Research Paper
Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. *, No. *, *, pp. 1-10
DOI:10.22034/CMDE.2024.60001.2558

Application of new Kudryashov method to Sawada-Kotera and Kaup-Kupershmidt equations

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Abstract

In this article, with the help of the new Kudryashov method, we examine general solutions to the (2+1)-dimensional Sawada-Kotera equation (SKE) and Kaup-Kupershmidt (KK) equation. Using Maple, a symbolic computing application, it was shown that all obtained solutions are given by hyperbolic, exponential and logaritmic function solutions which obtained solutions are useful for fluid dynamics, optics and so on. Finally, we have presented some graphs for general solutions of these equations with special parameter values. The reliability and scope of programming provide eclectic applicability to high-dimensional nonlinear evolution equations for the development of this method. The results found gave us important information regarding the applicability of the new Kudryashov method.

Keywords. Exact solutions, Travelling wave solutions, Nonlinear partial differential equations.

2010 Mathematics Subject Classification. 65L05, 34K06, 34K28.

1. Introduction

Partial differential equations are important component of applied mathematics and have been known by scientists for a long time. They have a significant role in many fields including elasticity theory, fluid dynamics, plasma physics and engineering [22]. The development of computer programs for engineers and applied scientists approach to problems in PDEs has had an a significal impact. Pachage programs such as Maple, Mathematica, Matlab have started to be used for large calculations in the solutions of problems. In this paper, we used the symbolic computer program Maple to determine the exact solutions of partial differential equations.

Different approaches to solving nonlinear partial differential equations have been developed in recent years by scientits. Such as (G'/G)-expansion method [28], symmetry method [2], tanh-coth method [23, 30], Hirota's bilinear method [12], sine-cosine function method [32], the homogenous balance method [7, 29], simplest equation method [5, 34], Backlund transformation method [24], exp-function method [10], variational iteration method [11] , sine-Gordon method [33], first integral method [8], P^6- model expansion method [1], Kudryashov method [35], unified Ricatti equation expansion method [36], trial function method [3], sine-Gordon expansion approach [6], and so on.

Kudryashov method is one of the methods to solve the nonlinear PDEs. Kudryashov method was proposed by N. A. Kudryashov in 2011 [16–18]. Many researchers developed new methods using Kudryashov method [25, 37]. Like as modified Kudryashov method [14, 15, 27], improved Kudryashov method, modified improved Kudryashov method, generalized Kudryashov method, modified genereralized Kudryashov method, improved generalized Kudryashov method, modified improved generalized Kudryashov method, extended Kudryashov method and the new Kudryashov method [19] that we use in this article.

In this paper we will explain the new Kudryashov method proposed by N.A. Kudryashov. Then we will apply this method to the (2+1)-dimensional Sawada Kotera equation and the (1+1)-dimensional Kaup-Kuperchmidt equation. The new Kudryashov method is introduced in section 2. The exact solutions to the (2+1)-dimensional Sawada Kotera

Received: 09 January 2024 ; Accepted: 15 June 2024.
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equation and the (1+1)-dimensional Kaup-Kupershmidt equation are the found and illustrated in section 3. Also, we present some visual representation of the solutions. The final section provided the conclusion.

2. The new Kudryashov method

In this section, we describe the main steps of the new Kudryashov method. Assume the next nonlinear partial differential equation (PDE):

$$\Omega\left(u, u_t, u_x, u_{tt}, u_{xy}, u_{xx}, \dots\right) = 0, \tag{2.1}$$

where Ω simply represents a polynomial.

