



Modeling and simulation of COVID-19 disease dynamics via Caputo Fabrizio fractional derivative

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Abstract

The motive of this paper is to investigate the SEIQRD model of COVID-19 outbreak in Indonesia with the help of fractional modeling approach. The model is described by the nonlinear system of six fractional order differential equations (DE) incorporating Caputo Fabrizio Fractional derivative (CFFD) operator. The existence and uniqueness of model are proved by applying the well-known Banach contraction theorem. The reproduction number (R_0) is calculated, and its sensitivity analysis is conducted concerning each parameters of the model for the prediction and persistence of the infection. Moreover, the numerical simulation for various fractional orders is performed using Adams-Bashforth technique to analyze the transmission behavior of disease and to get the approximated solutions. At last, we represent our numerical simulation graphically to illustrate our analytical findings.

Keywords. COVID-19, Caputo Fabrizio Fractional Derivative, Existence and Uniqueness, Sensitivity Analysis, Simulation and Discussion.

2010 Mathematics Subject Classification. 92D30, 34A08, 35A05, 65P99.

1. INTRODUCTION

The COVID-19 infection continues to shape the world in different ways and affected almost every facet of life. The SARS-CoV-2 virus was first found in Wuhan, China in 2019 and is still challenging the world's economy, medical systems, social life etc. Although the incubation period of COVID-19 is generally 5 to 6 days but it can range up to 14 days. Dry cough, fever, loss of taste and smell, weakness, discomfort in the muscles and joints, etc. are some of the primary signs and indications of COVID-19 [19]. Symptoms vary from person to person based upon their immunity. Age also plays the role as the children are effected rarely but the elders are more severely effected. Vaccinations and isolation are the significant factors to prevent the propagation of COVID-19. Many researchers are tracking the spread of this virus and figuring out to analyze the impact of various factors [11, 20, 21]. For effective analysis, mathematical modeling plays a crucial role. Many mathematical models are formulated by researchers to estimate disease transmission by calculating rate of recovery, death rate etc. [4, 7, 12, 17, 22]. The system of DE is constructed to investigate the drift of the biological system. The epidemic modeling works with the classification of population into several compartments that play distinct role in the transmission of disease [5]. Earliest approach in epidemic modeling was the well known Mckendrick's SIR model [9]. SIR type of models have been extensively analyzed by researchers. Then various modifications were performed in this by adding additional compartments such as exposed, quarantined, recovered and vaccinated etc. Different kernels were used with various operators for the analysis of disease [15, 28]. For example the SEIR and SEIRD models separates recovered and dead individuals from the population [24, 25]. Age based compartmental model was also developed for assessment of COVID-19 [8, 16]. Later vaccinated and quarantined individuals also got separated from population. In this

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direction, Peng et al. [23] formulated the SEIQRD epidemic model but this model was not considering birth and death parameters. Then authors in [18] modified the model by considering the effect of these two but they neglected the deaths caused by COVID-19. Then authors in [29] formulated SEIQRD model by taking separate compartment for the deaths caused by COVID-19 into consideration.

In this paper our goal is to analyze the transmission behavior of COVID-19 by studying SEIQRD model [10] with CFFD operator. To examine the prevalence of disease within the population, the basic reproduction number is calculated and by making use of its sensitivity of parameters is also checked. The following is an outline of our study. Section 2 contains some relevant definitions for our study. Section 3 presents the fractional order model consisting of six equations. The existence and uniqueness of our fractional order SEIQRD model is proposed in section 4. Section 5 estimates the basic reproduction number by employing the next generation matrix approach and sensitivity of model parameters is performed. Section 6 provides the numerical approximation scheme of CFFD. Numerical simulation and discussion is provided in section 7. Then the conclusions of our study are given in section 8.

2. MATHEMATICAL PRELIMINARIES

This section of article will cover fundamental definitions of fractional order Caputo and Caputo Fabrizio operator.

