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Finite-difference method for Hygrothermoelastic boundary value problem

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Abstract

A two-dimensional coupled hygrothermoelastic medium boundary problem using Finite difference method is discussed in the present work. Explicit and Implicit finite difference schemes for this problem are formed. The solutions of these schemes are carried out using numerical methods of finite difference. These solutions are compared of and analyzed and exciting similarities were found as result.

Keywords. Tridiagonal matrix algorithm (TDMA), Hygrothermoelasticity, Thermal conductivity, Explicit Method, Implicit Method, Thermal diffusivity, Moisture diffusivity.

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1. Introduction

Thermoelasticity concerns the dynamic system whose interactions with the environment involve not only mechanical and external work, but also the exchange of heat. Changes in temperatures cause thermal effects on materials. Among the thermal effects are thermal stress, deformation and deformation. Thermal deformation means change in the "thermal energy " (and temperature) of a material. This change increases the vibration of its atoms/molecules and which results in a stretching of the molecular bonds due to which material get expanded. Similarly ,a decrease in the thermal energy(and temperature) may lead to shrinking of material. Further , we can also say that the thermal energy will vary with expansion and compression of material. Finaly, we can declare the theory of thermoelasticity as a process of prediction the thermomechanical behaviour of material.

Duhamel [1], in 1838 founded theory of thermo elasticity. He derived the equations for the strain in an elastic body with temperature gradients. Neumann [2] reconfirmed these laws in 1841 with some additions. Thomson [3], was first to find stresses and strains in an elastic body under dynamic conditions in the light of laws of thermodynamics in 1857. thermo elasticity was classified as three types, i.e., uncoupled, coupled and generalized thermo elasticity. The classical uncoupled and coupled thermo elastic theories of Biot [4] and Nowacki [5] have an inherent paradox which wee removed by later theories .

The process of migration of atoms and molecules from highly concentrated region to region with lower concentration till equilibrium is attained, is known as diffusion. In equilibrium position, atoms and molecules occupy a definite position. This migration in result produce disturbance caused by glitch of mechanical condition and internal stresses.

In addition, an uneven moisture distribution will develop the concentration gradient that will cause moisture transfer. This means that the temperature and moisture proportion of the medium will change according to time and space. Here, the heat conduction theory and moisture diffusion can be regarded as proportionate. The distribution of temperature and humidity can vary significantly due to the mechanical stresses applied. This enables to explore the

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conection between mechanical distortion and diffusion because of temperature and humidity. Many technical problems of practical interest demonstrate this relation between moisture, heat and deformation. Any problem involving the study of solids under the influence of humidity and heat belongs to hygrothermoelasticity.

Szekeres [6] spoke about the problem of coupling generalized heat transfer and humidity. Gasch et al.[7] compared damage caused by temperature and humidity changes to mechanical loads and demonstrated that temperature and humidity cause more damage. Basic correlation was found in moisture and heat for developing governing relation for coupled hygrothermoelasticity by Szekeres and EngelBrecht [8]. Gigliotti et al. [9] investigated the cyclic and transient hygrothermoeclastic stress in laminated composite plates. Gawain et al.[10] developed a mathematical setup for investigating the behaviour of concrete under hygrothermal setup. Aboudi and Williams [11] used a mechanical micromacro method to treat the response of hygrothermolar composites. Rao and Sinha [12] analysed the effect of humidity and temperature on free vibration in 3D multidirectional composites. Ailawalia et al. [13] investigated surface waves in hygrothermoelastic half-space with hydrostatic initial stress. Qalandarov and Khaldjigitov [14] solved a Mathematical and numerical modeling of the coupled dynamic thermoelastic problems for isotropic bodies. Khaldjigitov et al. [15] developed Finite-Difference equations for 2D elasticity problems on a non-uniform grid. Singh and Lata [16] explored study of two temperature parameter effect for the axisymmetric deformation in a two-dimensional nonlocal homogeneous isotropic thick circular plate without energy dissipation. Singh and Lata [17] dealt with the two-dimensional deformation in a homogeneous isotropic nonlocal magnetothermoelastic solid with two temperatures under the effects of inclined load at different inclinationsin this work. Lata [18] studied the effect of frequency in a two-dimensional orthotropic thermoelastic rotating solid with fractional order heat transfer in generalized thermoelasticity with two-temperature due to inclined load. The work of Sefidab Amini and Reihani [19] deals with a nonlinear parabolic moving boundary problem raised from the mathematical modelling of the behavior of the breast avascular cancer tumors at their first stage. Pishnamaz Mohammadi and Ivaz [20] developed a numerical technique for the solution of the one-phase Stefan problem for the non-classical heat equation with a convective condition is discussed in this work.

This article deals with, a 2-D problem of hygerthermoelasticity concerning deformation theory of isotropic bodies. Finite-difference method is used to investigate the situation. Discrete equations are formed for the explicit and implicit schemes. Explicit schemes are solved by obtaining recurrence relations in terms of the displacement, temperature and moisture. Implicit schemes are efficiently solved using the TDMA along the valid directions. The problem is numerically solved using MatLab software for a specific material and the results are depicted by graphs. This article is an effort to through light on effect of time and space on hygrothermoelastic problems.

2. Basic Equations

We use the constitutive relations, field equations, heat conduction and moisture diffusion for homogeneous, isotropic hygrothermoelastic solid with hydrostatic initial stress and in absence of incremental body forces and heat sources given by Hosseini et al. [21] and Montanaro [22] which are

$$\sigma_{ji,j} = \rho u_{,i}, \tag{2.1}$$

$$D_T T_{,ii} + D_T^m m_{,ii} - \dot{T} - \frac{\beta_{ij}^T T_0}{\rho c}\dot{u}_{j,j} = 0, \tag{2.2}$$

$$D_m m_{,ii} + D_m^T T_{,ii} - \dot{m} - \frac{\beta_{ij}^m T_0}{k_m}\dot{u}_{j,j} = 0, \tag{2.3}$$

where $C_{ijkl} = \frac{2G\nu}{1-2\nu}\delta_{ij}\delta_{kl} + G\delta_{ik}\delta_{jl} + G\delta_{il}\delta_{jk}$, $\epsilon_{ij} = \frac{u_{j,i}-u_{i,j}}{2}$,

$$\sigma_{ij,j} = C_{ijkl}\epsilon_{ij} - \beta_{ij}^m m - \beta_{ij}^T T, \tag{2.4}$$

where $\beta_{ij}^T = \beta_T\delta_{ij}, \beta_T = (3\lambda + 2\mu)\,\alpha_t$, and $\beta_{ij}^m = \beta_m\delta_{ij}, \beta_m = (3\lambda + 2\mu)\,\alpha_m$.