Step 1 By the travelling wave transformation $\vartheta = x + y - \sigma t$ and with $u\left(x, y, t\right) = U\left(\vartheta\right)$, Equation (2.1) can be reduced to an ordinary differential equation (ODE)

$$G\left(U, U_{\vartheta}, U_{\vartheta\vartheta}, U_{\vartheta\vartheta\vartheta}, \dots\right) = 0. \tag{2.2}$$

Step 2 Consider the Equation (2.2) has a solution as:

$$U\left(\vartheta\right) = \sum_{i=0}^{N} c_i Q^i\left(\vartheta\right), \tag{2.3}$$

where c_i $(i = 0, 1, ..., N)$ are the coefficients of $Q^i\left(\vartheta\right)$ with $c_N{\neq}0$ and

$$Q\left(\vartheta\right) = \frac{1}{\left(aA^{(\Theta\vartheta)} + bA^{(-\Theta\vartheta)}\right)} \tag{2.4}$$

is the solution of the differential Equation (2.5)

$$\left(Q^{'}\left(\vartheta\right)\right)^2 = \left(\Theta\left(lnA\right)Q\left(\vartheta\right)\right)^2\left(1 - 4abQ^2\left(\vartheta\right)\right) \tag{2.5}$$

where a, b,Θ and$A$ are constants and non-zero arbitrary real parameters to be determined later, with $A{>}0$ and $A{\neq}0$. Also the positive integer N can be determined by using homogenous balance between the highest order derivative term with the highest order nonlinear term appearing in ODE (2.2).

Step 3 After substituting solution (2.3) into Equation (2.2), the left-hand side of Equation (2.2) can be converted into a polynomial in powers of $Q\left(\vartheta\right)$. Collecting the terms that include the same power of $Q\left(\vartheta\right)$ and equating each coefficient equal to zero, one obtains an algebraic equation system for c_0, c_1, c_2, A, a, b and Θ.

Step 4 We solve the algebratic equations in the Step 3 with the help of the Maple. Substituting the obtained values of c_0, c_1, c_2, A, a, b and Θ into solution (2.3) by considering Equation (2.4), the solutions of the NLPE in Equation (2.1) can be obtained.

3. Application of the new Kudryashov method

3.1. **Solutions of the (2+1)-dimensoinal Sawada-Kotera Equation.** One of the important model in mathematical physics is the classical Sawada-Kotera equation (SKE), which was put forth by Sawada and Kotera firstly [32]. Aplication for this equation include nonlinear optics, nonlinear acoustic waves in an inharmonic lattice and quantum mechanics. It also emerged to be used to describe how long waves move in shallow water [33, 34]. Numerous varieties of approximate and exact solutions are obtained, such as lump solutions, periodic solitary wave solutions, N soliton solutions, and two soliton solutions. Thus, investigating novel interaction solutions among soliton molecules, breather molecules and soliton-breather molecules of it also of great significance.

The following form of the (2+1)-dimensional Sawada-Kotera Equation was proposed by Ma et al. [21].

$$u_t + u_{xxy} - 3uu_y - 3u_xv + u_{5x} - 15\left(uu_{xx} - u^3\right)_x = 0, \text{with } v_x = u_y, \tag{3.1}$$

where $u = u\left(x, y, t\right)$ and $v = v\left(x, y, t\right)$ are the unknow function with a real value that results from the real independent variables x, y and t. Also x, y are dimensionless spatial variables and t is temporal variable.

Now, we apply the new Kudryashov method to find the exact-solution for the Equations (3.1).

Using the travelling wave transformation

$$u(x,y,t) = U(\vartheta), v(x,y,t) = V(\vartheta), \vartheta = x + y - \sigma t. \tag{3.2}$$

where σ is represents the wave velocity. The Equation (3.1) is carried to NODE

$$-\sigma U_\vartheta + U_{\vartheta\vartheta\vartheta} - 3UU_\vartheta - 3U_\vartheta V + U_{\vartheta\vartheta\vartheta\vartheta\vartheta} - 15\left(UU_{\vartheta\vartheta} - U^3\right)_\vartheta = 0. \tag{3.3}$$

$$U_\vartheta = V_\vartheta. \tag{3.4}$$

By applying integrate on Equation (3.4) with respect to ϑ and then taking the integral constant as zero, we get

$$U = V. \tag{3.5}$$

Substituting Equation (3.5) into Equation (3.3), after integrating with respect to ϑ,

$$-\sigma U + U_{\vartheta\vartheta} - 3U^2 + U_{\vartheta\vartheta\vartheta\vartheta} - 15\left(UU_{\vartheta\vartheta} - U^3\right) + B = 0. \tag{3.6}$$

where B is constant of integration.