Definition 2.1. [14] Let $\phi \in (0, 1)$ then the fractional Caputo derivative of order ϕ is expressed as:

$${}^c\mathfrak{D}_t^\phi f(t) = \frac{1}{\Gamma(\phi)} \int_0^t G(\lambda, f(\lambda))(t - \lambda)^{\phi-1} d\lambda.$$

Definition 2.2. [1] Let $\phi \in (0, 1)$ then the CFFD with order ϕ for the function f is given by,

$${}^{CF}\mathfrak{D}_t^\phi f(t) = \frac{\phi\mathcal{M}(\phi)}{(1-\phi)} \int_a^t \frac{df(x)}{dx} \exp\left[-\phi \frac{t-x}{1-\phi}\right] dx,$$

here $\mathcal{M}(\phi)$ is a normalization constant. $\mathcal{M}(0)=\mathcal{M}(1)=1$.

Definition 2.3. [1] Let $\phi \in (0, 1)$ then the Caputo Fabrizio Fractional Integral having order ϕ for the function f is defined by,

$${}^{CF}\mathfrak{I}_t^\phi f(t) = \frac{2(1-\phi)}{(2-\phi)\mathcal{M}(\phi)} f(t) + \frac{2\phi}{(2-\phi)\mathcal{M}(\phi)} \int_0^t f(s) ds,$$

here $\mathcal{M}(\phi)$ is a normalization constant. $\mathcal{M}(0)=\mathcal{M}(1)=1$.

Lemma 2.4. For $0 < \phi \leq 1$, the relation between CFFD and the corresponding integral is given by

$$({}^{CF}\mathfrak{I}_t^\phi)({}^{CF}\mathfrak{D}_t^\phi)f(t) = f(t) - f(a)$$

Lemma 2.5. [27] Let $\varrho(\theta) \in C([0, T])$, then the solution of the following CFFD equation

$${}^{CF}\mathfrak{D}_\theta^{\phi(\theta)} \varrho(\theta) = \omega(\theta), \theta \in [0, T], 0 < \phi(\theta) \leq 1,$$

$$\varrho(0) = \omega_0, \omega_0 \in \mathbb{R},$$

is given by

$$\varrho(\theta) = \omega_0 + \frac{(1-\phi(\theta))}{M(\phi(\theta))} \omega_0 + \frac{\phi(\theta)}{M(\phi(\theta))} \int_0^\theta \omega(a) da.$$



3. MATHEMATICAL MODEL

In this article , we present a *SEIQRD* model by envisioning the model formulated in [10]. Later we transform our model into fractional order, so that classical mathematical model can be precised on an upper level which is required to analyze these kind of illness. The integer order non linear model transmission is as follows.

Figure 1 depicts the schematic diagram of SEIQRD model. In our presented model, the entire population is categorized into six classes namely: Susceptible humans (*S*), Exposed humans (*E*), Infected humans (*I*), Quarantined humans (*Q*), Recovered humans (*R*) and Dead humans (*D*). At rate λ susceptible people are recruiting into the population, α and β are the progression rates by which individuals are moving from susceptible to exposed and from exposed to infected classes respectively. ξ is the rate by which infected people are getting recovered. Some of the infected people are moving to quarantine class by rate ν . μ is natural death rate and the disease induced death rate from infected and quarantined class is depicted by τ and δ respectively.

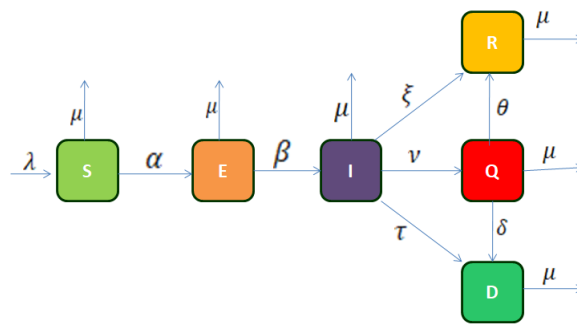


FIGURE 1. Schematic diagram of the SEIQRD model.