3. HYGERO-THERMOELASTICITY PROBLEM FOR RECTANGULAR REGION

We consider a rectangular rectangular region bouned by $x = 0$,$y = 0$, $x = l$ and $y = l$. The wave is propagating in the x–y plane. The displacement vector in two-dimensional space hygrothermoelastic material is given as as, $\vec{u} = (u, v, 0) where u = u(x, y, t), v = v(x, y, t)$. We have considered that the equations of motion and coupled equations of heat conduction and moisture diffusion (2.1)-(2.3) and the constitutive relations (2.4) in 2-D, takes the form as follows:

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2}(\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y}\mu\frac{\partial^2 u}{\partial y^2} - \beta_m\frac{\partial m}{\partial x} - \beta_T\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{3.1}$$

$$\mu\frac{\partial^2 v}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} + (\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} - \beta_m\frac{\partial m}{\partial y} - \beta_T\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{3.2}$$

$$D_T\nabla^2 T + D_T^m\nabla^2 m - \frac{\partial T}{\partial t} - \frac{\beta_T T_0}{\rho c}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \tag{3.3}$$

$$D_m\nabla^2 m + D_m^T\nabla^2 T - \frac{\partial m}{\partial t} - \frac{\beta_m m_0 D_m}{k_m}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0. \tag{3.4}$$

Appropriate Initial Boundary Conditions $u(x, y, t)|_{t=t_0} = \phi_1$, $v(x, y, t)|_{t=t_0} = \phi_2$, $m(x, y, t)|_{t=t_0} = m_0$, $T(x, y, t)|_{t=t_0} = T_0$, $\frac{\partial u}{\partial t}|_{t=t_0} = \psi_1$, $\frac{\partial v}{\partial t}|_{t=t_0} = \psi_2$
and Boundary Condition
$u(x, y, t)|_{x=x_0} = u_0, u(x, y, t)|_{x=l_1} = \overline{u_0}$, $u(x, y, t)|_{y=y_0} = u_0, u(x, y, t)|_{y=l_2} = \overline{u_0^{'}}$
$v(x, y, t)|_{x=x_0} = v_0, v(x, y, t)|_{x=l_1} = \overline{v_0}$, $v(x, y, t)|_{y=y_0} = v_0, v(x, y, t)|_{y=l_2} = \overline{v_0^{'}}$
$T(x, y, t)|_{x=x_0} = T_0, T(x, y, t)|_{x=l_1} = \overline{T_0}$, $T(x, y, t)|_{y=y_0} = T_0, T(x, y, t)|_{y=l_2} = \overline{T_0^{'}}$
$m(x, y, t)|_{x=x_0} = m_0, m(x, y, t)|_{x=l_1} = \overline{m_0}$, $m(x, y, t)|_{y=y_0} = m_0, m(x, y, t)|_{y=l_2} = \overline{m_0^{'}}$.

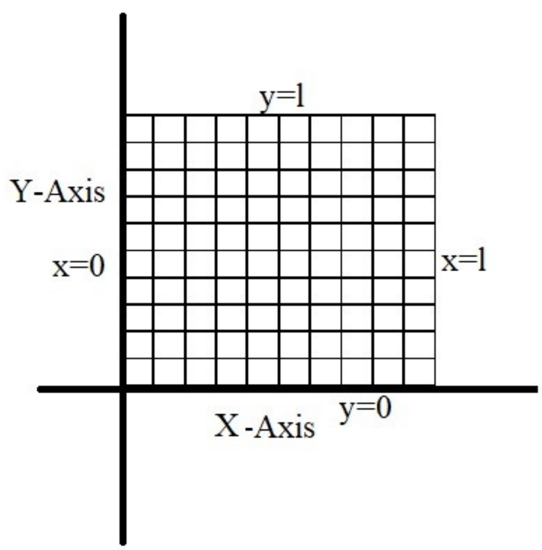

FIGURE 1. Hygerothermoelastic rectangular region.

4. Equation in finite difference environment

Rewriting(3.1) − (3.4) using central finite difference approximation using region $t \succeq 0, 0 \preceq x \preceq l_1, 0 \preceq yl_2$ and boundary points $x = ih_1, y = jh_2, t = k\tau$ where $i = 0ton_1, j = 0ton_2$ and k = 0 1 2 3 ... Then putting the finite difference approximation respective differential coefficient.

$$(\lambda+2\mu)\frac{(u^k_{i+1,j}-2u^k_{i,j}+u^k_{i-1,j})}{h_1^2}+(\lambda+\mu)\frac{(v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1})}{4h_1h_2}+$$

$$\mu\frac{(u^k_{i,j+1}-2u^k_{i,j}+u^k_{i,j-1})}{h_2^2}-\beta_m\frac{(m^k_{i+1,j}-m^k_{i-1,j})}{2h_1}-\beta_T\frac{(T^k_{i+1,j}-T^k_{i-1,j})}{2h_1}=\rho\frac{(u^{k+1}_{i,j}-2u^{k+1}_{i,j}+u^{k+1}_{i,j})}{\tau^2},$$

$$u^{k+1}_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{(u^k_{i+1,j}-2u^k_{i,j}+u^k_{i-1,j})}{h_1^2}+(\lambda+\mu)\frac{(v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1})}{4h_1h_2}\right)+$$

$$\frac{\tau^2}{\rho}\left(\mu\frac{(u^k_{i,j+1}-2u^k_{i,j}+u^k_{i,j-1})}{h_2^2}-\beta_m\frac{(m^k_{i+1,j}-m^k_{i-1,j})}{2h_1}-\beta_T\frac{(T^k_{i+1,j}-T^k_{i-1,j})}{2h_1}\right)+2u^k_{i,j}-u^{k-1}_{i,j}, \tag{4.1}$$