By balancing the highest order derivative term $U_{\vartheta\vartheta\vartheta\vartheta}$ and the highest order nonlinear term U^3,we get;

$$\begin{aligned} N + 4 &= 3N, \\ N &= 2. \end{aligned} \tag{3.7}$$

So from solution (2.3), we can write

$$U(\vartheta) = c_0 + c_1 Q(\vartheta) + c_2 Q^2(\vartheta) \tag{3.8}$$

where c_0, c_1 and c_2 are constants to be determined later.

Substituting solution (3.8) into the Equation (3.6) yields a polinamial in $Q(\vartheta)$. A system of algebraic equations is obtained by setting each coefficient of the equations to zero.

$$\begin{aligned}
Q^6 &: \; 1920\ln(A)^4 a^2b^2c_2\Theta^4 + 360\ln(A)^2 abc_2^2\Theta^2 + 15c_2^3, \\
Q^5 &: \; 384\ln(A)^4 a^2b^2c_1\Theta^4 + 480\ln(A)^2 abc_1c_2\Theta^2 + 45c_1c_2^2, \\
Q^4 &: \; -480\ln(A)^4 abc_2\Theta^4 + 360\ln(A)^2 abc_0c_2\Theta^2 + 120\ln(A)^2 abc_1^2\Theta^2 - 24\ln(A)^2 abc_2\Theta^2 \\
&\quad\; -60\ln(A)^2 c_2^2\Theta^2 + 45c_0c_2^2 + 45c_1^2c_2 - 3c_2^2, \\
Q^3 &: \; -80\ln(A)^4 abc_1\Theta^4 + 120\ln(A)^2 abc_0c_1\Theta^2 - 8\ln(A)^2 abc_1\Theta^2 - 75\ln(A)^2 c_1c_2\Theta^2 + 90c_0c_1c_2 + 15c_1^3 - 6c_1c_2, \\
Q^2 &: \; 16\ln(A)^4\Theta^4c_2 - 60\ln(A)^2\Theta^2c_0c_2 - 15\ln(A)^2\Theta^2c_1^2 + 4\ln(A)^2\Theta^2c_2 + 45c_0^2c_2 + 45c_1^2c_0 - \sigma c_2 - 6c_0c_2 - 3c_1^2, \\
Q^1 &: \; \ln(A)^4\Theta^4c_1 - 15\ln(A)^2\Theta^2c_0c_1 + \ln(A)^2\Theta^2c_1 + 45c_0^2c_1 - \sigma c_1 - 6c_0c_1, \\
Q^0 &: \; 15c_0^3 - \sigma c_0 - 3c_0^2 + B.
\end{aligned}$$

To solve the above system of algebraic equations we use Maple Software. Therefore we get following 2 solution sets;

Set 1

$$\begin{aligned}
&B = 16\ln(A)^4\Theta^4c_0 - 60\ln(A)^2\Theta^2c_0^2 + 4\ln(A)^2\Theta^2c_0 + 30c_0^3 - 3c_0^2, \\
&\sigma = 16\ln(A)^4\Theta^4 - 60\ln(A)^2\Theta^2c_0 + 4\ln(A)^2\Theta^2 + 45c_0^2 - 6c_0, \\
&c_0 = c_0, c_1 = 0, c_2 = -8\ln(A)^2\Theta^2ab.
\end{aligned} \tag{3.9}$$