$$\begin{aligned}
 \frac{dS(t)}{dt} &= \lambda - \frac{\alpha S(t)I(t)}{T(t)} - \mu S(t), \\
 \frac{dE(t)}{dt} &= \frac{\alpha S(t)I(t)}{T(t)} - (\mu + \beta)E(t), \\
 \frac{dI(t)}{dt} &= \beta E(t) - (\xi + \nu + \mu + \tau)I(t), \\
 \frac{dQ(t)}{dt} &= \nu I(t) - (\theta + \mu + \delta)Q(t), \\
 \frac{dR(t)}{dt} &= \xi I(t) + \theta Q(t) - \mu R(t), \\
 \frac{dD(t)}{dt} &= \delta Q(t) + \tau I(t).
 \end{aligned}
 \tag{3.1}$$

Then, the entire population $T(t)$ at time t i.e.

$$T(t) = S(t) + E(t) + I(t) + Q(t) + R(t) + D(t).$$

The specifications of parameters for our model are explained in the Table 1 and their values are taken from [10].



TABLE 1. Parameters denotations and values.

Parameter	Description	Value
λ	Transmission rate	3.52
α	Rate of infection	1.7
β	Progression rate	0.052
ν	Rate of quarantine	2
ξ	Recovery rate of infected	0.041
τ	Disease induced mortality rate	0.2
θ	Rate of recovery of quarantined people	0.041
δ	Death rate of quarantined individuals induced by disease	0.2
μ	Natural death rate	0.0001

The term $\alpha S(t)I(t)$ indicates the density of Susceptible individuals who have been infected (not infectious). Here it is assumed that after being isolated, these people do not infect other people. Now we take forward this model from the integer order to the fractional order via CFFD.

$$\begin{aligned}
{}^{CF}\mathfrak{D}_t^\phi S(t) &= \lambda - \frac{\alpha S(t)I(t)}{T(t)} - \mu S(t), \\
{}^{CF}\mathfrak{D}_t^\phi E(t) &= \frac{\alpha S(t)I(t)}{T(t)} - (\mu + \beta)E(t), \\
{}^{CF}\mathfrak{D}_t^\phi I(t) &= \beta E(t) - (\xi + \nu + \mu + \tau)I(t), \\
{}^{CF}\mathfrak{D}_t^\phi Q(t) &= \nu I(t) - (\theta + \mu + \delta)Q(t), \\
{}^{CF}\mathfrak{D}_t^\phi R(t) &= \xi I(t) + \theta Q(t) - \mu R(t), \\
{}^{CF}\mathfrak{D}_t^\phi D(t) &= \delta Q(t) + \tau I(t).
\end{aligned} \tag{3.2}$$

with positive initial conditions,

$$S_0 > 0, E_0 > 0, I_0 > 0, Q_0 > 0, R_0 > 0, D_0 > 0.$$

Here ϕ is the fractional order of Caputo Fabrizio Fractional operator and also, $\phi \in (0, 1]$.

4. THE EXISTENCE AND UNIQUENESS

We will conduct a qualitative analysis of model (3.2) in this part of our study. To check that our modeled problem is posed well and have unique solution, we will employ the well known Banach contraction theorem of fixed point theory [2, 26]. For this, first we let the vector $(S, E, I, Q, R, D) = \zeta(t)$. We apply the Picard's Operator approach to our considered model (3.2). Let's define the function as follows:

$$\begin{aligned}
f_1(t, \zeta(t)) &= \lambda - \frac{\alpha S(t)I(t)}{T(t)} - \mu S(t), \\
f_2(t, \zeta(t)) &= \frac{\alpha S(t)I(t)}{T(t)} - (\beta + \mu)E(t), \\
f_3(t, \zeta(t)) &= \beta E(t) - (\xi + \nu + \tau + \mu)I(t), \\
f_4(t, \zeta(t)) &= \nu I(t) - (\theta + \delta + \mu)Q(t), \\
f_5(t, \zeta(t)) &= \xi I(t) + \theta Q(t) - \mu R(t), \\
f_6(t, \zeta(t)) &= \delta Q(t) + \tau I(t).
\end{aligned} \tag{4.1}$$



