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The use of technological intelligence model in solving Terrorism dynamics: a case study of Nigeria

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Abstract

Nigeria is one of the most populated countries in West Africa and is in seventh position globally. The issue of terrorism has become a common problem in Nigeria, and the government has been applying local strategies to address the situation but has yet to produce good results. The challenges necessitate the effort in this paper to develop a new deterministic model to curb terrorism and insurgency through technology intelligence in Nigeria. This analysis indicates that Unmanned Aerial Vehicles (UAV) and the transmission rate per capita are the most sensitive parameters. Also pictured from the graphs in Figures 2, 3, and 4 were drone used to reduced the number of informants of both the terrorist and kidnapper individuals in Nigeria. Finally, this paper recommended the model adopted for controlling terrorism in Nigeria.

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1. Introduction

Information and Communication Technology (ICT) to counter terrorism and other criminals in Nigeria led to a new era of technology in military operations. The government has been addressing the security challenges through different methods and approaches, such as using police, military, vigilante and local hunters, yet yielded no success. However, deploying technological intelligence through ICT devices, such as combat drones for surveillance and intelligence gathering, would produce the desired result. The term "drone" refers to an unmanned aerial vehicle (UAV) that can be used for either military or commercial purposes. The United States has also used combat drone strikes to attack terrorist and insurgent groups, including those that target militants in Pakistan who support al-Qaeda and insurgents operating in Afghanistan, al-Qaeda in the Arabian Peninsula in Yemen, and the al-Shabaab campaign in Somalia, to name just a few [24]. These combat drone attacks are meant to keep insurgent/terrorist groups under control and stop them from growing. Drone punishes the groups by assassinating or killing their commanders and instilling uncertainty in the minds of their foot soldiers. It is crucial to describe how drone warfare helps to effectively combat terrorism and rebellion. How drone aircraft integrate into the military strategy was demonstrated in [5] study. The drone provides global intelligence and hostile strike capabilities, which aid in the counter-terrorism operation. The outcome demonstrated that the drone, with its special ability to utilize widespread reconnaissance and forceful air strikes, is a particularly successful military tool of counter-terrorism strategy. In addition, Yaacoub *et. al.,* (2020) examined the use of drone that is Unmanned Aerial Vehicles (UAV) for both the military and commercial sectors of aviation, involving information and communication technology as in [26]. Adamu and Ibrahim in [1] examined drones and Analysed counter-terrorism control techniques with five compartments in a population. The result showed the effective use of a drone to fight against terrorists and insurgency. Also, in Adamu and Ibrahim [2] examined the

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use of Elzaki transform method for analytic solution to solve deterministic modelling of terrorism in a population. Similarly, [19] examined the Counter-insurgency operations of the Nigerian military capable of defeating the Boko Haram insurgency. The results show how numerous factors, including corruption, have challenged military campaigns at the strategic, operational and tactical levels. A mathematical model of the dynamics behaviour of terrorism was developed in [3]. The model was developed to control the spread of terrorist ideologies and check the recruitment pool. Also, in [25] a model of terrorism with multiple ideologies was developed to describe individual radicalisation under the influence of two cooperative ideologies.

