



## Finite-difference method for Hygrothermoelastic boundary value problem

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### Abstract

A two-dimensional coupled hygrothermoelastic medium boundary problem using Finite difference method is discussed in the present work. Explicit and Implicit finite difference schemes for this problem are formed. The solutions of these schemes are carried out using numerical methods of finite difference. These solutions are compared of and analyzed and exciting similarities were found as result.

**Keywords.** Tridiagonal matrix algorithm (TDMA), Hygrothermoelasticity, Thermal conductivity, Explicit Method, Implicit Method, Thermal diffusivity, Moisture diffusivity.

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### 1. INTRODUCTION

Thermoelasticity concerns the dynamic system whose interactions with the environment involve not only mechanical and external work, but also the exchange of heat. Changes in temperatures cause thermal effects on materials. Among the thermal effects are thermal stress, deformation, and deformation. Thermal deformation means change in the "thermal energy" (and temperature) of a material. This change increases the vibration of its atoms/molecules and which results in a stretching of the molecular bonds due to which material gets expanded. Similarly, a decrease in the thermal energy (and temperature) may lead to the shrinking of material. Further, we can also say that the thermal energy will vary with the expansion and compression of the material. Finally, we can declare the theory of thermoelasticity as a process of predicting the thermomechanical behavior of material.

Duhamel [5], in 1838 founded the theory of thermo elasticity. He derived the equations for the strain in an elastic body with temperature gradients. Neumann [14] reconfirmed these laws in 1841 with some additions. Thomson [24], was first to find stresses and strains in an elastic body under dynamic conditions in the light of laws of thermodynamics in 1857. thermo elasticity was classified as three types, i.e., uncoupled, coupled and generalized thermo elasticity. The classical uncoupled and coupled thermo elastic theories of Biot [3] and Nowacki [15] have an inherent paradox which were removed by later theories.

The process of migration of atoms and molecules from highly concentrated region to region with lower concentration till equilibrium is attained, is known as diffusion. In equilibrium position, atoms and molecules occupy a definite position. This migration in result produce disturbance caused by glitch of mechanical conditions and internal stresses.

In addition, an uneven moisture distribution will develop the concentration gradient that will cause moisture transfer. This means that the temperature and moisture proportion of the medium will change according to time and space. Here, the heat conduction theory and moisture diffusion can be regarded as proportionate. The distribution of temperature and humidity can vary significantly due to the mechanical stresses applied. This enables to explore the connection between mechanical distortion and diffusion because of temperature and humidity. Many technical

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problems of practical interest demonstrate this relation between moisture, heat, and deformation. Any problem involving the study of solids under the influence of humidity and heat belongs to hygrothermoelasticity.

Szekeres [23] spoke about the problem of coupling generalized heat transfer and humidity. Gasch et al. [6] compared damage caused by temperature and humidity changes to mechanical loads and demonstrated that temperature and humidity cause more damage. The basic correlation was found in moisture and heat for developing governing relation for coupled hygrothermoelasticity by Szekeres and EngelBrecht [22]. Gigliotti et al. [12] investigated the cyclic and transient hygrothermoelastic stress in laminated composite plates. Gawain et al. [7] developed a mathematical setup for investigating the behavior of concrete under hygrothermal setup. Aboudi and Williams [1] used a mechanical micromacro method to treat the response of hygrothermolar composites. Rao and Sinha [17] analysed the effect of humidity and temperature on free vibration in 3D multidirectional composites. Ailawalia et al. [2] investigated surface waves in hygrothermoelastic half-space with hydrostatic initial stress. Qalandarov and Khaldjigitov [16] solved a Mathematical and numerical modeling of the coupled dynamic thermoelastic problems for isotropic bodies. Khaldjigitov et al. [9] developed Finite-Difference equations for 2D elasticity problems on a non-uniform grid. Singh and Lata [21] explored the study of two temperature parameters effect for the axisymmetric deformation in a two-dimensional nonlocal homogeneous isotropic thick circular plate without energy dissipation. Singh and Lata [20] dealt with the two-dimensional deformation in a homogeneous isotropic nonlocal magnetothermoelastic solid with two temperatures under the effects of inclined load at different inclinations in this work. Lata [10] studied the effect of frequency in a two-dimensional orthotropic thermoelastic rotating solid with fractional order heat transfer in generalized thermoelasticity with two-temperature due to inclined load. The work of Sefidab Amini and Reihani [18] deals with a nonlinear parabolic moving boundary problem raised from the mathematical modeling of the behavior of the breast avascular cancer tumors at their first stage. Pishnamaz Mohammadi and Ivaz [11] developed a numerical technique for the solution of the one-phase Stefan problem for the non-classical heat equation with a convective condition is discussed in this work.

This article deals with, a 2-D problem of hygrothermoelasticity concerning the deformation theory of isotropic bodies. Finite-difference method is used to investigate the situation. Discrete equations are formed for the explicit and implicit schemes. Explicit schemes are solved by obtaining recurrence relations in terms of displacement, temperature, and moisture. Implicit schemes are efficiently solved using the TDMA along the valid directions. The problem is numerically solved using MatLab software for a specific material and the results are depicted by graphs. This article is an effort to through light on effect of time and space on hygrothermoelastic problems.

