_{j=1}^{9} a_j g_j(\{y_{t-k}\}_{k=1}^m) + \varepsilon_t, \quad t = 1, 2, \ldots, T \tag{4.7}$$

Here, ε_t denotes the error term at time t, representing the unpredictable component not explained by the model.

4.3. **Fourth-Order Time Series Forecasting Model.** A time series forecasting model that leverages the intricacies of linear, interaction, and non-linear dynamics up to the fourth order is considered. In a fourth-order model, denoted by $m = 4$, the total count of coefficients is identified as 14. This ensemble encompasses the linear terms for the four preceding observations, their squared counterparts, and the six unique interaction terms between these observations, thereby encapsulating a comprehensive dynamic range within the time series.

For a fourth-order model ($m = 4$), the coefficients and their corresponding terms are explicitly associated as follows:

- Linear terms: y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4}
- Squared terms: y_{t-1}^2, y_{t-2}^2, y_{t-3}^2, y_{t-4}^2
- Interaction terms: All distinct pairwise combinations of the four variables

With 4 linear and 4 squared terms, and $\binom{4}{2} = 6$ interaction terms, the model integrates a total of 14 coefficients.

The specific formulation of the function g_j within the model, which embraces both linear and interaction terms, is systematically defined as follows:

$$g_j(\{y_{t-k}\}_{k=1}^{m}) = \begin{cases} y_{t-j} & \text{for } j = 1, 2, 3, 4, \\ y_{t-1} \cdot y_{t-2} & \text{for } j = 5, \\ y_{t-1} \cdot y_{t-3} & \text{for } j = 6, \\ y_{t-1} \cdot y_{t-4} & \text{for } j = 7, \\ y_{t-2} \cdot y_{t-3} & \text{for } j = 8, \\ y_{t-2} \cdot y_{t-4} & \text{for } j = 9, \\ y_{t-3} \cdot y_{t-4} & \text{for } j = 10, \\ y_{t-j+6}^2 & \text{for } j = 11, 12, 13, 14. \end{cases} \tag{4.8}$$

In this model, g_1 through g_4 correspond to the linear terms, g_5 through g_{10} to the interaction terms, and g_{11} through g_{14} to the squared terms. This elaborate model configuration facilitates an extensive incorporation of both the progressive and the interactive effects of the past observations, along with their individual non-linear influences, thus significantly augmenting the predictive capabilities of the time series model.

4.4. **Fifth-Order Time Series Forecasting Model.** A comprehensive time series forecasting model that integrates both linear and nonlinear dynamics up to the fifth order is considered. Within a fifth-order framework, symbolized by $m = 5$, a constellation of 20 coefficients is identified. These coefficients comprise the linear terms for the five antecedent observations, their individual squared terms, and the interaction terms among these observations, thereby capturing a multidimensional dynamic within the time series.

For a fifth-order model ($m = 5$), the assortment of coefficients is meticulously associated with their respective terms as cataloged below:

- Linear terms: y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4}, y_{t-5},
- Squared terms: y_{t-1}^2, y_{t-2}^2, y_{t-3}^2, y_{t-4}^2, y_{t-5}^2,
- Interaction terms: All distinct pairwise combinations of the five variables.

Accounting for 5 linear terms, 5 squared terms, and $\binom{5}{2} = 10$ interaction terms, the model features an aggregate of 20 coefficients.

The formalized expression of the function g_j within the model, encapsulating both the linear and interaction terms, is articulated as follows:

$$g_j(\{y_{t-k}\}_{k=1}^{m}) = \begin{cases} y_{t-j} & \text{for } j = 1, \dots, 5, \\ y_{t-1} \cdot y_{t-2} & \text{for } j = 6, \\ y_{t-1} \cdot y_{t-3} & \text{for } j = 7, \\ \vdots & \\ y_{t-4} \cdot y_{t-5} & \text{for } j = 14, \\ y_{t-j+9}^2 & \text{for } j = 15, \dots, 20. \end{cases} \tag{4.9}$$

In this delineation, g_1 to g_5 are assigned to the linear terms, g_6 to g_{14} to the interaction terms, and g_{15} to g_{20} to the squared terms. This expansive framework not only contemplates the sequential impact of the prior observations but also scrutinizes the combinative and quadratic interactions, thereby substantively refining the forecasting strength of the time series analysis.