A variety of strategies are used to counter the potential dangers rather than a single strategy to prevent security difficulties. The best way to deal with insecurity is to employ both military and civil intervention strategies. To stop the spread of terrorist cells and their influence, the military intervention concentrates on using armed drones to target top-level terrorist organization officials and front-line members. The civic intervention strategy relies on unrestricted financial assistance for the local populace to help with development initiatives, programme design for convicted criminals in correctional facilities, and certified or de-radicalized terrorists. Projects for the physical, social, and economic improvement of the local region will be funded with the help of this aid. As a result, the programme emphasizes on increasing access to education and training in [21]. The root of insecurity in Nigeria is the government's inability to address the situation early. Terrorist groups in Nigeria, such as Boko-Haram (BHT), Islamic State of West African Provence (ISWAP) and others, joined a coalition of Salafi-jihadist terrorist groups from the Sahel region to implement so-called Islamic governance. At the same time, criminal groups like IPOB, ESN, Cattle rustlers, Fulani extremists, Bandits, Kidnappers and others are making the Nigerian government miserable. Subsequently, as of November 2022, Nigeria moved from the third most impacted country to sixth, while Afgha and Iraq retained their position as first and second in [6, 7]. This rank showed the significant effects of using technology on criminals in Nigeria. Also, as of October 2021, the number of deaths from terrorism had increased to 497, but by 2022, that figure had dropped to 385, according to the Global Terrorism Index, which was published in 2023. With 120 incidents reported in 2022 compared to 214 in 2021, Nigeria saw a significant decline in the number of terrorist strikes. Since 2011, there have been fewer terror incidents and fatalities than at any other time. That said, in Nigeria in 2022, civilians accounted for half of all terrorism-related deaths. There was a significant decline in military deaths from 2021 to 2022, falling by 74 percent, while civilian deaths jumped by 78 percent to 196 deaths in [8]. The work of Okrinya and Consul in [10] built a basic deterministic model of kidnapping and incorporating control strategies to measure the threats in a population. According to a 2017 report by the United Nations Office on Drugs and Crime, criminal activity is a major contributing factor to a number of political and social issues, such as corruption, insecurity, and a wretched existence that results in civilian deaths, disrupts social progress and economic growth, and undermines governments. in [12]. A persistent issue that jeopardizes Nigerian citizens safety is insecurity. The complex phenomenon impedes progress and fuels the rise in criminal activity. distinct geopolitical zones have distinct dimensions of insecurity, which is a complex problem. Banditry, armed robberies, kidnappings, extra judicial executions, herder-farmer conflicts, cybercrime, and ritual killings are all on the rise in the South West. The South East has become a hotbed of ritual killings, banditry, kidnapping, herder-farmer conflicts, commercial crime, and secessionist activity. in [4]. Similarly, in [9] a compartmental deterministic model is employed by Kambai *et al.,* to examine the dynamics of small scale weapon proliferation and kidnapping in Nigeria. The findings indicate that limiting the importation of light and small arms can reduced kidnapping in a society, but the model failed to relying on technical intelligence instead of ignoring it can assist reduce the amount of kidnapping incidents in Nigeria. Raimundo, *et al.,* in [20] analysed a model of criminal careers with different levels of offence, which describes the interaction of mild, serious offenders and potential criminals.
Furthermore, in Mataru, *et al.,* (2023) in [15] developed a mathematical model for crimes in developing countries with control technique to eradicate level of unemployment-related crimes. The study adopts the epidemiological model concepts on model formulation and model analysis while considering unemployment as main driver of crime. Also, [18] developed a mathematical model to examine the dynamics of terrorism and likely counter-terrorism techniques that can moderate terrorism in a given population. The resulting made forecast the terrorism dynamics like Boko Haram and some related criminals in Nigeria. Moreover, Mohammad and Roslan, (2017) in [17] built a crime model using dynamical approach to examine the spread of the crime system in a population. The study revealed that bifurcation

analysis shows the number of criminals active that imprisoned is increases as well as parameter values increases in the model.

Therefore, this paper aims to extend the work of Wang and Bu in [25] by incorporating technological intelligence for control method together with the kidnapper compartment. This made the use of technology as a study control parameter necessary. The article is structured as follows: Section 1 provides background information of the paper. The introduction of the criminal mathematical model was covered in section 2. The stability equilibrium with no individual terrorist, kidnapper, and informant compartments is found in subsection 2. Next generation approach is also used to calculate the basic reproduction number. The globally stability analysis is asymptotically stable for $R_0 < 1$ and the equilibrium point, the endemic equilibrium, is globally asymptotically stable for $R_0 > 1$, as demonstrated by the Lyapunov functions and sensitivity analysis, also in subsection 2. The analytical results are supported by numerical simulations presented in section 3. Both the discussion of the results and conclusion are in sections 4,5 respectively. Lastly, acknowledgement and suggestion for future study.