## 2. BASIC EQUATIONS

We use the constitutive relations, field equations, heat conduction and moisture diffusion for homogeneous, isotropic hygrothermoelastic solid in the absence of incremental body forces and heat sources given by Hosseini et al. [8] and Montanaro [13] which are

$$\sigma_{ji,j} = \rho u_i, \quad (2.1)$$

$$D_T T_{,ii} + D_T^m m_{,ii} - \dot{T} - \frac{\beta_{ij}^T T_0}{\rho c} \dot{u}_{j,j} = 0, \quad (2.2)$$

$$D_m m_{,ii} + D_m^T T_{,ii} - \dot{m} - \frac{\beta_{ij}^m T_0}{k_m} \dot{u}_{j,j} = 0, \quad (2.3)$$

$$\text{where } C_{ijkl} = \frac{2G\nu}{1-2\nu} \delta_{ij} \delta_{kl} + G \delta_{ik} \delta_{jl} + G \delta_{il} \delta_{jk}, \quad \epsilon_{ij} = \frac{u_{j,i} - u_{i,j}}{2},$$

$$\sigma_{ij,j} = C_{ijkl} \epsilon_{ij} - \beta_{ij}^m m - \beta_{ij}^T T, \quad (2.4)$$

$$\text{where } \beta_{ij}^T = \beta_T \delta_{ij}, \quad \beta_T = (3\lambda + 2\mu) \alpha_t, \quad \text{and } \beta_{ij}^m = \beta_m \delta_{ij}, \quad \beta_m = (3\lambda + 2\mu) \alpha_m.$$



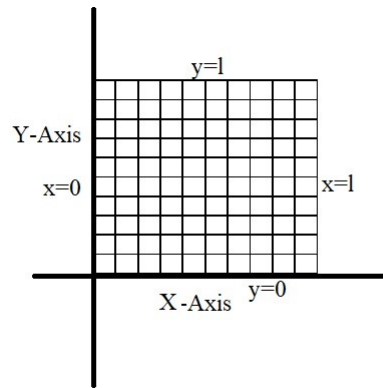


FIGURE 1. Hygrothermoelastic rectangular region.

### 3. HYGRO-THERMOELASTICITY PROBLEM FOR RECTANGULAR REGION

We consider a rectangular region bounded by  $x = 0, y = 0, x = l$  and  $y = l$ . The wave is propagating in the  $xy$  plane. The displacement vector in two-dimensional space hygrothermoelastic material is given as  $\vec{u} = (u, v, 0)$  where  $u = u(x, y, t), v = v(x, y, t)$ . We have considered that the equations of motion and coupled equations of heat conduction and moisture diffusion (2.1)-(2.3) and the constitutive relations (2.4) in 2-D, takes the form as follows:

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} + (\lambda + \mu) \frac{\partial^2 v}{\partial x \partial y} + \mu \frac{\partial^2 u}{\partial y^2} - \beta_m \frac{\partial m}{\partial x} - \beta_T \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{3.1}$$

$$\mu \frac{\partial^2 v}{\partial x^2} + (\lambda + \mu) \frac{\partial^2 u}{\partial x \partial y} + (\lambda + 2\mu) \frac{\partial^2 v}{\partial y^2} - \beta_m \frac{\partial m}{\partial y} - \beta_T \frac{\partial T}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{3.2}$$

$$D_T \nabla^2 T + D_m^T \nabla^2 m - \frac{\partial T}{\partial t} - \frac{\beta_T T_0}{\rho c} \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0, \tag{3.3}$$

$$D_m \nabla^2 m + D_m^T \nabla^2 T - \frac{\partial m}{\partial t} - \frac{\beta_m m_0 D_m}{k_m} \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0. \tag{3.4}$$

Appropriate Initial Boundary Conditions  $u(x, y, t)|_{t=t_0} = \phi_1, v(x, y, t)|_{t=t_0} = \phi_2, m(x, y, t)|_{t=t_0} = m_0, T(x, y, t)|_{t=t_0} = T_0, \frac{\partial u}{\partial t}|_{t=t_0} = \psi_1, \frac{\partial v}{\partial t}|_{t=t_0} = \psi_2$   
and Boundary Condition

$$\begin{aligned} u(x, y, t)|_{x=x_0} &= u_0, u(x, y, t)|_{x=l_1} = \bar{u}_0, & u(x, y, t)|_{y=y_0} &= u_0, u(x, y, t)|_{y=l_2} = \bar{u}_0, \\ v(x, y, t)|_{x=x_0} &= v_0, v(x, y, t)|_{x=l_1} = \bar{v}_0, & v(x, y, t)|_{y=y_0} &= v_0, v(x, y, t)|_{y=l_2} = \bar{v}_0, \\ T(x, y, t)|_{x=x_0} &= T_0, T(x, y, t)|_{x=l_1} = \bar{T}_0, & T(x, y, t)|_{y=y_0} &= T_0, T(x, y, t)|_{y=l_2} = \bar{T}_0, \\ m(x, y, t)|_{x=x_0} &= m_0, m(x, y, t)|_{x=l_1} = \bar{m}_0, & m(x, y, t)|_{y=y_0} &= m_0, m(x, y, t)|_{y=l_2} = \bar{m}_0. \end{aligned}$$



## 4. EQUATION IN FINITE DIFFERENCE ENVIRONMENT

Rewriting (3.1)-(3.4) using central finite difference approximation using region  $t \geq 0, 0 \leq x \leq l_1, 0 \leq y \leq l_2$  and boundary points  $x = ih_1, y = jh_2, t = k\tau$  where  $i = 0$  to  $n_1, j = 0$  to  $n_2$  and  $k = 0, 1, 2, 3$ , Then putting the finite difference approximation respective differential coefficient.