Now,

$$\delta_j = \sup_{C \in [m, b_i]} \|f_1(t, \zeta(t))\| \quad \text{for } j = 1, 2, 3, 4, 5, 6. \tag{4.2}$$

$$C \in [m, b_i] = [t - m, t + m] \times [u - c_j, u + c_j] = Z \times Z_j \quad \text{for } j = 1, 2, 3, 4, 5, 6. \tag{4.3}$$

Now by applying Banach fixed point theorem, we express the norm on $C \in [m, b_i]$ for $j=1,2,\dots,6$ as follows

$$\|V\|_\infty = \sup_{t \in [t-m, t+m]} |\phi(t)|. \tag{4.4}$$

We define Picard's operator as

$P : C(Z, Z_1, Z_2, Z_3, Z_4, Z_5, Z_6) \rightarrow C(Z, Z_1, Z_2, Z_3, Z_4, Z_5, Z_6)$. Applying Caputo Fabrizio integral i.e. ${}^{CF}I^\phi$ on the model equations, we get

$$\begin{aligned} S(t) - S_0 &= {}^{CF}I_t^\phi [f_1(t, \zeta(t))], \\ E(t) - E_0 &= {}^{CF}I_t^\phi [f_2(t, \zeta(t))], \\ I(t) - I_0 &= {}^{CF}I_t^\phi [f_3(t, \zeta(t))], \\ Q(t) - Q_0 &= {}^{CF}I_t^\phi [f_4(t, \zeta(t))], \\ R(t) - R_0 &= {}^{CF}I_t^\phi [f_5(t, \zeta(t))], \\ D(t) - D_0 &= {}^{CF}I_t^\phi [f_6(t, \zeta(t))]. \end{aligned} \tag{4.5}$$

On solving the right hand side of Equation (4.5) by using Lemma 2.5, we get

$$V(t) = V_0(t) + [\Xi(t, V(t)) - \Xi(t)] \frac{1 - \phi}{M(\phi)} + \frac{1 - \phi}{M(\phi)} \int_0^t \Xi(\rho, V(\rho)) d\rho, \tag{4.6}$$

where, $V(t) = (S, E, I, Q, R, D)^T$ and $V_0(t) = (S_0, E_0, I_0, Q_0, R_0, D_0)^T$.

$$\Xi(t, V(t)) = \begin{cases} f_1(t, \zeta(t)) \\ f_2(t, \zeta(t)) \\ f_3(t, \zeta(t)) \\ f_4(t, \zeta(t)) \\ f_5(t, \zeta(t)) \\ f_6(t, \zeta(t)) \end{cases} \tag{4.7}$$

and

$$\Xi_0(t, V(t)) = \begin{cases} f_1(0, \zeta_0(t)) \\ f_2(0, \zeta_0(t)) \\ f_3(0, \zeta_0(t)) \\ f_4(0, \zeta_0(t)) \\ f_5(0, \zeta_0(t)) \\ f_6(0, \zeta_0(t)) \end{cases} \tag{4.8}$$

So, the Picard's operator takes the form as



$$\mathbf{P}V(t) = V_0(t) + [\Xi(t, V(t)) - \Xi_0(t, V(t))] \frac{1-\phi}{M(\phi)} + \frac{1-\phi}{M(\phi)} \int_0^t \Xi(\rho, V(\rho)) d\rho. \quad (4.9)$$

We assume that $\|V\|_\infty \leq \max\{k_1, k_2, k_3, k_4, k_5, k_6\}$.