2. Formulation of the Model and Description

The use of technological intelligence model in solving terrorism in Nigeria was developed, and the corresponding diagram shown in Figure 1;

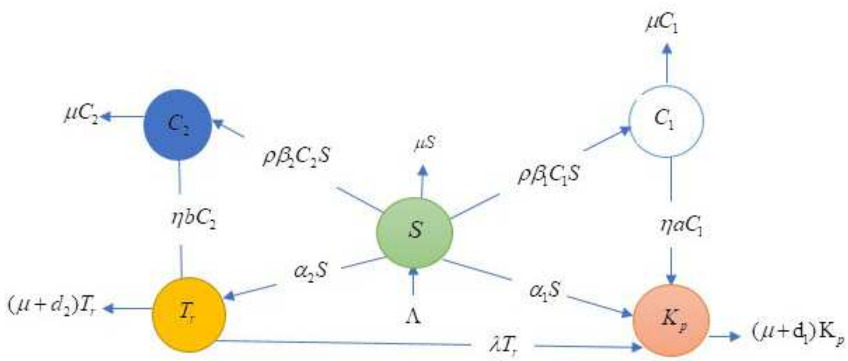

FIGURE 1. flow network of crime dynamics in a population.

TABLE 1. Description of Variables.

Variables	**Description**
$S(t)$	The Susceptible group
$T_r(t)$	The Terrorist group
$K_p(t)$	The Kidnapper group
$C_1(t)$	The Kidnapper informant group
$C_2(t)$	The Terrorist informant group

Table 2. Description of Parameters.

Parameters	Description
ρ	The transmission rate of both terrorist and kidnapper informant groups, respectively
β_1, β_2	Contact rates with the criminal individuals
a	The activities rate of kidnapper informant to kidnapper group
b	The activities rate of the terrorist informant to the terrorist group
α_1, α_2	The rates at which other criminals join terrorist, kidnapper groups
λ	The rate at which terrorist individuals turn to kidnapper
d_1	The removal rate of kidnapper individuals
d_2	The removal rate of terrorist individuals
η	The proportion rates at which UAV monitor terrorist and kidnapper groups, respectively
Λ	The recruitment rate per-capital

The total population $N(t)$ divided into five compartments with those who are susceptible to crime groups are $S(t)$, those who are terrorist groups $T_r(t)$, those who are kidnapper groups $K_p(t)$, those who are kidnapper informant group $C_1(t)$ and those who are terrorist informant group $C_2(t)$ at time $t \geq 0$. Assume that there is a positive recruitment Λ into the susceptible group by immigration. From this group, other criminals proportion join both the terrorist and the kidnapper group, respectively, with parameters α_1 and α_2. The study assumes that the mortality rates μ is for all humans in times. Susceptible individuals will have a contact rate with the terrorist individuals at rate β_1 also with kidnapper individuals at rate β_2 and with criminal transmission probability ρ per contact and moved to the terrorist informant and kidnapper informant. From these individuals, a, b progressed at both terrorist and kidnapper compartments and reduced the activities rate of both terrorist and kidnapper groups with η. The criminal individuals get knowledge about the effect of crime in jail and move to the detention facilities, while d_1, d_2. are the removal rates of a criminal through technology. It also assumes that $\frac{1}{\eta}$ is a fraction with an interval [1,0] that enters both terrorist and kidnapper groups, respectively. It assumes that η is the UAV technique used to monitor criminals along the borderline and finally, all the parameters listed in Table 1, 3, and 4 are well described.