$$\begin{aligned}
& (\lambda + 2\mu) \frac{(u_{i+1,j}^k - 2u_{i,j}^k + u_{i-1,j}^k)}{h_1^2} + (\lambda + \mu) \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1h_2} + \\
& \mu \frac{(u_{i,j+1}^k - 2u_{i,j}^k + u_{i,j-1}^k)}{h_2^2} - \beta_m \frac{(m_{i+1,j}^k - m_{i-1,j}^k)}{2h_1} - \beta_T \frac{(T_{i+1,j}^k - T_{i-1,j}^k)}{2h_1} = \rho \frac{(u_{i,j}^{k+1} - 2u_{i,j}^{k+1} + u_{i,j}^{k+1})}{\tau^2}, \\
& u_{i,j}^{k+1} = \frac{\tau^2}{\rho} \left( (\lambda + 2\mu) \frac{(u_{i+1,j}^k - 2u_{i,j}^k + u_{i-1,j}^k)}{h_1^2} + (\lambda + \mu) \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1h_2} \right) + \\
& \frac{\tau^2}{\rho} \left( \mu \frac{(u_{i,j+1}^k - 2u_{i,j}^k + u_{i,j-1}^k)}{h_2^2} - \beta_m \frac{(m_{i+1,j}^k - m_{i-1,j}^k)}{2h_1} - \beta_T \frac{(T_{i+1,j}^k - T_{i-1,j}^k)}{2h_1} \right) + 2u_{i,j}^k - u_{i,j}^{k-1}, \quad (4.1)
\end{aligned}$$

$$\begin{aligned}
& \mu \frac{(v_{i+1,j}^k - 2v_{i,j}^k + v_{i-1,j}^k)}{h_1^2} + (\lambda + \mu) \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1h_2} + \\
& (\lambda + 2\mu) \frac{(v_{i,j+1}^k - 2v_{i,j}^k + v_{i,j-1}^k)}{h_2^2} - \beta_m \frac{(m_{i,j+1}^k - m_{i,j-1}^k)}{2h_2} - \beta_T \frac{(T_{i,j+1}^k - T_{i,j-1}^k)}{2h_2} = \rho \frac{(v_{i,j}^{k+1} - 2v_{i,j}^{k+1} + v_{i,j}^{k+1})}{\tau^2} \\
& v_{i,j}^{k+1} = \frac{\tau^2}{\rho} \left( \mu \frac{(v_{i+1,j}^k - 2v_{i,j}^k + v_{i-1,j}^k)}{h_1^2} + (\lambda + \mu) \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1h_2} \right) + \\
& \frac{\tau^2}{\rho} \left( (\lambda + 2\mu) \frac{(v_{i,j+1}^k - 2v_{i,j}^k + v_{i,j-1}^k)}{h_2^2} - \beta_m \frac{(m_{i,j+1}^k - m_{i,j-1}^k)}{2h_2} - \beta_T \frac{(T_{i,j+1}^k - T_{i,j-1}^k)}{2h_2} \right) + 2v_{i,j}^k - v_{i,j}^{k-1} \quad (4.2)
\end{aligned}$$

$$\begin{aligned}
& D_T \frac{(T_{i+1,j}^k - 2T_{i,j}^k + T_{i-1,j}^k)}{h_1^2} + D_T \frac{(T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k)}{h_2^2} + \\
& D_T^m \frac{(m_{i+1,j}^k - 2m_{i,j}^k + m_{i-1,j}^k)}{h_1^2} + D_T^m \frac{(m_{i,j+1}^k - 2m_{i,j}^k + m_{i,j-1}^k)}{h_2^2} - \frac{(T_{i,j}^{k+1} - T_{i,j}^k)}{\tau} - \\
& \frac{\beta_T T_0}{\rho c} \left( \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1h_2} - \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1h_2} \right) = 0, \\
& T_{i,j}^{k+1} = \tau \left( D_T \frac{(T_{i+1,j}^k - 2T_{i,j}^k + T_{i-1,j}^k)}{h_1^2} + D_T \frac{(T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k)}{h_2^2} \right) + \\
& \tau \left( D_T^m \frac{(m_{i+1,j}^k - 2m_{i,j}^k + m_{i-1,j}^k)}{h_1^2} + D_T^m \frac{(m_{i,j+1}^k - 2m_{i,j}^k + m_{i,j-1}^k)}{h_2^2} \right) - \\
& \tau \frac{\beta_T T_0}{\rho c} \left( \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1h_2} - \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1h_2} \right) + T_{i,j}^k, \quad (4.3)
\end{aligned}$$



$$\begin{aligned}
 & D_m \frac{(m_{i+1,j}^k - 2m_{i,j}^k + m_{i-1,j}^k)}{h_1^2} + D_m \frac{(m_{i,j+1}^k - 2m_{i,j}^k + m_{i,j-1}^k)}{h_2^2} + \\
 & D_m^T \frac{(T_{i+1,j}^k - 2T_{i,j}^k + T_{i-1,j}^k)}{h_1^2} + D_m^T \frac{(T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k)}{h_2^2} - \frac{(m_{i,j}^{k+1} - m_{i,j}^k)}{\tau} - \\
 & \frac{\beta_m D_m m_0}{k_m} \left( \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1 h_2} - \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1 h_2} \right) = 0, \\
 & m_{i,j}^{k+1} = \tau \left( D_m \frac{(m_{i+1,j}^k - 2m_{i,j}^k + m_{i-1,j}^k)}{h_1^2} + D_m \frac{(m_{i,j+1}^k - 2m_{i,j}^k + m_{i,j-1}^k)}{h_2^2} \right) + \\
 & \tau \left( D_m^T \frac{(T_{i+1,j}^k - 2T_{i,j}^k + T_{i-1,j}^k)}{h_1^2} + D_m^T \frac{(T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k)}{h_1^2} \right) - \\
 & \tau \frac{\beta_m D_m m_0}{k_m} \left( \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1 h_2} + \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1 h_2} \right) + m_{i,j}^k,
 \end{aligned} \tag{4.4}$$