$$\mu\frac{(v^k_{i+1,j}-2v^k_{i,j}+v^k_{i-1,j})}{h_1^2}+(\lambda+\mu)\frac{(u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1})}{4h_1h_2}+$$

$$(\lambda+2\mu)\frac{(v^k_{i,j+1}-2v^k_{i,j}+v^k_{i,j-1})}{h_2^2}-\beta_m\frac{(m^k_{i,j+1}-m^k_{i,j-1})}{2h_2}-\beta_T\frac{(T^k_{i,j+1}-T^k_{i,j-1})}{2h_2}=\rho\frac{(v^{k+1}_{i,j}-2v^{k+1}_{i,j}+v^{k+1}_{i,j})}{\tau^2}$$

$$v^{k+1}_{i,j}=\frac{\tau^2}{\rho}\left(\mu\frac{(v^k_{i+1,j}-2v^k_{i,j}+v^k_{i-1,j})}{h_1^2}+(\lambda+\mu)\frac{(u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1})}{4h_1h_2}\right)+$$

$$\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{(v^k_{i,j+1}-2v^k_{i,j}+v^k_{i,j-1})}{h_2^2}-\beta_m\frac{(m^k_{i,j+1}-m^k_{i,j-1})}{2h_2}-\beta_T\frac{(T^k_{i,j+1}-T^k_{i,j-1})}{2h_2}\right)+2v^k_{i,j}-v^{k-1}_{i,j} \tag{4.2}$$

$$D_T\frac{(T^k_{i+1,j}-2T^k_{i,j}+T^k_{i-1,j})}{h_1^2}+D_T\frac{(T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1})}{h_2^2}+$$

$$D^m_T\frac{(m^k_{i+1,j}-2m^k_{i,j}+m^k_{i-1,j})}{h_1^2}+D^m_T\frac{(m^k_{i,j+1}-2m^k_{i,j}+m^k_{i,j-1})}{h_2^2}-\frac{(T^{k+1}_{i,j}-T^k_{i,j})}{\tau}-$$

$$\frac{\beta_TT_0}{\rho c}\left(\frac{(u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1})}{4h_1h_2}-\frac{(v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1})}{4h_1h_2}\right)=0,$$

$$T^{k+1}_{i,j}=\tau\left(D_T\frac{(T^k_{i+1,j}-2T^k_{i,j}+T^k_{i-1,j})}{h_1^2}+D_T\frac{(T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1})}{h_2^2}\right)+$$

$$\tau\left(D^m_T\frac{(m^k_{i+1,j}-2m^k_{i,j}+m^k_{i-1,j})}{h_1^2}+D^m_T\frac{(m^k_{i,j+1}-2m^k_{i,j}+m^k_{i,j-1})}{h_2^2}\right)-$$

$$\tau\frac{\beta_TT_0}{\rho c}\left(\frac{(u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1})}{4h_1h_2}-\frac{(v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1})}{4h_1h_2}\right)+T^k_{i,j}, \tag{4.3}$$

$$D_m\frac{(m^k_{i+1,j}-2m^k_{i,j}+m^k_{i-1,j})}{h_1^2}+D_m\frac{(m^k_{i,j+1}-2m^k_{i,j}+m^k_{i,j-1})}{h_2^2}+$$
$$D_m^T\frac{(T^k_{i+1,j}-2T^k_{i,j}+T^k_{i-1,j})}{h_1^2}+D_m^T\frac{(T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1})}{h_2^2}-\frac{(m^{k+1}_{i,j}-m^k_{i,j})}{\tau}-$$
$$\frac{\beta_m D_m m_0}{k_m}\left(\frac{(u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1})}{4h_1h_2}-\frac{(v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1})}{4h_1h_2}\right)=0,$$

$$m^{k+1}_{i,j}=\tau\left(D_m\frac{(m^k_{i+1,j}-2m^k_{i,j}+m^k_{i-1,j})}{h_1^2}+D_m\frac{(m^k_{i,j+1}-2m^k_{i,j}+m^k_{i,j-1})}{h_2^2}\right)+$$
$$\tau\left(D_m^T\frac{(T^k_{i+1,j}-2T^k_{i,j}+T^k_{i-1,j})}{h_1^2}+D_m^T\frac{(T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1})}{h_1^2}\right)-$$
$$\tau\frac{\beta_m D_m m_0}{k_m}\left(\frac{(u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1})}{4h_1h_2}+\frac{(v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1})}{4h_1h_2}\right)+m^k_{i,j}, \tag{4.4}$$

Reviewing equations (4.1) − (4.4) we can find values $u(x,y,t), v(x,y,t), T(x,y,t)$ and $m(x,y,t)$ at the layer t_{k+1} . We can get the values of $u(x,y,t)$ and $v(x,y,t)$ for two primary layers $k=0$ and $k=1$ from the initial conditions for $k=0$ $u(0,0)=\phi_1$, $v(0,0)=\phi_2$, $m(0,0)=m_0$, $T(0,0)=T_0$,

$$\frac{\partial u}{\partial t}|_{t=t_0}=\psi_1(\,i,j)\Rightarrow\frac{u^1_{i,j}-u^{-1}_{i,j}}{2\tau}=\psi_1(\,x_i,y_j)\Rightarrow u^1_{i,j}=u^{-1}_{i,j}+2\tau\psi_1(\,x_i,y_j)\Rightarrow u^{-1}_{i,j}=u^1_{i,j}-2\tau\psi_1(\,x_i,y_j)\,,$$
$$\frac{\partial v}{\partial t}|_{t=t_0}=\psi_2(\,x_i,y_j)\Rightarrow\frac{v^1_{i,j}-v^{-1}_{i,j}}{2\tau}=\psi_2(\,x_i,y_j)\Rightarrow v^1_{i,j}=v^{-1}_{i,j}+2\tau\psi_2(\,x_i,y_j)\Rightarrow v^{-1}_{i,j}=v^1_{i,j}-2\tau\psi_2(\,x_i,y_j)\,.$$