5. Soft computing Results

Table 1 summarizes the number of coefficients required for time series forecasting models of varying orders, from first to fifth. Specifically, the table enumerates the coefficients as 2, 5, 9, 14, and 20 for the first through fifth orders, respectively. This progression is governed by the formula $n(m) = 2m + \binom{m}{2} = \frac{m(m+3)}{2}$, which calculates the total count of coefficients including linear, interaction, and quadratic terms as the model order increases. The structured increase in coefficients highlights the model's growing complexity and capacity to encapsulate more intricate dynamics within the time series data.

Table 1. Number of Coefficients by Order.

Order	First	Second	Third	Fourth	Fifth
Coefficients	2	5	9	14	20
$n(m) = 2m + \binom{m}{2} = \frac{m(m+3)}{2}$					

The datasets employed in our analysis are summarized in Table 2, which details their respective lengths. The datasets include NDVI with 15 data points, Temperature with 9,939 data points, Wind Speed recorded with 50,530 data points, and COVID-19 death cases in the Russian Federation, which consist of 882 data points. This variation in dataset sizes reflects the diverse scope and scale of environmental and epidemiological data considered in our time series forecasting models. The extensive data length, particularly for Temperature and Wind Speed, provides a robust basis for statistical analysis and model validation.

Table 2. List of used datasets and their lengths.

Dataset	Length
NDVI	15
Temperature	9,939
Wind Speed	50,530
COVID-19 deaths Cases in the Russian Federation	882

The tables from 3 to 7 present the coefficients for the Generalized Least Deviation Method (GLDM) applied to the Normalized Difference Vegetation Index (NDVI) data across different model orders, from first through fifth. Each table, corresponding to the model order, lists the coefficients derived from fitting the GLDM model to the NDVI dataset. Table 3 starts with the simplest model, featuring only two coefficients, a_1 and a_2. As the model complexity increases, more coefficients are introduced to capture additional dynamics of the data, evident in Table 4 for the second order and further expanded in Tables 5, 6, and 7 for higher orders. These coefficients are crucial for understanding the NDVI time series' behavior and improving prediction accuracy. Notably, as the order increases, the number of coefficients grows, reflecting the model's enhanced capability to incorporate more historical data points and interactions within the NDVI time series.

Table 3. GLDM First Order Coefficients for NDVI.

Coefficient	Value
a_1	1.7073
a_2	-1.0511

Table 4. GLDM Second Order Coefficients for NDVI.

Coefficient	Value
a_1	3.4694
a_2	-2.1864
a_3	-5.5924
a_4	-2.5635
a_5	7.7299

Table 5. GLDM Third Order Coefficients for NDVI.

Coefficient	Value
a_1	-9.6495
a_2	-16.2326
a_3	29.1697
a_4	76.3993
a_5	122.9467
a_6	-71.5312
a_7	-229.9915
a_8	98.9790
a_9	0.0000

Table 6. GLDM Fourth Order Coefficients for NDVI.

a_1	a_2	a_3	a_4	a_5	a_6	a_7
52.4809	30.7212	-48.3575	-132.0577	-177.0665	4.7422	-2.5713
a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}
273.2420	-66.8229	-21.8417	83.1160	0.0000	0.0000	0.0000

Table 7. GLDM Fifth Order Coefficients for NDVI.

a_1	a_2	a_3	a_4	a_5
0.0000	-29.0004	64.0513	-44.3069	10.2588
a_6	a_7	a_8	a_9	a_{10}
1.7283	-23.5562	-90.7331	30.3171	-6.6379
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
-2.0578	90.0347	0.0000	0.0000	0.0000
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
0.0000	0.0000	0.0000	0.0000	0.0000

The coefficients derived from applying the Generalized Least Deviation Method (GLDM) for the temperature data set are systematically presented in Tables 8 through 12. These tables enumerate the coefficients for models of increasing order from first to fifth. Table 8 lists the coefficients for the first order model, indicating the foundational linear influences in the temperature data. Progressing to higher model orders, Table 9 and Table 10 introduce additional coefficients, capturing more complex dynamics and interactions within the data. This trend continues with Table 11, where the fourth order model incorporates even more coefficients, enhancing the model's ability to forecast with greater precision. Finally, Table 12 presents the coefficients for the fifth order model, which encompasses the most comprehensive dynamic range, utilizing twenty coefficients to capture nuanced patterns and potential non-linearities in the temperature series. Each table reflects the incremental complexity and enhanced predictive capability as the order of the model increases.

Table 8. GLDM First Order Coefficients for Temperature.

Coefficient	Value
a_1	1.0159
a_2	-0.0009

Table 9. GLDM Second Order Coefficients for Temperature.

Coefficient	Value
a_1	1.0498
a_2	-0.0302
a_3	0.0229
a_4	0.0098
a_5	-0.0340

Table 10. GLDM Third Order Coefficients for Temperature.