Substituting Equations (3.9) into (3.8) with using (2.4), we obtain the exact solution of the (2+1)-dimensional SKE as follows:

$$U_1 = V_1 = c_0 - \frac{8\ln(A)^2\Theta^2ab}{\left(aA^{(\Theta\vartheta)} + bA^{(-\Theta\vartheta)}\right)^2}, \tag{3.10}$$

where

$$\vartheta = -\left(16\ln(A)^4\Theta^4 - 60\ln(A)^2\Theta^2 c_0 + 4\ln(A)^2\Theta^2 + 45c_0^2 - 6c_0\right)t + x + y. \tag{3.11}$$

Set 2

$$\begin{aligned} &B = \frac{-\left(128\ln(A)^6\Theta^6\right)}{9} + \frac{\left(16\ln(A)^4\Theta^4\right)}{15} - \frac{1}{225}, \sigma = 16\ln(A)^4\Theta^4 - \frac{1}{5}, \\ &c_0 = \frac{\left(4\ln(A)^2\Theta^2\right)}{3} + \frac{1}{5}, c_1 = 0, c_2 = -16\ln(A)^2\Theta^2 ab. \end{aligned} \tag{3.12}$$

Substituting Equations (3.12) into (3.8) with using ((2.4), we obtain exact solution of the (2+1)-dimensional SKE as follows:

$$U_2 = V_2 = \frac{\left(4\ln(A)^2\Theta^2\right)}{3} + \frac{1}{15} - \frac{16\ln(A)^2\Theta^2 ab}{\left(aA^{(\Theta\vartheta)} + bA^{(-\Theta\vartheta)}\right)^2}, \tag{3.13}$$

where

$$\vartheta = -\left(16\ln(A)^4\Theta^4 - \frac{1}{5}\right)t + x + y. \tag{3.14}$$

In particular, if we have $a = b$ in solution U_2 ,then Equation (3.3) has the following bright-soliton solution

$$u_{2.1}(x,y,t) = \frac{1}{15} + \frac{4\ln(A)^2\Theta^2\left(1 - 3sech^2\left(\Theta\left(80t\ln(A)^4\Theta^4 - t - 5x - 5y\right)\frac{\ln(A)}{5}\right)\right)}{3}. \tag{3.15}$$

On the other hand if we take $a = -b$ in in solution U_2, then Equation (3.3) has the following singular-soliton solution

$$u_{2.2}(x,y,t) = \frac{4\ln(A)^2\Theta^2}{3} + 4\ln(A)^2\Theta^2 csch^2\left(\Theta\left(80t\ln(A)^4\Theta^4 - t - 5x - 5y\right)\frac{\ln(A)}{5}\right) + \frac{1}{15}. \tag{3.16}$$

Figure 1 and Figure 2 as given below, eight graphs in total are drawn with the help of the Maple for the exact solutions $U_1 = V_1$ (3.10) and $U_2 = V_2$ (3.13). For the graphs of these exact solutions are drawn for the Sawada-Kotera equation, the parameters and the range of x and t values are specially chosen.

3.2. **Solutions of the (1+1)-dimensional Kaup-Kupershmidt** (KK) **equation.** The standard fifth-order KdV equation (fKdV) of the form [31],

$$u_t + \alpha u^2 u_x + \beta u_x u_{xx} + \gamma u u_{3x} + u_{5x} = 0. \tag{3.17}$$

where α, β and γ are arbitrary nonzero and real parameters and $u = u(x,t)$ is a differentiable function. With taking different values of the parameters α, β and γ, the Equation (3.17) change the characteristics of the fKdV equation.

Kaup-Kupershmidt equation is characterized by

$$\beta = \frac{5}{2}\gamma, \alpha = \frac{1}{5}\gamma^2.$$

For $\gamma = 10$, then $\beta = 25$ and $\alpha = 20$, therefore Equation (3.17) reduces to standard KK equation:

$$u_t + 20u^2 u_x + 25u_x u_{xx} + 10u u_{3x} + u_{5x} = 0. \tag{3.18}$$

where $u = u(x,t)$ is the unknow function with a real value that results from the real independent variables x and y.