Replacing these values in Equation (4.1) − (4.4) we can get relation to calculate the values of $u(x,y,t)$, $v(x,y,t)$, $T(x,y,t)$ and $m(x,y,t)$, for $k=1$,

$$u^1_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{(u^0_{i+1,j}-2u^0_{i,j}+u^0_{i-1,j})}{h_1^2}+(\lambda+\mu)\frac{(v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1})}{4h_1h_2}\right)+$$
$$\frac{\tau^2}{\rho}\left(\mu\frac{(u^0_{i,j+1}-2u^0_{i,j}+u^0_{i,j-1})}{h_2^2}-\beta_m\frac{(m^0_{i+1,j}-m^0_{i-1,j})}{2h_1}-\beta_T\frac{(T^0_{i+1,j}-T^0_{i-1,j})}{2h_1}\right)+2u^0_{i,j}-u^{-1}_{i,j},$$
$$u^1_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{(u^0_{i+1,j}-2u^0_{i,j}+u^0_{i-1,j})}{h_1^2}+(\lambda+\mu)\frac{(v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1})}{4h_1h_2}\right)+$$
$$\frac{\tau^2}{\rho}\left(\mu\frac{(u^0_{i,j+1}-2u^0_{i,j}+u^0_{i,j-1})}{h_2^2}-\beta_m\frac{(m^0_{i+1,j}-m^0_{i-1,j})}{2h_1}-\beta_T\frac{(T^0_{i+1,j}-T^0_{i-1,j})}{2h_1}\right)+2u^0_{i,j}-u^1_{i,j}+2\tau\psi_1(\,x_i,y_j)\,,$$

$$u^1_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{(u^0_{i+1,j}-2u^0_{i,j}+u^0_{i-1,j})}{h_1^2}+(\lambda+\mu)\frac{(v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1})}{4h_1h_2}\right)+$$

$$\frac{\tau^2}{\rho}\left(\mu\frac{(u^0_{i,j+1}-2u^0_{i,j}+u^0_{i,j-1})}{h_2^2}-\beta_m\frac{(m^0_{i+1,j}-m^0_{i-1,j})}{2h_1}-\beta_T\frac{(T^0_{i+1,j}-T^0_{i-1,j})}{2h_1}\right)+u^0_{i,j}+\tau\psi_1\left(\,x_i,y_j\right), \tag{4.5}$$

$$v^1_{i,j}=\frac{\tau^2}{\rho}\left(\mu\frac{(v^0_{i+1,j}-2v^0_{i,j}+v^0_{i-1,j})}{h_1^2}+(\lambda+\mu)\,\frac{(u^0_{i+1,j+1}-u^0_{i-1,j+1}-u^0_{i+1,j-1}+u^0_{i-1,j-1})}{4h_1h_2}\right)+$$

$$\frac{\tau^2}{\rho}\left((\lambda+2\mu)\,\frac{(v^0_{i,j+1}-2v^0_{i,j}+v^0_{i,j-1})}{h_2^2}-\beta_m\frac{(m^0_{i,j+1}-m^0_{i,j-1})}{2h_2}-\beta_T\frac{(T^0_{i,j+1}-T^0_{i,j-1})}{2h_2}\right)+2v^0_{i,j}-v^{-1}_{i,j},$$

$$v^1_{i,j}=\frac{\tau^2}{\rho}\left(\mu\frac{(v^0_{i+1,j}-2v^0_{i,j}+v^0_{i-1,j})}{h_1^2}+(\lambda+\mu)\,\frac{(u^0_{i+1,j+1}-u^0_{i-1,j+1}-u^0_{i+1,j-1}+u^0_{i-1,j-1})}{4h_1h_2}\right)+$$

$$\frac{\tau^2}{\rho}\left((\lambda+2\mu)\,\frac{(v^0_{i,j+1}-2v^{-}_{i,j}+v^0_{i,j-1})}{h_2^2}-\beta_m\frac{(m^0_{i,j+1}-m^0_{i,j-1})}{2h_2}-\beta_T\frac{(T^0_{i,j+1}-T^0_{i,j-1})}{2h_2}\right)+v^0_{i,j}+\tau\psi_2\left(\,x_i,y_j\right),$$

$$v^1_{i,j}=\frac{\tau^2}{\rho}\left(\mu\frac{(v^0_{i+1,j}-2v^0_{i,j}+v^0_{i-1,j})}{h_1^2}+(\lambda+\mu)\,\frac{(u^0_{i+1,j+1}-u^0_{i-1,j+1}-u^0_{i+1,j-1}+u^0_{i-1,j-1})}{4h_1h_2}\right)+$$

$$\frac{\tau^2}{\rho}\left((\lambda+2\mu)\,\frac{(v^0_{i,j+1}-2v^0_{i,j}+v^0_{i,j-1})}{h_2^2}-\beta_m\frac{(m^0_{i,j+1}-m^0_{i,j-1})}{2h_2}-\beta_T\frac{(T^0_{i,j+1}-T^0_{i,j-1})}{2h_2}\right)+v^0_{i,j}+\tau\psi_2\left(\,x_i,y_j\right), \tag{4.6}$$

$$T^1_{i,j}=\tau\left(D_T\frac{(T^0_{i+1,j}-2T^0_{i,j}+T^0_{i-1,j})}{h_1^2}+D_T\frac{(T^0_{i,j+1}-2T^0_{i,j}+T^0_{i,j-1})}{h_2^2}\right)+$$

$$\tau\left(D^m_T\frac{(m^0_{i+1,j}-2m^0_{i,j}+m^0_{i-1,j})}{h_1^2}+D^m_T\frac{(m^0_{i,j+1}-2m^0_{i,j}+m^0_{i,j-1})}{h_2^2}\right)-$$

$$\tau\frac{\beta_T T_0}{\rho c}\left(\frac{(u^0_{i+1,j+1}-u^0_{i-1,j+1}-u^0_{i+1,j-1}+u^0_{i-1,j-1})}{4h_1h_2}-\frac{(v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1})}{4h_1h_2}\right)+T^0_{i,j}, \tag{4.7}$$

$$m^1_{i,j}=\tau\left(D_m\frac{(m^0_{i+1,j}-2m^0_{i,j}+m^0_{i-1,j})}{h_1^2}+\frac{(m^0_{i,j+1}-2m^0_{i,j}+m^0_{i,j-1})}{h_2^2}\right)+$$

$$\tau\left(D^T_m\frac{(T^0_{i+1,j}-2T^0_{i,j}+T^0_{i-1,j})}{h_1^2}+D^T_m\frac{(T^0_{i,j+1}-2T^0_{i,j}+T^0_{i,j-1})}{h_1^2}\right)-$$

$$\tau\frac{\beta_m D_m m_0}{k_m}\left(\frac{(u^0_{i+1,j+1}-u^0_{i-1,j+1}-u^0_{i+1,j-1}+u^0_{i-1,j-1})}{4h_1h_2}+\frac{(v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1})}{4h_1h_2}\right)+m^0_{i,j}, \tag{4.8}$$