Coefficient	Value
Coefficient	Value
a_1	-0.1658
a_2	0.0395
a_3	1.1547
a_4	0.0362
a_5	0.0298
a_6	0.0175
a_7	-0.0489
a_8	-0.0365
a_9	0.0000

Tables 13 and 14 detail the coefficients for first and second order models applied to wind speed data using a specific modeling technique. Table 13 displays the coefficients for the first order model, capturing the most immediate past influence with coefficients a_1 and a_2. Moving to a more complex model, Table 14 lists the coefficients for the second order model, which considers additional past values to better capture the dynamics and potential patterns in

TABLE 11. GLDM Fourth Order Coefficients for Temperature.

a_1	a_2	a_3	a_4	a_5	a_6	a_7
1.1661	-0.3931	1.6191	-1.2894	-0.0031	0.1141	-0.1237
a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}
-0.0320	-0.0594	0.1424	-0.0881	-0.2193	0.1068	0.1502

TABLE 12. GLDM Fifth Order Coefficients for Temperature.

a_1	a_2	a_3	a_4	a_5
0.0000	1.0667	-0.4329	1.4878	-0.7536
a_6	a_7	a_8	a_9	a_{10}
-0.2734	0.0154	0.0804	0.0679	0.0827
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
0.0083	-0.0745	-0.0029	0.0609	-0.0092
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
-0.1386	0.0510	0.0560	-0.1933	0.0442

wind speed variations. This model includes more coefficients (a_1 to a_5), thereby providing a richer, more nuanced understanding of the influence of past wind speeds on future predictions. The expansion in the number of coefficients from the first to the second order model reflects an increase in model complexity and potential predictive power.

TABLE 13. First Order Coefficients for Wind Speed.

Coefficient	Value
a_1	1.0092
a_2	-0.0011

TABLE 14. Second Order Coefficients for Wind Speed.

Coefficient	Value
a_1	0.9300
a_2	0.0764
a_3	0.0248
a_4	0.0241
a_5	-0.0499

Tables 15, 16, and 17 illustrate the coefficients determined by the Generalized Least Deviation Method (GLDM) for analyzing death cases in Russia across three different model orders. Table 15 lists the coefficients for the first order model, suggesting a simplistic model where the primary coefficient a_1 is 1.0000, indicating a direct influence of the immediate past value on the future value with minimal adjustment (a_2 is 0.0000). As the model complexity increases, Table 16 provides five coefficients for a second order model, incorporating more nuanced interactions and trends in the data. The third order model, shown in Table 17, further expands this complexity by including nine coefficients, thus offering a more detailed and intricate depiction of the dynamics influencing the death rates. These tables collectively represent a progression in model sophistication and predictive potential, adapting to the increasing complexity required to accurately model the temporal dynamics of death cases.

6. Conclusion

For the problem (issue) of multi-criteria sequencing $1/(Et_{max}, \sum_{j=1}^{n}(C_{t_j})^2, \sum_{j=1}^{n} L_{tj})$, for the hierarchical (lexicographical) scenario, an approach to find the best solution (outcome) is put forth. It is hoped that this paper's contribution would encourage more study in the zone of multi-measuring ruling take on, particularly three hierarchically ranked criteria. Experimentation with the following machine sequencing problem will be a future research topic: $1/Lex(Et_{max}, \sum_{j=1}^{n} L_{tj}, \sum_{j=1}^{n} C_{t_j})$. In addition, we have rigorously analyzed time series data from various domains,

Table 15. First Order GLDM Model Coefficients for Death Cases in Russia.

Coefficient	Value
a_1	1.0000
a_2	0.0000

Table 16. Second Order GLDM Model Coefficients for Death Cases in Russia.

Coefficient	Value
a_1	0.7265
a_2	0.2610
a_3	0.0020
a_4	0.0016
a_5	-0.0036

Table 17. Third Order GLDM Model Coefficients for Death Cases in Russia.

Coefficient	Value
a_1	0.5970
a_2	-0.3694
a_3	0.7396
a_4	0.0083
a_5	0.0101
a_6	-0.0009
a_7	-0.0185
a_8	0.0010
a_9	0.0000

including environmental and epidemiological fields, employing the Generalized Least Deviation Method to identify the optimal model order for forecasting. Our results demonstrate that the complexity required for a predictive model is highly contingent on the dataset's characteristics, such as the nature of the data, its underlying dynamics, and the presence of non-linear patterns, rather than solely on the quantity of data available.

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