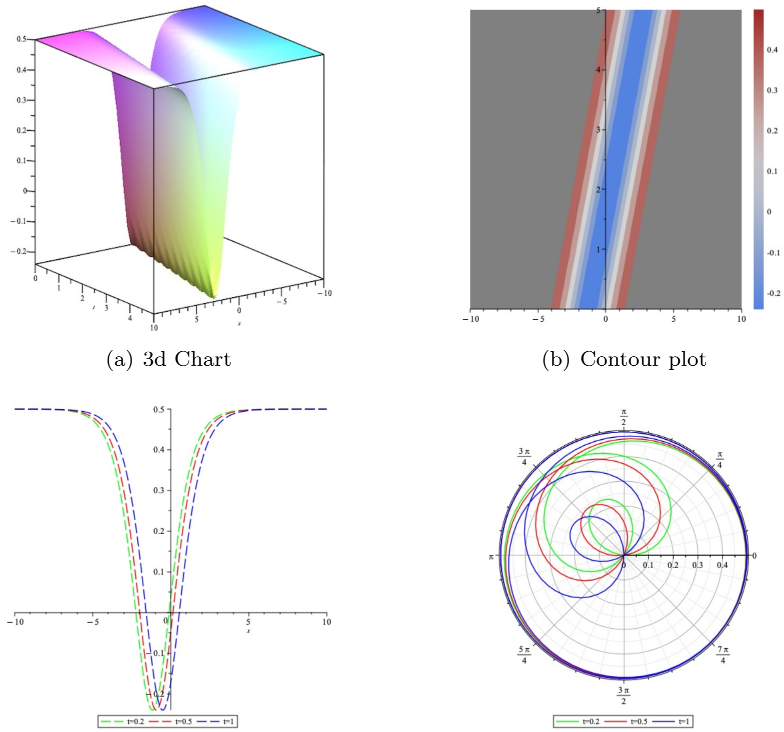
The Kaup-Kupershmidt equation was proposed by Kaup in 1980 [13, 20].

To find the exact-solution for the Kaup-Kupershmidt Equation (3.18), we apply the new Kudryashov method.

Consider the following travelling wave transformation

$$u(x,t) = U(\vartheta), \vartheta = x - \sigma t. \tag{3.19}$$

where σ is represents the wave velocity.



(a) 3d Chart (b) Contour plot

(c) 2d Chart (d) Polar coordinates

FIGURE 1. New exact solution $U_1 = V_1(3.6)$ of the (2+1)-dimensional SKE (3.10), for $A = 1.5, \Theta = 1.5, a = 3, b = 2, c_0 = 0.5, y = 1$, when in the range of t $\in [0, 10]$, x $\in [-10, 10]$.

The Equation (3.18) is carried to NODE

$$-\sigma U_\vartheta + 20U^2U_\vartheta + 25U_\vartheta U_{\vartheta\vartheta} + 10U_\vartheta U_{\vartheta\vartheta\vartheta} + U_{5\vartheta} = 0, \tag{3.20}$$

integrating with respect to ϑ

$$-\sigma U + \frac{20}{3}U^3 + \frac{15}{2}(U_\vartheta)^2 + 10UU_{\vartheta\vartheta} + U_{\vartheta\vartheta\vartheta\vartheta} + B = 0, \tag{3.21}$$