Reviewing Equations (4.1)-(4.4) we can find values  $u(x, y, t), v(x, y, t), T(x, y, t)$  and  $m(x, y, t)$  at the layer  $t_{k+1}$ . We can get the values of  $u(x, y, t)$  and  $v(x, y, t)$  for two primary layers  $k = 0$  and  $k = 1$  from the initial conditions for  $k = 0$   $u(0, 0) = \phi_1, v(0, 0) = \phi_2, m(0, 0) = m_0, T(0, 0) = T_0$ ,

$$\begin{aligned}
 \frac{\partial u}{\partial t} \Big|_{t=t_0} = \psi_1(i, j) & \Rightarrow \frac{u_{i,j}^1 - u_{i,j}^{-1}}{2\tau} = \psi_1(x_i, y_j) \Rightarrow u_{i,j}^1 = u_{i,j}^{-1} + 2\tau\psi_1(x_i, y_j) \Rightarrow u_{i,j}^{-1} = u_{i,j}^1 - 2\tau\psi_1(x_i, y_j), \\
 \frac{\partial v}{\partial t} \Big|_{t=t_0} = \psi_2(x_i, y_j) & \Rightarrow \frac{v_{i,j}^1 - v_{i,j}^{-1}}{2\tau} = \psi_2(x_i, y_j) \Rightarrow v_{i,j}^1 = v_{i,j}^{-1} + 2\tau\psi_2(x_i, y_j) \Rightarrow v_{i,j}^{-1} = v_{i,j}^1 - 2\tau\psi_2(x_i, y_j).
 \end{aligned}$$

Replacing these values in Equation (4.1)-(4.4) we can get relation to calculate the values of  $u(x, y, t), v(x, y, t), T(x, y, t)$  and  $m(x, y, t)$ , for  $k = 1$ ,

$$\begin{aligned}
 u_{i,j}^1 &= \frac{\tau^2}{\rho} \left( (\lambda + 2\mu) \frac{(u_{i+1,j}^0 - 2u_{i,j}^0 + u_{i-1,j}^0)}{h_1^2} + (\lambda + \mu) \frac{(v_{i+1,j+1}^0 - v_{i-1,j+1}^0 - v_{i+1,j-1}^0 + v_{i-1,j-1}^0)}{4h_1 h_2} \right) + \\
 & \frac{\tau^2}{\rho} \left( \mu \frac{(u_{i,j+1}^0 - 2u_{i,j}^0 + u_{i,j-1}^0)}{h_2^2} - \beta_m \frac{(m_{i+1,j}^0 - m_{i-1,j}^0)}{2h_1} - \beta_T \frac{(T_{i+1,j}^0 - T_{i-1,j}^0)}{2h_1} \right) + 2u_{i,j}^0 - u_{i,j}^{-1}, \\
 u_{i,j}^1 &= \frac{\tau^2}{\rho} \left( (\lambda + 2\mu) \frac{(u_{i+1,j}^0 - 2u_{i,j}^0 + u_{i-1,j}^0)}{h_1^2} + (\lambda + \mu) \frac{(v_{i+1,j+1}^0 - v_{i-1,j+1}^0 - v_{i+1,j-1}^0 + v_{i-1,j-1}^0)}{4h_1 h_2} \right) + \\
 & \frac{\tau^2}{\rho} \left( \mu \frac{(u_{i,j+1}^0 - 2u_{i,j}^0 + u_{i,j-1}^0)}{h_2^2} - \beta_m \frac{(m_{i+1,j}^0 - m_{i-1,j}^0)}{2h_1} - \beta_T \frac{(T_{i+1,j}^0 - T_{i-1,j}^0)}{2h_1} \right) + 2u_{i,j}^0 - u_{i,j}^1 + 2\tau\psi_1(x_i, y_j), \\
 u_{i,j}^1 &= \frac{\tau^2}{\rho} \left( (\lambda + 2\mu) \frac{(u_{i+1,j}^0 - 2u_{i,j}^0 + u_{i-1,j}^0)}{h_1^2} + (\lambda + \mu) \frac{(v_{i+1,j+1}^0 - v_{i-1,j+1}^0 - v_{i+1,j-1}^0 + v_{i-1,j-1}^0)}{4h_1 h_2} \right) +
 \end{aligned}$$



$$\frac{\tau^2}{\rho} \left( \mu \frac{(u_{i,j+1}^0 - 2u_{i,j}^0 + u_{i,j-1}^0)}{h_2^2} - \beta_m \frac{(m_{i+1,j}^0 - m_{i-1,j}^0)}{2h_1} - \beta_T \frac{(T_{i+1,j}^0 - T_{i-1,j}^0)}{2h_1} \right) + u_{i,j}^0 + \tau\psi_1(x_i, y_j), \quad (4.5)$$

$$v_{i,j}^1 = \frac{\tau^2}{\rho} \left( \mu \frac{(v_{i+1,j}^0 - 2v_{i,j}^0 + v_{i-1,j}^0)}{h_1^2} + (\lambda + \mu) \frac{(u_{i+1,j+1}^0 - u_{i-1,j+1}^0 - u_{i+1,j-1}^0 + u_{i-1,j-1}^0)}{4h_1h_2} \right) +$$