Then considering $\Delta = \max\{\Delta_j\}$ for $j=1,2,\dots,6$ and $t_0 = \max\{t \in D\}$ we get,

$$\begin{aligned} \|\mathbf{P}V(t) - V_0(t)\| &= \left\| [\Xi(t, V(t)) - \Xi_0(t, V(t))] \frac{1-\phi}{M(\phi)} + \frac{1-\phi}{M(\phi)} \int_0^t \Xi(\rho, V(\rho)) d\rho \right\| \\ &\leq \frac{1-\phi}{M(\phi)} \|\Xi(t, V(t))\| + \frac{\phi}{M(\phi)} \int_0^t \|\Xi(\rho, V(\rho))\| d\rho \\ &\leq \frac{1-\phi}{M(\phi)} \Delta + \frac{\phi}{M(\phi)} \Delta t \\ &\leq \Delta k \leq \max\{k_1, k_2, k_3, k_4, k_5, k_6\} = \bar{k}. \end{aligned} \quad (4.10)$$

and also, $k \leq \frac{\bar{k}}{\Delta}$.

Moreover, to evaluate inequality,

$$\begin{aligned} \|\mathbf{P}V_1(t) - \mathbf{P}V_2(t)\|_\infty &= \sup_{t \in Z} \|V_1(t) - V_2(t)\| \\ \|\mathbf{P}V_1(t) - \mathbf{P}V_2(t)\| &= \left\| \frac{1-\phi}{M(\phi)} (\Xi(\rho, V_1(t)) - \Xi(\rho, V_2(t))) + \frac{\phi}{M(\phi)} \int_0^t (\Xi(\rho, V_1(\rho)) - \Xi(\rho, V_2(\rho))) d\rho \right\| \\ &\leq \frac{1-\phi}{M(\phi)} l \|V_1(t) - V_2(t)\| + \frac{sl}{M(s)} \int_0^t \|V_1(t) - V_2(t)\| \end{aligned} \quad (4.11)$$

with, $l = \sup_{t \in Z} \Xi(t, V_1(t))$ and also, $l < 1$,

$$\begin{aligned} &\leq \left(\frac{1-\phi}{M(\phi)} l + \frac{\phi t_0}{M(\phi)} l \right) \|V_1(t) - V_2(t)\| \\ &\leq kl \|V_1(t) - V_2(t)\|. \end{aligned}$$

For \mathbf{P} to be contracted kl should be less than 1. So, the defined operator \mathbf{P} is contracted. Consequently our system has unique solution.

5. REPRODUCTION NUMBER AND SENSITIVITY ANALYSIS

5.1. Reproduction Number. The number of secondary cases transmit by one infected individual is called Reproduction number (R_0). R_0 helps to analyze the effort which is required to eradicate the disease from population. In this part, we find the disease-free equilibrium (DFE) point and R_0 by employing Next Generation Matrix Approach. By DFE we refer to the situation where disease no longer remains in the population i.e. $E = I = Q = R = D = 0$ Hence DFE is given by

$$\chi_0 = (S^0, E^0, I^0, Q^0, R^0, D^0) = \left(\frac{\lambda}{\mu}, 0, 0, 0, 0, 0 \right)$$

Now to calculate R_0 , the considered system is

$${}^{CF}\mathfrak{D}_t^\phi \zeta(t) = F(\zeta(t)) - V(\zeta(t))$$



here,

$$F(\zeta(t)) = \begin{pmatrix} \frac{\alpha SI}{T} \\ 0 \\ 0 \end{pmatrix} \text{ and, } V(\zeta(t)) = \begin{pmatrix} (\beta + \mu)E \\ -\beta E + (\xi + \nu + \mu + \tau)I \\ -\nu I + (\delta + \mu + \theta)Q \end{pmatrix}$$

At χ_0 the Jacobian matrices of F and V are

$$J_F = \begin{pmatrix} 0 & \alpha & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ and, } J_V = \begin{pmatrix} \beta + \mu & 0 & 0 \\ -\beta & (\xi + \mu + \tau + \nu) & 0 \\ 0 & 0 & (\theta + \delta + \mu) \end{pmatrix}.$$