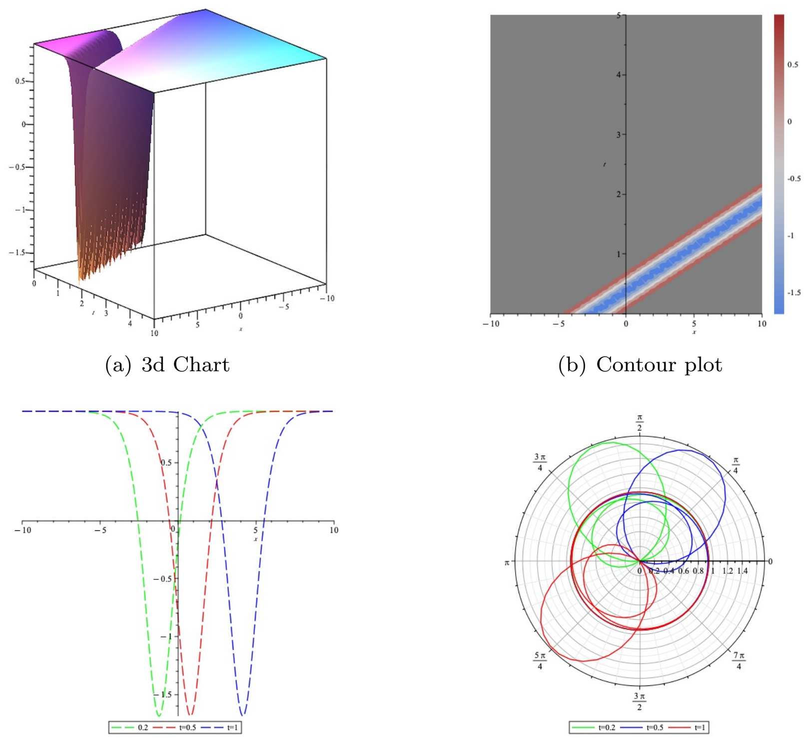
where B is constant of integration. By balancing the highest order derivative term $U_{\vartheta\vartheta\vartheta\vartheta}$ and the highest order nonlinear term U^3, we get,

$$\begin{aligned} N + 4 &= 3N, \\ N &= 2. \end{aligned} \tag{3.22}$$

So from (2.3), we can write

$$U(\vartheta) = c_0 + c_1Q(\vartheta) + c_2Q^2(\vartheta), \tag{3.23}$$

where c_0, c_1 and c_2 are constants to be determined later.



(a) 3d Chart (b) Contour plot

(c) 2d Chart (d) Polar coordinates

FIGURE 2. New exact solution $U_2 = V_2(3.13)$ of the (2+1)-dimensional SKE (3.6), for $A = 1.5, \Theta = 2, a = 5, b = 2, c_0 = 0.5, y = 2$ when in the range of t $\in [0, 10]$, x $\in [-10, 10]$.

Substituting (3.23) into the Equation (3.21) yields a polinamial in $Q(\vartheta)$. A system of algebraic equations is obtained by setting each coefficient of the equations to zero.

$$\begin{aligned}
Q^6 &: \quad 1920 \ln(A)^4 \Theta^4 a^2 b^2 c_2 - 360\Theta^2 \ln(A)^2 abc_2^2 + \frac{(20c_2^3)}{3}, \\
Q^5 &: \quad 384 \ln(A)^4 \Theta^4 a^2 b^2 c_1 - 440\Theta^2 \ln(A)^2 abc_1c_2 + 20c_1c_2^2, \\
Q^4 &: \quad -480 \ln(A)^4 \Theta^4 abc_2 - 240\Theta^2 \ln(A)^2 abc_0c_2 - 110\Theta^2 \ln(A)^2 abc_1^2 + 70\Theta^2 \ln(A)^2 c_2^2 + 20c_0c_2^2 + 20c_1^2c_2, \\
Q^3 &: \quad 80\Theta^2 \ln(A)^2 c_1c_2 - 80 \ln(A)^4 \Theta^4 abc_1 + 40c_0c_1c_2 + \frac{20c_1^3}{3} - 80\Theta^2 \ln(A)^2 abc_0c_1, \\
Q^2 &: \quad 40\Theta^2 \ln(A)^2 c_0c_2 + 20c_0c_1^2 - \sigma c_2 + 20c_0^2c_2 + 16 \ln(A)^4 \Theta^4 c_2 + \frac{35\Theta^2 \ln(A)^2 c_1^2}{2}, \\
Q^1 &: \quad \ln(A)^4 \Theta^4 c_1 + 10\Theta^2 \ln(A)^2 c_0c_1 + 20c_0^2c_1 - \sigma c_1, \\
Q^0 &: \quad B - \sigma c_0 + \frac{20c_0^3}{3}.
\end{aligned}$$