$$\frac{\tau^2}{\rho} \left( (\lambda + 2\mu) \frac{(v_{i,j+1}^0 - 2v_{i,j}^0 + v_{i,j-1}^0)}{h_2^2} - \beta_m \frac{(m_{i,j+1}^0 - m_{i,j-1}^0)}{2h_2} - \beta_T \frac{(T_{i,j+1}^0 - T_{i,j-1}^0)}{2h_2} \right) + 2v_{i,j}^0 - v_{i,j}^{-1},$$

$$v_{i,j}^1 = \frac{\tau^2}{\rho} \left( \mu \frac{(v_{i+1,j}^0 - 2v_{i,j}^0 + v_{i-1,j}^0)}{h_1^2} + (\lambda + \mu) \frac{(u_{i+1,j+1}^0 - u_{i-1,j+1}^0 - u_{i+1,j-1}^0 + u_{i-1,j-1}^0)}{4h_1h_2} \right) +$$

$$\frac{\tau^2}{\rho} \left( (\lambda + 2\mu) \frac{(v_{i,j+1}^0 - 2v_{i,j}^- + v_{i,j-1}^0)}{h_2^2} - \beta_m \frac{(m_{i,j+1}^0 - m_{i,j-1}^0)}{2h_2} - \beta_T \frac{(T_{i,j+1}^0 - T_{i,j-1}^0)}{2h_2} \right) + v_{i,j}^0 + \tau\psi_2(x_i, y_j),$$

$$v_{i,j}^1 = \frac{\tau^2}{\rho} \left( \mu \frac{(v_{i+1,j}^0 - 2v_{i,j}^0 + v_{i-1,j}^0)}{h_1^2} + (\lambda + \mu) \frac{(u_{i+1,j+1}^0 - u_{i-1,j+1}^0 - u_{i+1,j-1}^0 + u_{i-1,j-1}^0)}{4h_1h_2} \right) +$$

$$\frac{\tau^2}{\rho} \left( (\lambda + 2\mu) \frac{(v_{i,j+1}^0 - 2v_{i,j}^0 + v_{i,j-1}^0)}{h_2^2} - \beta_m \frac{(m_{i,j+1}^0 - m_{i,j-1}^0)}{2h_2} - \beta_T \frac{(T_{i,j+1}^0 - T_{i,j-1}^0)}{2h_2} \right) + v_{i,j}^0 + \tau\psi_2(x_i, y_j), \quad (4.6)$$

$$T_{i,j}^1 = \tau \left( D_T \frac{(T_{i+1,j}^0 - 2T_{i,j}^0 + T_{i-1,j}^0)}{h_1^2} + D_T \frac{(T_{i,j+1}^0 - 2T_{i,j}^0 + T_{i,j-1}^0)}{h_2^2} \right) +$$

$$\tau \left( D_T^m \frac{(m_{i+1,j}^0 - 2m_{i,j}^0 + m_{i-1,j}^0)}{h_1^2} + D_T^m \frac{(m_{i,j+1}^0 - 2m_{i,j}^0 + m_{i,j-1}^0)}{h_2^2} \right) -$$

$$\tau \frac{\beta_T T_0}{\rho c} \left( \frac{(u_{i+1,j+1}^0 - u_{i-1,j+1}^0 - u_{i+1,j-1}^0 + u_{i-1,j-1}^0)}{4h_1h_2} - \frac{(v_{i+1,j+1}^0 - v_{i-1,j+1}^0 - v_{i+1,j-1}^0 + v_{i-1,j-1}^0)}{4h_1h_2} \right) + T_{i,j}^0, \quad (4.7)$$

$$m_{i,j}^1 = \tau \left( D_m \frac{(m_{i+1,j}^0 - 2m_{i,j}^0 + m_{i-1,j}^0)}{h_1^2} + \frac{(m_{i,j+1}^0 - 2m_{i,j}^0 + m_{i,j-1}^0)}{h_2^2} \right) +$$

$$\tau \left( D_m^T \frac{(T_{i+1,j}^0 - 2T_{i,j}^0 + T_{i-1,j}^0)}{h_1^2} + D_m^T \frac{(T_{i,j+1}^0 - 2T_{i,j}^0 + T_{i,j-1}^0)}{h_1^2} \right) -$$

$$\tau \frac{\beta_m D_m m_0}{k_m} \left( \frac{(u_{i+1,j+1}^0 - u_{i-1,j+1}^0 - u_{i+1,j-1}^0 + u_{i-1,j-1}^0)}{4h_1h_2} + \frac{(v_{i+1,j+1}^0 - v_{i-1,j+1}^0 - v_{i+1,j-1}^0 + v_{i-1,j-1}^0)}{4h_1h_2} \right) + m_{i,j}^0, \quad (4.8)$$



5. IMPLICIT SOLUTION

In the formulae given above, we have used the explicit method. Now to deduce an implicit method for solving coupled hygrothermoelasticity problems numerically, we need to rewrite Equations (4.1)-(4.4) in the following form. Solving (4.1)

$$\begin{aligned} & \frac{(\lambda + 2\mu)}{h_1^2} u_{i+1,j}^{k+1} - \left( \frac{(\lambda + 2\mu)}{h_1^2} + \frac{\rho}{\tau^2} \right) u_{i,j}^{k+1} + \frac{(\lambda + 2\mu)}{h_1^2} u_{i-1,j}^{k+1} = \frac{\rho}{\tau^2} \left( u_{i,j}^{k-1} - 2u_{i,j}^k - \mu \frac{(u_{i,j+1}^k - 2u_{i,j}^k + u_{i,j-1}^k)}{h_2^2} \right) - \\ & (\lambda + \mu) \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1h_2} + \beta_m \frac{(m_{i+1,j}^k - m_{i-1,j}^k)}{2h_1} + \beta_T \frac{(T_{i+1,j}^k - T_{i-1,j}^k)}{2h_1}, \\ & a_i u_{i+1,j}^{k+1} + b_i u_{i,j}^{k+1} + c_i u_{i-1,j}^{k+1} = f_{i,j}, \end{aligned} \tag{5.1}$$