The differential equations corresponding to the flow diagram for the transmission dynamics of crime given in Figure 1 are as follows;

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda - (\rho\beta_1 C_1 + \rho\beta_2 C_2)S - (\alpha_1 + \alpha_2 + \mu)S,\\
\frac{dC_1}{dt} &= \rho\beta_1 C_1 S - (\eta a + \mu)C_1,\\
\frac{dC_2}{dt} &= \rho\beta_2 C_2 S - (\eta b + \mu)C_2,\\
\frac{dT_r}{dt} &= \alpha_2 S + \eta b C_2 - (\lambda + d_2 + \mu)T_r,\\
\frac{dK_p}{dt} &= \alpha_1 S + \eta a C_1 + \lambda T_r - (d_1 + \mu)K_p.
\end{aligned}\right\}. \tag{2.1}$$

2.1. Basic Properties.

2.1.1. ***Invariant Region***. It is necessary to prove that all solutions of system (2.1) above with positive initial data will stay positive for all times $t > 0$. The following lemma will establish this.

Theorem 2.1. *All feasible solution S(t), $C_1(t)$, $C_2(t)$, $T_r(t)$, and $K_p(t)$ of system Equation (1) are bounded by the region.*

$$\Omega = \left\{ S, C_1, C_2, T_r, K_p) \epsilon \Re^5_+ : S + C_1 + C_2 + T_r + K_p \leq \frac{\Lambda}{\alpha_1 + \alpha_2 + \mu} \right\} \tag{2.2}$$

Proof. From the system Equation (2.1) □

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dC_1}{dt} + \frac{dC_2}{dt} + \frac{dT_r}{dt} + \frac{dK_p}{dt}. \tag{2.3}$$

Here, it is necessary to note that in the absence of the crimes,

$$\frac{dN}{dt} = \Lambda - \mu N, \tag{2.4}$$

$$\frac{dN(t)}{dt} = \Lambda - \mu N(t), \tag{2.5}$$

and it follows that

$$N(t) \leq N(0) \exp -\mu t, \tag{2.6}$$

where N(0) is the initial population; thus,

$$\lim_{t\to+\infty} \sup N(t) \leq \frac{\Lambda}{\alpha_1 + \alpha_2 + \mu}, \tag{2.7}$$

hence,

$$S + C_1 + C_2 + T_r + K_p \leq \frac{\Lambda}{\alpha_1 + \alpha_2 + \mu}. \tag{2.8}$$

Hence, for the analysis of system (2.1), we get the region given by the set

$$\Omega = \left\{ (S, C_1, C_2, T_r, K_p) \epsilon \Re^5_+ : S + C_1 + C_2 + T_r + K_p \leq \frac{\Lambda}{\alpha_1 + \alpha_2 + \mu} \right\}, \tag{2.9}$$

which is a positively invariant set. Thus, the model analysis restrict to the region Ω. (where the model shows criminals behaviour in nature)

2.2. Positivity of Solutions of the Model.

Theorem 2.2. *If $S(0) > 0$, $C_1 \geq 0$, $C_2 \geq 0$, $T_r \geq 0$, $K_p \geq 0$ then the solution of system Equation (2.1) S, C_1, C_2, T_r, and K_p are positive for all $t > 0$.*

Proof. From the first equation of system (2.1), gives □

$$\frac{dS}{dt} \geq \Lambda - [\alpha_1 + \alpha_2 + \mu] S, \tag{2.10}$$

gives

$$S(t) \geq \frac{\Lambda}{\alpha_1 + \alpha_2 + \mu} + S(0) \exp - (\alpha_1 + \alpha_2 + \mu) t \geq 0. \tag{2.11}$$

So, the solution S(t) is positive. Similarly, from the second equation of system (2.1), thus

$$C_1(t) \geq C_1(0) \exp - (b + \mu) t, \geq 0. \tag{2.12}$$

also similarly, from the third, fourth, and fifth equations of system (2.1), gives

$$C_2(t) \geq C_2(0) \exp - (a + \mu) t \geq 0. \tag{2.13}$$

$$T_r(t) \geq T_r(0) \exp - (\lambda + \delta_2 + \mu) t \geq 0. \tag{2.14}$$

$$K_p(t) \geq K_p(0)\exp - (\delta_1 + \mu)\, t. \geq 0. \tag{2.15}$$

Therefore, $S(t) > 0$, $C_1(t) > 0$, $C_2(t) > 0$, $T_r(t) > 0$, and $K_p(t) > 0\ \forall$ t ≥ 0, and this completes the proof.