Now the next generation matrix FV^{-1} is given by,

$$FV^{-1} = \begin{pmatrix} \frac{\alpha\beta}{(\beta+\mu)(\xi+\mu+\tau+\nu)} & \frac{\alpha}{(\xi+\mu+\tau+\nu)} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{5.1}$$

The maximum of the absolute values of the eigenvalues of (5.1) is our required R_0 . So,

$$R_0 = \frac{\alpha\beta}{(\beta + \mu)(\xi + \nu + \tau + \mu)} \tag{5.2}$$

5.2. Sensitivity Analysis. By sensitivity analysis, we can tell about the most critical parameter of our model which has substantial influence in disease transmission. By doing so, we are able to find that which parameter has high impact on R_0 . The sensitivity indices are very useful to estimate the variation in the state variable caused by the parameter change. The parameter values are used to calculate sensitivity index. Sensitivity index could be both positive or negative. A positive sensitive index implies that disease will spread on increase of the respective parameter. The normalized forward sensitivity index of R_0 for the parameter ϕ is given by [6]

$$S_{\phi}^{R_0} = \frac{\partial R_0}{\partial \phi} \times \frac{\phi}{R_0}.$$

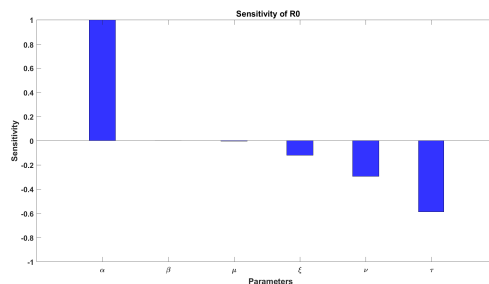


FIGURE 2. Sensitivity indices of parameters.



So, by using the above formula, we get

$$\begin{aligned}
S_{\alpha}^{R_0} &= \frac{\beta}{(\mu + \beta)(\xi + \nu + \tau + \mu)} \times \frac{\alpha}{R_0} = 1, \\
S_{\beta}^{R_0} &= \frac{\alpha\mu}{(\mu + \beta)^2(\xi + \nu + \tau + \mu)} \times \frac{\beta}{R_0} = \frac{\mu}{(\mu + \beta)}, \\
S_{\mu}^{R_0} &= \frac{-\alpha\beta(\xi + \nu + \tau + 2\mu + \beta)}{(\mu + \beta)^2(\xi + \nu + \tau + \mu)^2} \times \frac{\mu}{R_0} = \frac{-\mu(\xi + \nu + \tau + 2\mu + \beta)}{\xi + \nu + \tau + \mu}, \\
S_{\xi}^{R_0} &= \frac{-\alpha\beta}{(\mu + \beta)(\xi + \nu + \tau + \mu)^2} \times \frac{\xi}{R_0} = \frac{-\xi}{(\xi + \nu + \tau + \mu)}, \\
S_{\nu}^{R_0} &= \frac{-\alpha\beta}{(\mu + \beta)(\xi + \nu + \tau + \mu)^2} \times \frac{\nu}{R_0} = \frac{-\nu}{(\xi + \nu + \tau + \mu)}, \\
S_{\tau}^{R_0} &= \frac{-\alpha\beta}{(\mu + \beta)(\xi + \mu + \tau + \nu)^2} \times \frac{\tau}{R_0} = \frac{-\tau}{(\xi + \nu + \tau + \mu)}.
\end{aligned}$$

By putting the parameters values from the Table 1 we get the following Sensitivity indices given in Table 2:

TABLE 2. Parameters versus Sensitivity indices.