To solve the above system of algebraic equations, we use Maple Software and we get the following two solution sets;
Set 1

$$B = \frac{\Theta^6 \ln(A)^6}{3}, \sigma = \ln(A)^4 \Theta^4, c_0 = \frac{-\Theta^2 \ln(A)^2}{2}, c_1 = 0, c_2 = 6\Theta^2 ab \ln(A)^2. \tag{3.24}$$

Substituting Equations (3.24) into (3.23) using (2.4), we obtain the exact solution of the KK as follows:

$$U_3 = \frac{-\Theta^2 \ln(A)^2}{2} + \frac{6\ln(A)^2\,\Theta^2 ab}{\left(aA^{(\Theta\vartheta)} + bA^{(-\Theta\vartheta)}\right)^2}, \tag{3.25}$$

where

$$\vartheta = -\ln(A)^4\,\Theta^4 t + x. \tag{3.26}$$

In particular, if we have $a = b$ in solution U_2, then Equation (3.3) has the following bright-soliton solution

$$u_{1.1}(x,t) = \Theta^2 \ln(A)^2\, \frac{3sech^2\left(\Theta^2\left(\ln(A)^4\,\Theta^8 t - x\right)\ln(A)\right) - 1}{2}. \tag{3.27}$$

On the other hand if we take $a = -b$ in in solution U_2, then Equation (3.3) has the following singular-soliton solution

$$u_{1.2}(x,t) = -\Theta^2 \ln(A)^2\, \frac{1 + 3csch^2\left(\Theta\left(\ln(A)^4\,\Theta^4 t - x\right)\ln(A)\right)}{2}. \tag{3.28}$$

Set 2

$$B = \frac{-832\Theta^6 \ln(A)^6}{3}, \sigma = 176\ln(A)^4\,\Theta^4, c_0 = -4\Theta^2\ln(A)^2, c_1 = 0, c_2 = 48\Theta^2 ab\ln(A)^2. \tag{3.29}$$

Substituting Equations (3.29) into (3.23) with using (2.4), we obtain the exact solution of the KK as follows:

$$U_4 = -4\Theta^2 \ln(A)^2 + \frac{48\ln(A)^2\,\Theta^2 ab}{\left(aA^{(\Theta\vartheta)} + bA^{(-\Theta\vartheta)}\right)^2}, \tag{3.30}$$