2.3. **Equilibria and Stability Analysis.** In this model, there are two equilibrium points: the crime-free and the crime-present equilibrium points. The equilibrium points are found by setting the right-hand side of Equation (2.1) equal to zero. The crime-free equilibrium $E^0(\frac{\Lambda}{\alpha_1+\alpha_2+\mu}, 0, 0, 0, 0)$ is achieved in the absence of crime ($C_1 = C_2 = T_r = K_p = 0$). The crime-present equilibrium at $E^*(S^*, C_1^*, C_2^*, T_r^*, K_p^*)$ is achieved when crime exist $S \neq 0$, $C_1 \neq 0$, $C_2 \neq 0$, $T_r \neq 0$, and $K_p \neq 0$.
where;

$$\begin{aligned} C_1 &= \tfrac{a\eta\,\mu - a\eta\,\alpha_1 - a\eta\,\alpha_2 + \rho\,\Lambda\,\beta_1 - \mu^2 - \mu\,\alpha_1 - \mu\,\alpha_2}{(a\eta+\mu)\rho\,\beta_1}, \\ C_2 &= \tfrac{\mu^2 - (-b\eta - \alpha_1 - \alpha_2)\mu + b(\alpha_1+\alpha_2)\eta + \rho\,\Lambda\,\beta_2}{(b\eta+\mu)\rho\,\beta_2}, \\ K_p &= \tfrac{\Lambda\,(\mu\,\alpha_1 + (\lambda+d_2)\alpha_1 + \lambda\,\alpha_2)}{(\mu+\alpha_1+\alpha_2)(\mu+\lambda+d_2)(d_1+\mu)}, \\ S &= \tfrac{\Lambda}{\mu+\alpha_1+\alpha_2}\ S = \tfrac{a\eta+\mu}{\rho\,\beta_1},\ S = \tfrac{b\eta+\mu}{\rho\,\beta_2}, \\ T_r &= \tfrac{\Lambda\,\alpha_2}{(\mu+\alpha_1+\alpha_2)(\mu+\lambda+d_2)}. \end{aligned} \tag{2.16}$$

2.4. **Basic Reproduction Number.** The next-generation matrix method can determine the basic reproduction number R_0. The following basic reproduction numbers are obtained using the idea in [22, 23] as;

$$F = \begin{bmatrix} \rho\,\beta_1 S_0 & 0 & 0 & 0 \\ 0 & \beta_2\rho\,S_0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and } V = \begin{bmatrix} a\eta+\mu & 0 & 0 & 0 \\ 0 & b\eta+\mu & 0 & 0 \\ 0 & -b\eta & \lambda+d_2+\mu & 0 \\ a\eta & 0 & \lambda & -d_1-\mu \end{bmatrix},$$

thus;

$$V^{-1} = \begin{bmatrix} (a\eta+\mu)^{-1} & 0 & 0 & 0 \\ 0 & (b\eta+\mu)^{-1} & 0 & 0 \\ 0 & \frac{b\eta}{(b\eta+\mu)(\lambda+d_2+\mu)} & (\lambda+d_2+\mu)^{-1} & 0 \\ \frac{a\eta}{(a\eta+\mu)(d_1+\mu)} & \frac{\lambda\,\eta\,b}{(b\eta+\mu)(\lambda+d_2+\mu)(d_1+\mu)} & \frac{\lambda}{(\lambda+d_2+\mu)(d_1+\mu)} & -(d_1+\mu)^{-1} \end{bmatrix},$$