5. Implicit Solution

In the formulae given above, we have used an explicit. Now to deduce an implicit method for solving coupled hygerothermoelasticity problems numerically, we need to rewrite equations (4.1) − (4.4) in the following form problems. solving (4.1)

$$\frac{(\lambda+2\mu)}{h_1^2}u_{i+1,j}^{k+1}-\left(\frac{(\lambda+2\mu)}{h_1^2}+\frac{\rho}{\tau^2}\right)u_{i,j}^{k+1}+\frac{(\lambda+2\mu)}{h_1^2}u_{i-1,j}^{k+1}=\frac{\rho}{\tau^2}\left(u_{i,j}^{k-1}-2u_{i,j}^{k}-\mu\frac{(u_{i,j+1}^{k}-2u_{i,j}^{k}+u_{i,j-1}^{k})}{h_2^2}\right)-$$

$$(\lambda+\mu)\frac{(v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k})}{4h_1h_2}+\beta_m\frac{(m_{i+1,j}^{k}-m_{i-1,j}^{k})}{2h_1}+\beta_T\frac{(T_{i+1,j}^{k}-T_{i-1,j}^{k})}{2h_1},$$

$$a_iu_{i+1,j}^{k+1}+b_iu_{i,j}^{k+1}+c_iu_{i-1,j}^{k+1}=f_{i,j}, \tag{5.1}$$

$$a_i=\frac{(\lambda+2\mu)}{h_1^2},b_i=-\left(\frac{2(\lambda+2\mu)}{h_1^2}+\frac{\rho}{\tau^2}\right),c_i=\frac{(\lambda+2\mu)}{h_1^2},$$

$$f_{i,j}=\frac{\rho}{\tau^2}\left(u_{i,j}^{k-1}-2u_{i,j}^{k}\right)-\mu\frac{(u_{i,j+1}^{k}-2u_{i,j}^{k}+u_{i,j-1}^{k})}{h_2^2}$$

$$-(\lambda+\mu)\frac{(v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k})}{4h_1h_2}+\beta_m\frac{(m_{i+1,j}^{k}-m_{i-1,j}^{k})}{2h_1}+\beta_T\frac{(T_{i+1,j}^{k}-T_{i-1,j}^{k})}{2h_1},$$

Solving (4.2)

$$\mu\frac{(v_{i+1,j}^{k+1}-2v_{i,j}^{k+1}+v_{i-1,j}^{k+1})}{h_1^2}+(\lambda+\mu)\frac{(u_{i+1,j+1}^{k}-u_{i-1,j+1}^{k}-u_{i+1,j-1}^{k}+u_{i-1,j-1}^{k})}{4h_1h_2}+(\lambda+2\mu)$$

$$\frac{(v_{i,j+1}^{k}-2v_{i,j}^{k}+v_{i,j-1}^{k})}{h_2^2}-\beta_m\frac{(m_{i,j+1}^{k}-m_{i,j-1}^{k})}{2h_2}-\beta_T\frac{(T_{i,j+1}^{k}-T_{i,j-1}^{k})}{2h_2}=\rho\frac{(v_{i,j}^{k+1}-2v_{i,j}^{k+1}+v_{i,j}^{k+1})}{\tau^2},$$

$$a_iv_{i+1,j}^{k+1}+b_iv_{i,j}^{k+1}+c_iv_{i-1,j}^{k+1}=f_{i,j}, \tag{5.2}$$

$$a_i=\frac{\mu}{h_1^2},b_i=-\left(\frac{2\mu}{h_1^2}+\frac{\rho}{\tau^2}\right),c_i=\frac{\mu}{h_1^2},f_{i,j}=\frac{\rho}{\tau^2}\left(v_{i,j}^{k-1}-2v_{i,j}^{k}\right)-\mu\frac{(v_{i,j+1}^{k}-2v_{i,j}^{k}+v_{i,j-1}^{k})}{h_2^2}$$

$$-(\lambda+\mu)\frac{(u_{i+1,j+1}^{k}-u_{i-1,j+1}^{k}-u_{i+1,j-1}^{k}+u_{i-1,j-1}^{k})}{4h_1h_2}+\beta_m\frac{(m_{i,j+1}^{k}-m_{i,j-1}^{k})}{2h_2}+\beta_T\frac{(T_{i,j+1}^{k}-T_{i,j-1}^{k})}{2h_2},$$

solving (4.3)

$$\frac{D_T}{h_1^2}T_{i+1,j}^{k+1}-\left(2\frac{D_T}{h_1^2}+\frac{1}{\tau}\right)T_{i,j}^{k+1}+\frac{D_T}{h_1^2}T_{i-1,j}^{k+1}=$$

$$\frac{\beta_TT_0}{\rho c}\left(\frac{(u_{i+1,j+1}^{k}-u_{i-1,j+1}^{k}-u_{i+1,j-1}^{k}+u_{i-1,j-1}^{k})}{4h_1h_2}-\frac{(v_{i+1,j+1}^{k}-v_{i-1,j+1}^{k}-v_{i+1,j-1}^{k}+v_{i-1,j-1}^{k})}{4h_1h_2}\right)$$

$$D_T\frac{(T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k})}{h_2^2}-D_T^m\frac{(m_{i+1,j}^{k}-2m_{i,j}^{k}+m_{i-1,j}^{k})}{h_1^2}-D_T^m\frac{(m_{i,j+1}^{k}-2m_{i,j}^{k}+m_{i,j-1}^{k})}{h_2^2}+\frac{T_{i,j}^{k}}{\tau},$$