Parameter	Sensitivity index
α	1
β	0.001919
μ	-0.000102
ξ	-0.018294
ν	-0.892418
τ	-0.892418

6. NUMERICAL SIMULATION

This section of our study is specified to verify our findings by Numerical Simulation of the presented model. The parameters used in simulations are provided in Table 1. The initial populations in each compartment of the model is given as $S_0 = 38000$, $E_0 = 2000$, $I_0 = 1000$, $Q_0 = 3000$, $R_0 = 50$, $D_0 = 1000$ [10]. The total population of Indonesia is = 280000000. We simulate six categories of the model by using two step Adams-Bashforth approach via CFFD. By the graphs, we can observe that dynamics of disease transmission can be better depicted by fractional orders rather than integer ones.

Figure ?? is representing the dynamics of Susceptible individuals. It is observed that Susceptible are decreasing as the people in this class are adhere to prevent disease by getting quarantined. we can see that as we are decreasing the fractional order, number of susceptible are increasing. This means that number of susceptible is inversely proportional to fractional order.

Figure 4 is representing the behavior of Exposed individuals which shows that by the passage of time the number of people who are moving into Exposed class decreases. This class is also inversely proportional to fractional order.



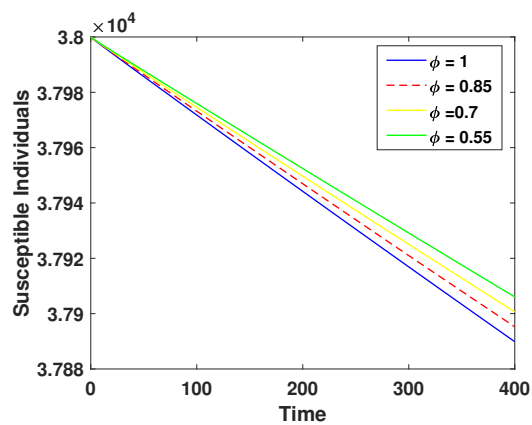


FIGURE 3. Susceptible individuals versus time.

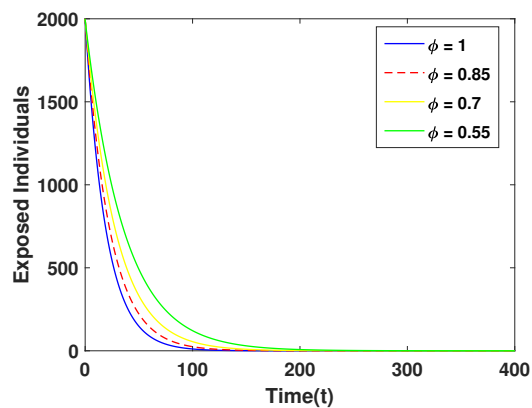


FIGURE 4. Exposed individuals versus time.

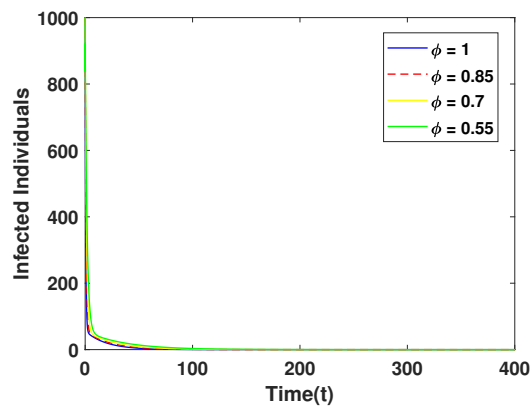


FIGURE 5. Infected Individuals versus time.



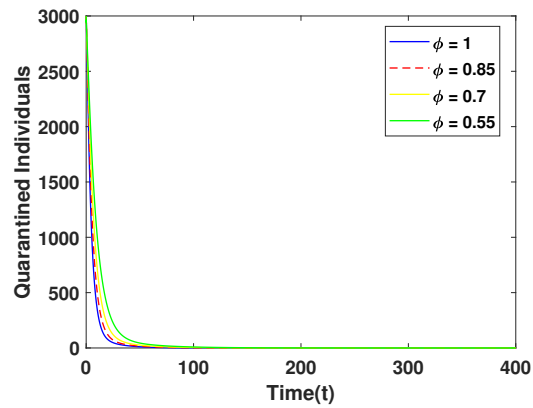


FIGURE 6. Quarantined Individuals versus time.