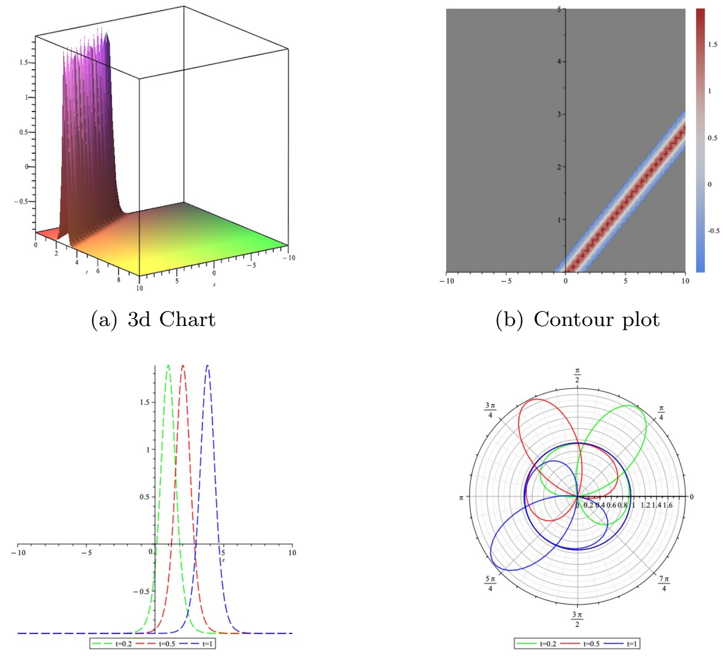
where

$$\vartheta = -176\ln(A)^4\,\Theta^4 t + x. \tag{3.31}$$

The graphs are drawn with help of the Maple for the U_3 (3.25) and U_4 (3.30) by choosing the parameters and x, t values in Figure 3 and Figure 4.

4. Conclusion

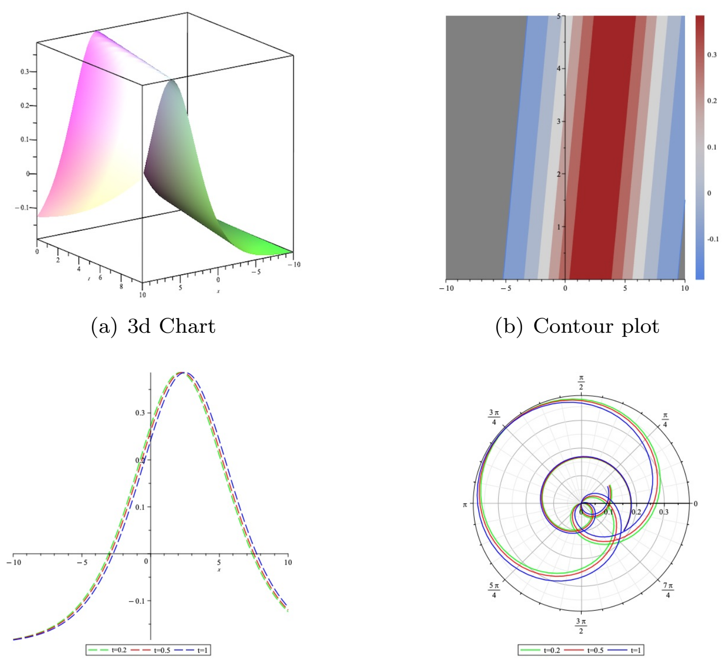
In this study, we discussed (2+1)-dimensional Sawada Kotera equation and the (1+1)-dimensional Kaup-Kupershmidt equation, both of which have several uses in mathematics and physics. We utilized the new Kudryashov method after reducing the PDE to ODE using traveling wave transformation. We observed different bright soliton solutions for the (2+1)-dimensional Sawada Kotera equation and the (1+1)-dimensional Kaup-Kupershmidt equation using the $Q(\vartheta)$ function, which is in exponential form and is the solution of the auxiliary equation. Various three and two dimensional plots, polar coordinate plots and contour plots, and dynamical characteristics of these waves are shown well using Maple. The newly found hyperbolic function solutions may be useful for understanding physical phenomena especially in long-wave propagation, dynamics of shallow water wave, and plasma fluid. Additionally, it is clear that the approach used in this article is a powerful and practical mathematical tool that may be used to provide exact solutions to various other different NLPDEs in the future.



(a) 3d Chart (b) Contour plot

(c) 2d Chart (d) Polar coordinates

FIGURE 3. New exact solution U_3(3.25) of the KK Equation (3.21), for $A = 2.5, \Theta = 1.5, a = 1, b = 2$ when in the range of t $\in [0, 10]$, x $\in [-10, 10]$.



(a) 3d Chart (b) Contour plot

(c) 2d Chart (d) Polar coordinates

FIGURE 4. New exact solution U_4(3.30) of the KK Equation (3.21), for $A = 3, \Theta = 0.2, a = 2, b = 5$ when in the range of t $\in [0, 10]$, x $\in [-10, 10]$.

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