$$\begin{aligned} a_i &= \frac{(\lambda + 2\mu)}{h_1^2}, b_i = - \left( \frac{2(\lambda + 2\mu)}{h_1^2} + \frac{\rho}{\tau^2} \right), c_i = \frac{(\lambda + 2\mu)}{h_1^2}, \\ f_{i,j} &= \frac{\rho}{\tau^2} (u_{i,j}^{k-1} - 2u_{i,j}^k) - \mu \frac{(u_{i,j+1}^k - 2u_{i,j}^k + u_{i,j-1}^k)}{h_2^2} \\ & - (\lambda + \mu) \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1h_2} + \beta_m \frac{(m_{i+1,j}^k - m_{i-1,j}^k)}{2h_1} + \beta_T \frac{(T_{i+1,j}^k - T_{i-1,j}^k)}{2h_1}, \end{aligned}$$

solving (4.2)

$$\begin{aligned} & \mu \frac{(v_{i+1,j}^{k+1} - 2v_{i,j}^{k+1} + v_{i-1,j}^{k+1})}{h_1^2} + (\lambda + \mu) \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1h_2} + (\lambda + 2\mu) \\ & \frac{(v_{i,j+1}^k - 2v_{i,j}^k + v_{i,j-1}^k)}{h_2^2} - \beta_m \frac{(m_{i,j+1}^k - m_{i,j-1}^k)}{2h_2} - \beta_T \frac{(T_{i,j+1}^k - T_{i,j-1}^k)}{2h_2} = \rho \frac{(v_{i,j}^{k+1} - 2v_{i,j}^{k+1} + v_{i,j}^{k+1})}{\tau^2}, \\ & a_i v_{i+1,j}^{k+1} + b_i v_{i,j}^{k+1} + c_i v_{i-1,j}^{k+1} = f_{i,j}, \end{aligned} \tag{5.2}$$

$$\begin{aligned} a_i &= \frac{\mu}{h_1^2}, b_i = - \left( \frac{2\mu}{h_1^2} + \frac{\rho}{\tau^2} \right), c_i = \frac{\mu}{h_1^2}, f_{i,j} = \frac{\rho}{\tau^2} (v_{i,j}^{k-1} - 2v_{i,j}^k) - \mu \frac{(v_{i,j+1}^k - 2v_{i,j}^k + v_{i,j-1}^k)}{h_2^2} \\ & - (\lambda + \mu) \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1h_2} + \beta_m \frac{(m_{i,j+1}^k - m_{i,j-1}^k)}{2h_2} + \beta_T \frac{(T_{i,j+1}^k - T_{i,j-1}^k)}{2h_2}, \end{aligned}$$

solving (4.3)

$$\begin{aligned} & \frac{D_T}{h_1^2} T_{i+1,j}^{k+1} - \left( 2 \frac{D_T}{h_1^2} + \frac{1}{\tau} \right) T_{i,j}^{k+1} + \frac{D_T}{h_1^2} T_{i-1,j}^{k+1} = \\ & \frac{\beta_T T_0}{\rho c} \left( \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1h_2} - \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1h_2} \right) \\ & D_T \frac{(T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k)}{h_2^2} - D_T^m \frac{(m_{i+1,j}^k - 2m_{i,j}^k + m_{i-1,j}^k)}{h_1^2} - D_T^m \frac{(m_{i,j+1}^k - 2m_{i,j}^k + m_{i,j-1}^k)}{h_2^2} + \frac{T_{i,j}^k}{\tau}, \\ & a_i T_{i+1,j}^{k+1} + b_i T_{i,j}^{k+1} + c_i T_{i-1,j}^{k+1} = f_{i,j}, \end{aligned} \tag{5.3}$$



$$\begin{aligned}
a_i &= \frac{D_T}{h_1^2}, b_i = -\left(2\frac{D_T}{h_1^2} + \frac{1}{\tau}\right), c_i = \frac{D_T}{h_1^2}, \\
f_{i,j} &= \frac{\beta_T T_0}{\rho c} \left( \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1 h_2} - \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1 h_2} \right) \\
&\quad D_T \frac{(T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k)}{h_2^2} - D_T^m \frac{(m_{i+1,j}^k - 2m_{i,j}^k + m_{i-1,j}^k)}{h_1^2} - \\
&\quad D_T^m \frac{(m_{i,j+1}^k - 2m_{i,j}^k + m_{i,j-1}^k)}{h_2^2} + \frac{T_{i,j}^k}{\tau},
\end{aligned}$$

solving (4.4)

$$\begin{aligned}
&\frac{D_m}{h_1^2} m_{i+1,j}^{k+1} - \left(2\frac{D_m}{h_1^2}\right) m_{i,j}^{k+1} + \frac{D_m}{h_1^2} m_{i-1,j}^{k+1} = \\
&\frac{\beta_m D_m m_0}{k_m} \left( \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1 h_2} - \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1 h_2} \right) - \\
&D_m \frac{(m_{i,j+1}^k - 2m_{i,j}^k + m_{i,j-1}^k)}{h_2^2} - D_m^T \frac{(T_{i+1,j}^k - 2T_{i,j}^k + T_{i-1,j}^k)}{h_1^2} - D_m^T \frac{(T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k)}{h_2^2} - \frac{m_{i,j}^k}{\tau}, \\
&a_i m_{i+1,j}^{k+1} + b_i m_{i,j}^{k+1} + c_i m_{i-1,j}^{k+1} = f_{i,j}, \tag{5.4}
\end{aligned}$$