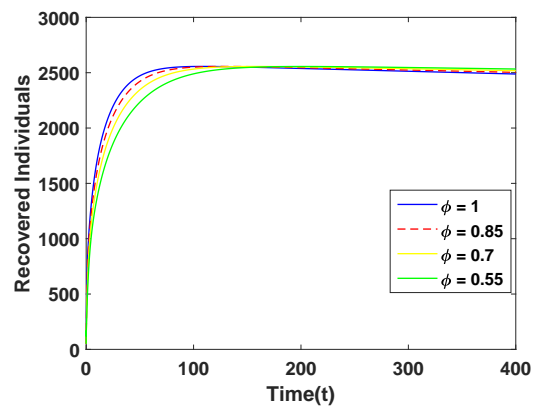


FIGURE 7. Recovered Individuals versus time.

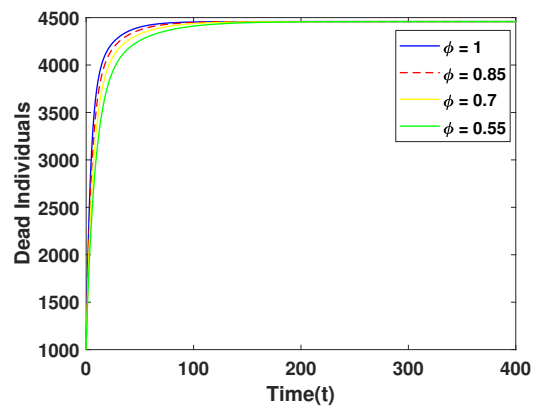


FIGURE 8. Dead Individuals versus time.



Figure 5 is the representation of Infected individuals. It shows that initially this class attains its maximum value then decreases very rapidly. Figure 6 is representing Quarantined class. Similar to Infected, this class also decreases very rapidly. The dynamics of both Infected and Quarantined individuals depicts that they are inversely proportional to fractional order.

Figure 7 is depicting the individuals who get recovered from the disease. It increases for almost 70 days then attains the stability. Figure 8 is showing the dynamics of dead individuals varying with time. After two months Dead population is converging towards stability. Number of deaths are in proportion to fractional order.

7. CONCLUSIONS

The present study examines the SEIQRD epidemic model, employing the CFFD in simulating the spread of the COVID-19 pandemic. At first some fundamental definitions of fractional calculus which are being used in this analysis are given. The Existence and Uniqueness for the model has been proved by applying Banach Contraction theorem. This shows that the model is biologically stable. The widely utilized Next Generation Matrix approach has been employed in determining the R_0 of the suggested model. This depicts the pandemic's potential behavior. The key parameters which highly impact the dynamics of COVID-19 are also investigated by performing sensitivity analysis. We observed that rate of infection is the most sensitive parameter while the incubation rate does not impact remarkably. The remaining parameters are negatively impacting R_0 i.e on increasing the values of these parameters, R_0 decreases.

Numerical Scheme for the Fractional order Caputo Fabrizio derivative has been provided by using two step Adams-Bashforth technique. The numerical simulation has been done using MATLAB software. Numerical results are produced for various values of fractional orders to countenance the importance of fractional order over integer order. The graphs are providing required information about the model's behavior and feasibility. It is observed that Quarantine plays an effective and positive role in eradicating the infection of COVID-19.

CONFLICT OF INTEREST

The authors state that they have no competing interests to declare.

AUTHORS CONTRIBUTION

All the authors made an equal contribution to this paper, and have read and approved the final manuscript.

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ETHICAL APPROVAL

No personal or sensitive information is disclosed or compromised.

DATA AVAILABILITY AND ACCESS

Not applicable.

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