$$a_iT_{i+1,j}^{k+1}+b_iT_{i,j}^{k+1}+c_iT_{i-1,j}^{k+1}=f_{i,j}, \tag{5.3}$$

$$a_i = \frac{D_T}{h_1^2, b_i = -\left(2\frac{D_T}{h_1^2} + \frac{1}{\tau}\right)}, c_i = \frac{D_T}{h_1^2},$$

$$f_{i,j} = \frac{\beta_T T_0}{\rho c}\left(\frac{(u^k_{i+1,j+1} - u^k_{i-1,j+1} - u^k_{i+1,j-1} + u^k_{i-1,j-1})}{4h_1h_2} - \frac{(v^k_{i+1,j+1} - v^k_{i-1,j+1} - v^k_{i+1,j-1} + v^k_{i-1,j-1})}{4h_1h_2}\right)$$

$$D_T\frac{(T^k_{i,j+1} - 2T^k_{i,j} + T^k_{i,j-1})}{h_2^2} - D_T^m\frac{(m^k_{i+1,j} - 2m^k_{i,j} + m^k_{i-1,j})}{h_1^2} -$$

$$D_T^m\frac{(m^k_{i,j+1} - 2m^k_{i,j} + m^k_{i,j-1})}{h_2^2} + \frac{T^k_{i,j}}{\tau},$$

solving (4.4)

$$\frac{D_m}{h_1^2}m^{k+1}_{i+1,j} - \left(2\frac{D_m}{h_1^2}\right)m^{k+1}_{i,j} + \frac{D_m}{h_1^2}m^{k+1}_{i-1,j} =$$

$$\frac{\beta_m D_m m_0}{k_m}\left(\frac{(u^k_{i+1,j+1} - u^k_{i-1,j+1} - u^k_{i+1,j-1} + u^k_{i-1,j-1})}{4h_1h_2} - \frac{(v^k_{i+1,j+1} - v^k_{i-1,j+1} - v^k_{i+1,j-1} + v^k_{i-1,j-1})}{4h_1h_2}\right) -$$

$$D_m\frac{(m^k_{i,j+1} - 2m^k_{i,j} + m^k_{i,j-1})}{h_2^2} - D_m^T\frac{(T^k_{i+1,j} - 2T^k_{i,j} + T^k_{i-1,j})}{h_1^2} - D_m^T\frac{(T^k_{i,j+1} - 2T^k_{i,j} + T^k_{i,j-1})}{h_2^2} - \frac{m^k_{i,j}}{\tau},$$

$$a_i m^{k+1}_{i+1,j} + b_i m^{k+1}_{i,j} + c_i m^{k+1}_{i-1,j} = f_{i,j}, \tag{5.4}$$

$$a_i = \frac{D_m}{h_1^2}, b_i = -\left(2\frac{D_m}{h_1^2} + \frac{1}{\tau}\right), c_i = \frac{D_m}{h_1^2},$$

$$f_{i,j} = \frac{\beta_m D_m m_0}{k_m}\left(\frac{(u^k_{i+1,j+1} - u^k_{i-1,j+1} - u^k_{i+1,j-1} + u^k_{i-1,j-1})}{4h_1h_2} - \frac{(v^k_{i+1,j+1} - v^k_{i-1,j+1} - v^k_{i+1,j-1} + v^k_{i-1,j-1})}{4h_1h_2}\right) -$$

$$D_m\frac{(m^k_{i,j+1} - 2m^k_{i,j} + m^k_{i,j-1})}{h_2^2} - D_m^T\frac{(T^k_{i+1,j} - 2T^k_{i,j} + T^k_{i-1,j})}{h_1^2} - D_m^T\frac{(T^k_{i,j+1} - 2T^k_{i,j} + T^k_{i,j-1})}{h_2^2} - \frac{m^k_{i,j}}{\tau}.$$

For the values of $u(x,y,t), v(x,y,t), m(x,y,t)$ and $T(x,y,t)$ at the first two time layers, we can use initial conditions and formulae (4.5) – (4.8) for calculations. Values on internal node of subsequent layers can be found by using (5.1) – (5.4) with the TDMA making use of given initial and boundary conditions.

6. Particular Case

Neglecting moisture effect, the problem reduces to the numerical solution of a two dimensional thermoelastic problem by finite difference method. Silveira et al. [23] discussed one dimensional themoelastic problem by finite difference method

7. Numerical Verification

Numerical problem satisfying (2.1) – (2.4) is simulated using the recurrence $T(x,y,t) = T_0 \sin\left(\frac{\pi x}{l_1}\right)\sin\left(\frac{\pi y}{l_2}\right)$ and boundary conditions are $u(x,y,t) = 0$, $v(x,y,t) = 0$, $m(x,y,t) = 0$ and $T(x,y,t) = 0$. formulas and TDMA as test with values of constants W. J. Chang [24], initial and boundary conditions given as per Table 1.

8. TABLE

SNo	Type	Symbol	Unit
1.	Lame's constant	λ	$46.9 * 10^9 N/m^2$
2.	Lame's constant	μ	$24.17 * 10^9 N/m^2$
3.	Density	ρ	370 Kg/m_3
4.	$Poisson'sration$	ν	.33
5.	Reference Moisture	m_0	10%
6.	Coefficient of Moisture expansion	α^m	$2.68 * 10^{-3} cm/cm(H_2O)$
7.	Coefficient of linear thermal expansion	α^T	$31.3 * 10 * -6 cm/cm(H_2O)$
8.	Temperature	T_0	283 K
9.	Heat Capacity	c	$2500 j/Kg(^0K)$
10.	Moisture diffusivity	k_m	$2.2 * 10^{-8} Kg/msM$
11.	Diffusion coefficient of moisture	D_m	$2.16 * 10^{-6}/s(^0K)$
12.	Coupled diffusivity	D_T^m	$2.1 * 10^{-7} m^2(^0K)/s(H_2O)$
13.	Temperature diffusivity	D_T	$k/\rho c$

$l_1 = 1, l_2 = 1$,at t=0 $u(x, y, t) = \phi_1 = 0, v(x, y, 0) = \phi_2 = 0, m(x, y, t) = 0$,and where $\lceil$ boundary of the body. Figures 1-8 show the displacement components $u(x, y, t)$, $v(x, y, t)$, temperature $T(x, y, t)$ and moisture $m(x, y, t)$ distributions in 3D space. Every unknown value was calculated using the recurrence formulas and TDMA with help of MatLab software.