$$\begin{aligned}
a_i &= \frac{D_m}{h_1^2}, b_i = -\left(2\frac{D_m}{h_1^2} + \frac{1}{\tau}\right), c_i = \frac{D_m}{h_1^2}, \\
f_{i,j} &= \frac{\beta_m D_m m_0}{k_m} \left( \frac{(u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k)}{4h_1 h_2} - \frac{(v_{i+1,j+1}^k - v_{i-1,j+1}^k - v_{i+1,j-1}^k + v_{i-1,j-1}^k)}{4h_1 h_2} \right) - \\
&D_m \frac{(m_{i,j+1}^k - 2m_{i,j}^k + m_{i,j-1}^k)}{h_2^2} - D_m^T \frac{(T_{i+1,j}^k - 2T_{i,j}^k + T_{i-1,j}^k)}{h_1^2} - D_m^T \frac{(T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k)}{h_2^2} - \frac{m_{i,j}^k}{\tau}.
\end{aligned}$$

For the values of  $u(x, y, t)$ ,  $v(x, y, t)$ ,  $m(x, y, t)$  and  $T(x, y, t)$  at the first two time layers, we can use initial conditions and formulae (4.5)-(4.8) for calculations. Values on the internal node of subsequent layers can be found by using (5.1)-(5.4) with the TDMA making use of given initial and boundary conditions.

## 6. PARTICULAR CASE

Neglecting moisture effect, the problem reduces to the numerical solution of a two-dimensional thermoelastic problem by finite difference method. Silveira et al. [19] discussed one-dimensional thermoelastic problem by finite difference method

## 7. NUMERICAL VERIFICATION

Numerical problem satisfying (2.1)-(2.4) is simulated using the recurrence  $T(x, y, t) = T_0 \sin\left(\frac{\pi x}{l_1}\right) \sin\left(\frac{\pi y}{l_2}\right)$  and boundary conditions are  $u(x, y, t) = 0$ ,  $v(x, y, t) = 0$ ,  $m(x, y, t) = 0$  and  $T(x, y, t) = 0$ . formulas and TDMA as test with values of constants W. J. Chang [4], initial and boundary conditions given as per Table 1.





TABLE 1.

SNo	Type	Symbol	Unit
1.	Lame's constant	$\lambda$	$46.9 * 10^9 N/m^2$
2.	Lame's constant	$\mu$	$24.17 * 10^9 N/m^2$
3.	Density	$\rho$	$370 Kg/m_3$
4.	Poisson's ratio	$\nu$	.33
5.	Reference Moisture	$m_0$	10%
6.	Coefficient of Moisture expansion	$\alpha^m$	$2.68 * 10^{-3} cm/cm(H_2O)$
7.	Coefficient of linear thermal expansion	$\alpha^T$	$31.3 * cm/cm(H_2O)$
8.	Temperature	$T_0$	283 K
9.	Heat Capacity	$c$	$2500j/Kg(^0K)$
10.	Moisture diffusivity	$k_m$	$2.2 * 10^{-8} Kg/msM$
11.	Diffusion coefficient of moisture	$D_m$	$2.16 * 10^{-6}/s(^0K)$
12.	Coupled diffusivity	$D_T^m$	$2.1 * 10^{-7} m^2(^0K)/s(H_2O)$
13.	Temperature diffusivity	$D_T$	$k/\rho c$

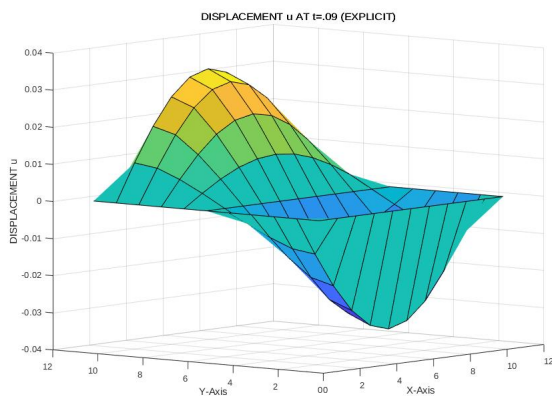


FIGURE 2. Horizontal displacement u using explicit method.

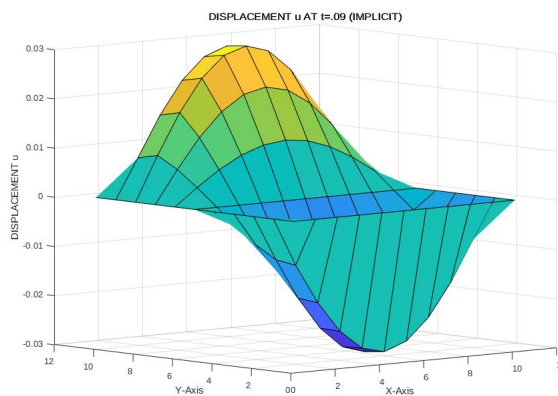


FIGURE 3. Horizontal displacement u using implicit method.