where $\mathit{\Gamma} FV^{-1}$ is given as a spectral radio and obtained;

$$\begin{bmatrix} \frac{\rho\,\beta_1 S_0}{a\eta+\mu} & 0 & 0 & 0 \\ 0 & \frac{\beta_2\rho\,S_0}{b\eta+\mu} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Subsequently, The characteristic equation of the jacobian matrix and the basic reproduction number R_0 obtained as follows;

$$R_0^{K_p} = \frac{\rho\,\beta_1\,S_0}{(\eta a+\mu)}, \qquad R_0^{T_r} = \frac{\rho\,\beta_2\,S_0}{(\eta b+\mu)}. \tag{2.17}$$

2.5. **Local Stability Analysis of Crime-Free Equilibrium.** The stability behaviour of equilibria E^0 and E^* can be analysed following the idea of Van den Driessche and Watmough in [22] the crime-free equilibrium $E^0(\frac{\Lambda}{\alpha_1+\alpha_2+\mu}, 0, 0, 0, 0)$ of system (2.1) is asymptotically stable iff $R_0 < 1$ and unstable iff $R_0 > 1$.
The Jacobian matrix at E^0 is given by

$$J(E^0) = \begin{bmatrix} K_1 & -\rho\,\beta_1\,S & -\rho\,\beta_2\,S & 0 & 0 \\ C_1\,\rho\,\beta_1 & K_2 & 0 & 0 & 0 \\ C_2\,\rho\,\beta_2 & 0 & K_3 & 0 & 0 \\ \alpha_2 & 0 & b\eta & -\lambda-\mu-\delta_2 & 0 \\ \alpha_1 & a\eta & 0 & \lambda & -\delta_1-\mu \end{bmatrix}, \tag{2.18}$$

where $K_1 = -\rho\,\beta_1\,C_1 - \rho\,\beta_2\,C_2 - \alpha_1 - \alpha_2 - \mu,\ \ K_2 = \rho\,\beta_1\,S - a\eta - \mu\ \ K_3 = \rho\,\beta_2\,S - b\eta - \mu.$
The Jacobian matrix for the crime-free equilibrium is given as;

$$J(E^0) = \begin{bmatrix} -\mu-\alpha_1-\alpha_2 & -\rho\,\beta_1\,S_0 & -\rho\,\beta_2\,S_0 & 0 & 0 \\ 0 & \rho\,\beta_1\,S_0 - c - \mu & 0 & 0 & 0 \\ 0 & 0 & \rho\,\beta_2\,S_0 - c - \mu & 0 & 0 \\ \alpha_2 & 0 & c & -\lambda-\mu & 0 \\ \alpha_1 & c & 0 & \lambda & -\mu \end{bmatrix}, \tag{2.19}$$

where $S_0 = \frac{\Lambda}{\alpha_1+\alpha_2+\mu} = N$, The determinant of this matrix is given by $\det\big(J(E^0) - \lambda I_5\big) = 0$ where I_5 is a square identity matrix of order 5.
Thus, eigenvalues of the determinant $J(E^0)$ are;

$$\begin{array}{ll} \lambda_1 = -\left(\alpha_1+\alpha_2+\mu\right), \qquad \lambda_2 = -\left(\mu+\delta_1\right), \qquad \lambda_3 = -\left(\lambda+\mu+\delta_2\right), \\ \lambda_4 = -\left(b\eta+\mu-\rho\,\beta_2\,S_0\right), \quad \lambda_5 = -\left(a\eta+\mu-\rho\,\beta_2\,S_0\right). \end{array} \tag{2.20}$$