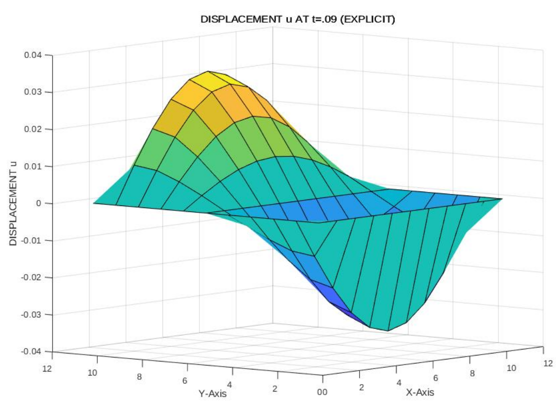

FIGURE 2. Horizontal displacement u using explicit method.

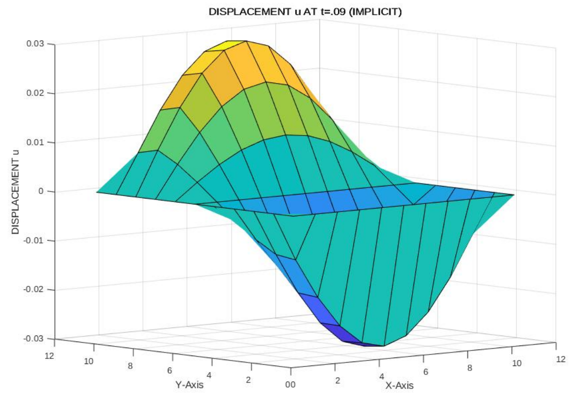

FIGURE 3. Horizontal displacement u using implicit method.

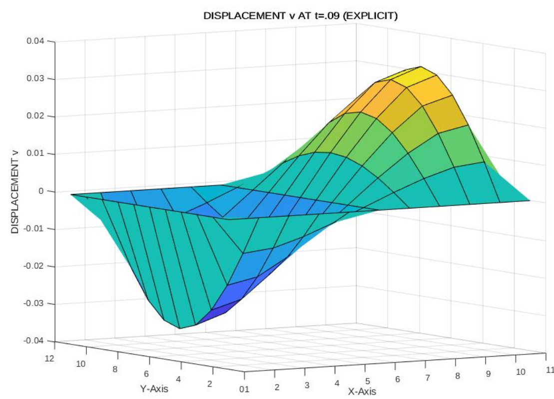

FIGURE 4. Normal displacement v using explicit method.

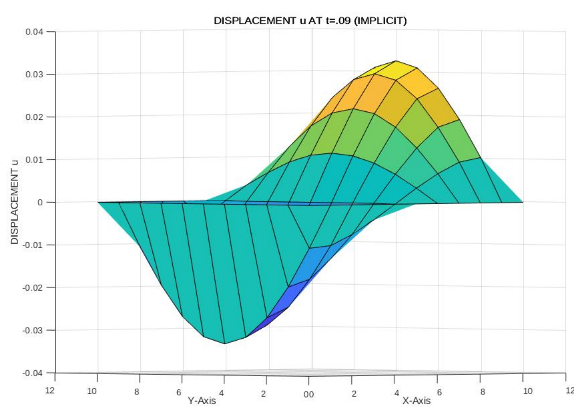

FIGURE 5. Normal displacement v using implicit method.

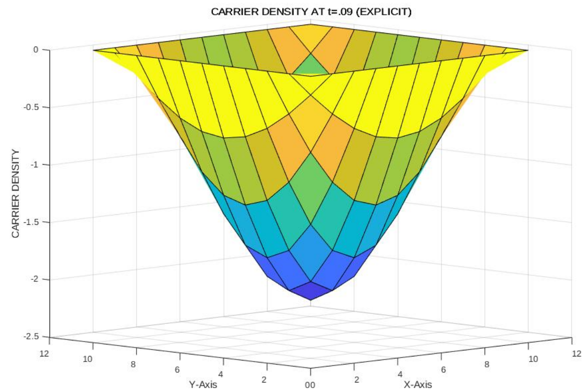

FIGURE 6. Carrier density N using explicit method.

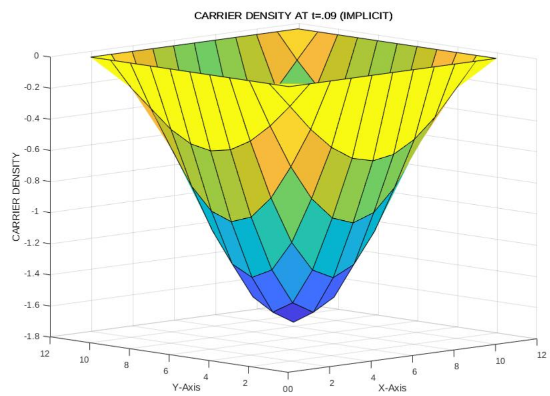

FIGURE 7. Carrier density N using implicit method

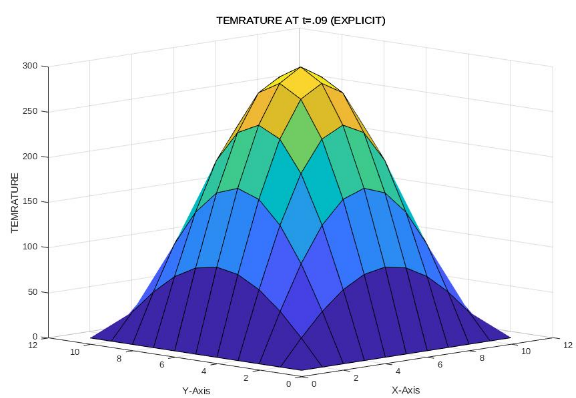

FIGURE 8. Temperature N using explicit method.