8. TABLE

$l_1 = 1, l_2 = 1, \text{ at } t=0 \ u(x, y, t) = \phi_1 = 0, v(x, y, 0) = \phi_2 = 0, m(x, y, t) = 0, \text{ and where } \Gamma \text{ boundary of the body. Figures 1-8 show the displacement components } u(x, y, t), v(x, y, t), \text{ temperature } T(x, y, t) \text{ and moisture } m(x, y, t) \text{ distributions in 3D space. Every unknown value was calculated using the recurrence formulas and TDMA with help of MatLab software.}$



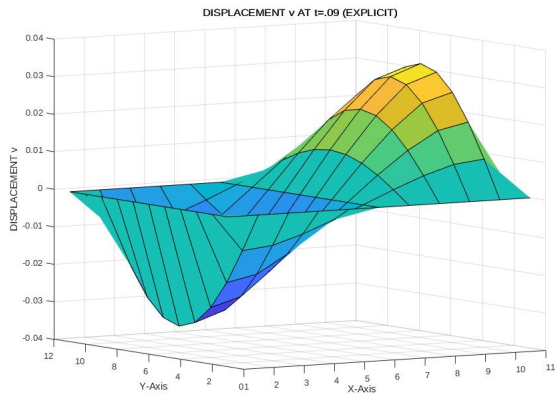


FIGURE 4. Normal displacement  $v$  using explicit method.

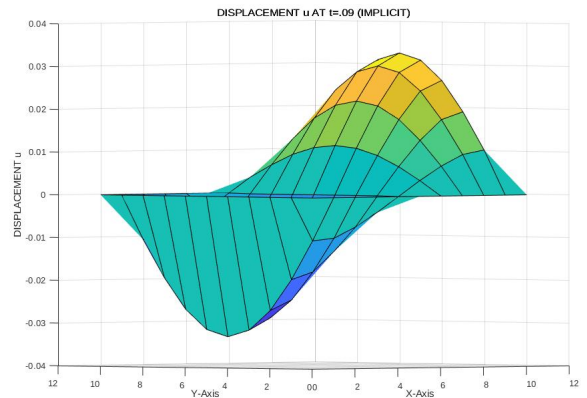


FIGURE 5. Normal displacement  $v$  using implicit method.

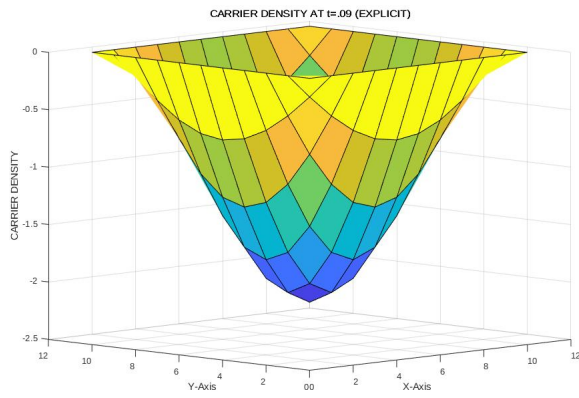


FIGURE 6. Carrier density  $N$  using explicit method.

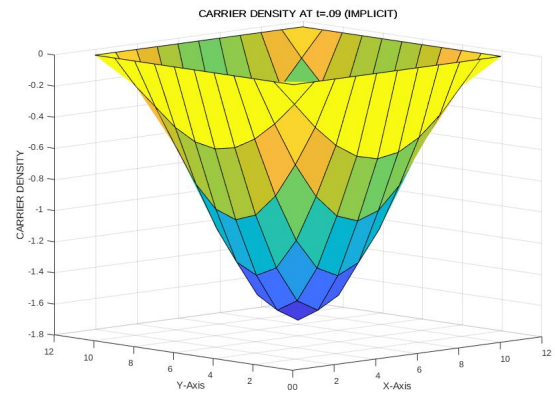


FIGURE 7. Carrier density  $N$  using implicit method.

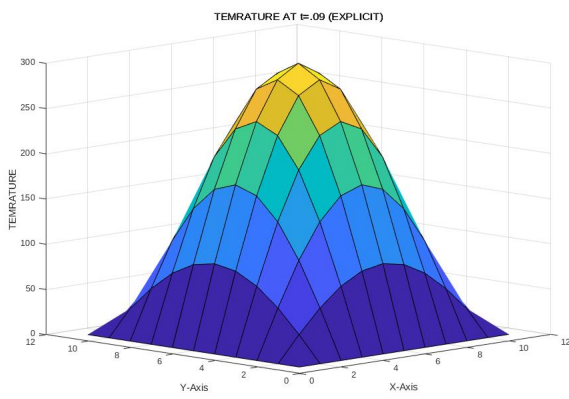


FIGURE 8. Temperature  $N$  using explicit method.

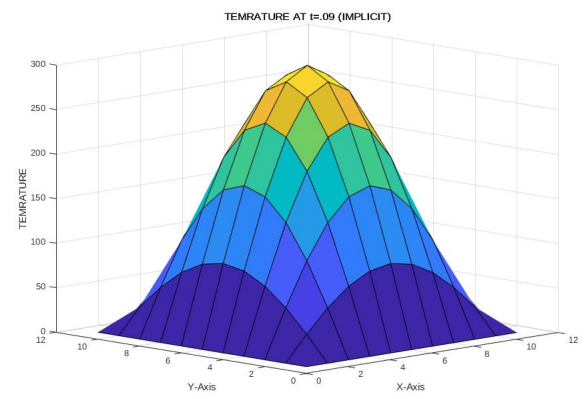


FIGURE 9. Temperature  $N$  using implicit method.



## 9. CONCLUSION

The authors have developed explicit and implicit finite difference methods for the two-dimensional coupled hygro-thermodynamic boundary value problem. The solution of these schemes was found using the using recurrence formulae and tridiagonal matrix algorithm. Numerically obtained solutions are compared and found almost similar.

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## 11. CONFLICT OF INTEREST

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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