Also, λ_1, λ_2, λ_3, are clearly real and negative. Now, crime free equilibrium, E^0 is stable if $\lambda_4 < 0$, $\lambda_5 < 0$. Both implies

$$\lambda_4 = -b\eta - \mu + \rho\beta_2 S_0 < 0 \ \Rightarrow\ \rho\beta_2 S_0 < b\eta + \mu \ \Rightarrow\ \frac{\rho\beta_2 S_0}{b\eta+\mu} \ \Rightarrow\ R_0^{T_r} < 1,$$

$$\lambda_5 = -a\eta - \mu + \rho\beta_1 S_0 < 0 \ \Rightarrow\ \rho\beta_1 S_0 < a\eta + \mu \ \Rightarrow\ \frac{\rho\beta_1 S_0}{a\eta+\mu} \ \Rightarrow\ R_0^{K_p} < 1.$$

Therefore, all the eigenvalues of the determinant $J(E^0)$ are clearly real and negative if $R_0^{K_p} < 1$ and $R_0^{T_r} < 1$. This conclude that the crime-free equilibrium is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

2.6. **Global Stability Analysis of Crime-Free Equilibrium.** To show the system (2.1) is globally asymptotically stable, it most certified the use of Lyapunov function theory. First, the global stability of the crime-free equilibrium E^0 is stable when $R_0 \leq 1$.

Theorem 2.3. *The crime-free equilibrium E^0 of the system (2.1) is globally asymptotically stable (GAS) in Ω if $R_0 \leq 1$ and unstable if $R_0 > 1$.*

Proof. Let $V(C_1, C_2)$ be a Lyapunov function for some non-negative values of h_1 and h_2 □

$$V = h_1 C_1 + h_2 C_2, \tag{2.21}$$

gives the time derivative of V

$$\begin{aligned}\frac{dV}{dt} &= h_1\frac{dC_1}{dt} + h_2\frac{dC_2}{dt}\\ &= h_1\left[\rho\beta_1 S - (\eta a+\mu)\right]C_1 + h_2\left[\rho\beta_2 S - (\eta b+\mu)\right]C_2\\ &= h_1(\eta a+\mu)\left[\frac{\rho\beta_1 S}{(\eta a+\mu)} - 1\right]C_1 + h_2(\eta b+\mu)\left[\frac{\rho\beta_2 S}{(\eta b+\mu)} - 1\right]C_2\\ &\leq \left(R_0^{K_p} - 1\right)C_1 + \left(R_0^{T_r} - 1\right)C_2,\end{aligned}$$

where;
$h_1 = \dfrac{1}{(\eta a+\mu)}$ and $h_2 = \dfrac{1}{(\eta b+\mu)}$, which also can be recall as;
$R_0^{K_p} = \frac{\rho\,\beta_1\,S_0}{(\eta a+\mu)}$ and $R_0^{T_r} = \frac{\rho\,\beta_2\,S_0}{(\eta b+\mu)}$. Therefore, $\left(\dfrac{dV}{dt}\right) \leq 0$ if and only if $R_0 \leq 1$.
Hence E^0 is globally asymptotically stable.

Hence, by Lasalle invariance principle, E^0 is global asymptotic stable with respect to the invariant set Ω. The idea can be seen in [11, 13, 14] for the proof and applications of the notion of asymptotic stability with respect to invariant sets.

2.7. **Local Stability Analysis of the Crime Endemic Equilibrium.** The Jacobian matrix approach is used to justified the local stability of the crime endemic equilibrium.

Theorem 2.4. *If E^* is locally asymptotically stable and $R_0 > 1$ in the model represents the endemic stability equilibrium, then there is crime in the population.*