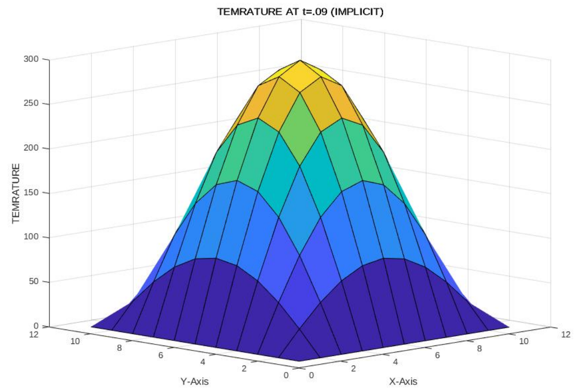

Figure 9. Temperature N using implicit method.

9. CONCLUSIONS

The authors have developed explicit and implicit finite difference methods for the two- dimensional coupled hygro-thermodynamic boundary value problem. Solution of these schemes was found using the using recurrence formulae and tridiagonal matrix algorithm. Numerically obtained solutions are compared and found almost similar.

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11. Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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References

[1] J. Duhamel, *Mémoire sur le calcul des actions moléculaires développées par les changements de température dans les corps solides*, Mathématiques et Physiques, *5* (1836), 440–498.

[2] K. E. Neumann, *Die gesetze der doppelbrechung des lichts in comprimirten oder ungleichförmig erwärmten unkrystallinischen körpern*, Ann.Phys., *130* (1841), 449.

[3] W. Thomson, *On the thermo-elastic and thermo-magnetic properties of matter*, Quarterly Journal of Mathematics, *1* (1857), 57–77.

[4] M. A. Biot, *On the theory of propagation of elastic waves in a fluid-saturated porous solid. i.low frequency range. ii. higher frequency range*, Journal of Acoustical Society of America, *28* (1956), 168–191.

[5] W. Nowacki, *Couple stresses in the theory of thermo elasticity i*, Bulletin L'Academie Polonaise des Science, Serie des Sciences Technology, *14*(3) (1966), 263–272.

[6] A. Szekeres, *Cross-coupled heat and moisture transport part 1—theory*, Journal of Thermal Stresses, *35*(1-3) (2012) 248–268.

[7] R. T. M. Gasch and A. Ansell, *A coupled hygro-thermo-mechanical model for concrete subjected to variable environmental conditions*, International Journal of Solids Structures, *91* (2016), 143–156.

[8] A. Szekeres and J. Engelbrecht, *Coupling of generalized heat and moisture transfer*, Periodica Polytechnica Mechanical Engineering, *44*(1) (2000) 161–170.

[9] M. F. J.Molimard and A. Vautrin, *Transient and cyclical hygrothermoelastic stress in laminated composite plates modelling and experimental assessment*, Mechanics of Materials, *39*(8) (2007), 729–245.

[10] D. Gawin and F. Pesavento, *Modelling of hygro-thermal behavior of concrete at high temperature with thermo-chemical and mechanical material degradation and engineering*, International Journal of Solids and Structures, *192*(13-14) (2003), 1731–1771.

[11] J. Aboudi and T. Williams, *Modelling of hygro-thermal behavior of concrete at high temperature with thermo-chemical and mechanical material degradation and engineering*, Computer Methods in Applied Mechanics, *37*(30) (2000), 4149–4179.

[12] V. Rao and P. Sinha, *Dynamic response of multidirectional composites in hygrothermal environments*, Composite Structures, *64*(3-4) (2004), 329–339.

[13] D. P. Ailawalia and V. Sharma, *Surface waves in hygrothermoelastic half-space with hydrostatic initial stress*, Mechanics of Advanced Materials Structures, *29*(16) (2022), 2380–2389.

[14] A. A. Qalandarov and A. Khaldjigitov, *Mathematical and numerical modeling of the coupled dynamic thermoelastic problems for isotropic bodies*, TWMS Journal of Pure and Applied Mathematics, *11* (2020), 119–126.

[15] A. K. A. Khaldjigitov and U. Djumayozov, *Finite- difference equations for 2d elasticity problems on a non-uniform grid*, In AIP Conference Proceedings, *2637*(1) (2022), 30005.

[16] S. Singh and P. Lata, *Effect of two temperature and nonlocality in an isotropic thermoelastic thick circular plate without energy dissipation*, Partial Differential Equations in Applied Mathematics, *7* (2023), 100512, doi:10.1016/j.padiff.2023.100512.

[17] S. Singh and P. Lata, *Effect of rotation and inclined load in a nonlocal magnetothermoelastic solid with two temperature*, Advances in Materials Research, *11*(1) (2022), 23–39.

[18] P. Lata, *Time harmonic interactions due to inclined load in an orthotropic thermoelastic rotating media with fractional order heat transfer and two-temperature*, Coupled Systems Mechanics, *14*(4) (2022), 297–313.

[19] P. B. Sefidab Amini and P. Reihani, *On a moving boundary problem associated with a mathematical model of breast cancer*, Computational Methods for Differential Equations, *2307* (2023), 1–11, doi.org/10.22034/CMDE.2023.55447

[20] S. Mohammadi and K. Ivaz, *Numerical solution of one-phase stefan problem for the non-classical heat equation with a convective condition*, Computational Methods for Differential Equations, *114* (2023), 776–784, doi:10.22034/cmde.2023.54177.2266.

[21] S. H. J. S. V. Sladek, *Application of meshless local integral equations to two dimensional analysis of coupled non-fick diffusionelasticity*, Engineering Analysis with Boundary Elements, *37*(3) (2013), 603–615, doi:10.1080/01495739.2016.1224134

[22] A. Montanaro,*On singular surfaces in isotropic linear thermoelasticity with initial stress*, Journal of Acoustical Society of America, *106* (1999), 1586–1588, doi: 10.1121/1.427154.

[23] J. C. A. M. D. J. M. M. M. H. Silveira Jonior, C. E. Piaz, and O. Hacke, *The finite difference method applied to the 1d thermoelastic problem*, International Proceedings of the XXXVII Iberian Latin-American Congress on Computational Methods in Engineering Suzana Moreira Ávila (Editor), ABMEC, Brasília, DF, Brazil, (6-9) (2016).

[24] W. Chang and C. Weng, *An analytical solution to coupled heat and moisture diffusion transfer in porous materials*, International Journal of Heat and Mass Transfer, *43*(19) (2000), 3621–3632, doi: 10.1016/S0017-9310(00)00003-X