Proof. Based on the signs of the eigenvalues of the Jacobian matrix obtained at the crime endemic equilibrium E^*, the local stability analysis of the crime endemic equilibrium E^* is established. At E^*, the model's Jacobian matrix can now be found as follows:

$$J(E^*) = \begin{bmatrix} -\psi_1 & -\beta_1\rho S^* & -\beta_2\rho S^* & 0 & 0\\ \beta_1\rho C_1^* & \psi_2 & 0 & 0 & 0\\ \beta_2\rho C_2^* & 0 & \psi_3 & 0 & 0\\ \alpha_2 & 0 & \eta b & -\psi_4 & 0\\ \alpha_1 & \eta a & 0 & \lambda & -\psi_5 \end{bmatrix},$$

where;

$$\left.\begin{aligned}\psi_1 &= -(\beta_1\rho C_1^* + \beta_2\rho C_2^* + \mu + \alpha_1 + \alpha_2),\\ \psi_2 &= \beta_1\rho S^* - \mu - \eta a,\\ \psi_3 &= \beta_2\rho S^* - \mu - \eta b,\\ \psi_4 &= -\lambda - d_2 - \mu,\\ \psi_5 &= -d_1 - \mu.\end{aligned}\right\}. \tag{2.22}$$

Therefore, by using the idea of matrix form of echelon reduction or elimination method, starting eliminations from right columns of zeros to rows by repeating the operations twice and obtained;

$$J(E^*) = \begin{bmatrix} -\psi_1 & -\beta_1\rho S^* & -\beta_2\rho S^*\\ \rho C_1^*\beta_1 & \psi_2 & 0\\ \rho C_2^*\beta_2 & 0 & \psi_3 \end{bmatrix},$$

Subsequently, the jacobian matrix of local stability analysis of endemic crime equilibrium resolved to 3×3 matrix. The characteristic polynomial is obtained with the help of mathematical software called Maple20, which gives;

$$-\lambda^3 + (-\psi_1 + \psi_2 + \psi_3)\lambda^2 + \left((\psi_1 - \psi_3)\psi_2 + \psi_1\psi_3 - \rho S^* \left(\rho C_1^* {\beta_1}^2 + \rho C_2^* {\beta_2}^2\right)\right)\lambda$$
$$+ \left(\rho S^* \rho C_2^* {\beta_2}^2 - \psi_1\psi_3\right)\psi_2 + \rho C_1^* {\beta_1}^2 \rho S^* \psi_3.$$

Collecting the coefficient of the eigenvalues λ and characteristic polynomial gives;

$$a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0. \tag{2.23}$$

Where;
$a_3 = 1$,
$a_2 = (-\psi_1 + \psi_2 + \psi_3)$,
$a_1 = (\psi_1 - \psi_3)\psi_2 + \psi_1\psi_3 - \rho S\left(\rho C_1 {\beta_1}^2 + \rho C_2 {\beta_2}^2\right)$,
$a_0 = \rho C_1^* {\beta_1}^2 \rho S^* \psi_3 + \left(\rho S^* \rho C_2^* {\beta_2}^2 - \psi_1\psi_3\right)\psi_2$.

Routh-Hurwitz criteria are used to analysed the characteristic polynomial of (2.23). The characteristic polynomial's real positive coefficients are $a_3; a_2; a_1; a_0$.

where $a_1 > 0, a_2 > 0, a_3 > 0, a_4 > 0, a_5 > 0, a_1a_2a_3 > a_3^2 + a_1^2 a_4$, and $(a_1a_4 - a_5)(a_1a_2a_3 - a_3^2 + a_1^2a_4) > a_5(a_1a_2 - a_3)^2 + a_1a_5^2$ for $R_0 > 1$, which satisfied the above assertion, then endemic equilibrium exists. Consequently, all necessary conditions have been satisfied for the stability analysis of the endemic criminal equilibrium point. Thus, all of the characteristic polynomial's eigenvalues are negative according to the Routh-Hurwitz criteria in [16]. Therefore, the endemic equilibrium of crime E^* is locally asymptotically stable iff $R_0 > 1$ and this complete the proved. □

2.8. **Sensitivity Analysis.** This section focuses on sensitivity analysis of the basic reproduction number R_0. To determine the parameters that are significantly impact the model's threshold ratio R_0. Such parameters must deserve the attention of management and control strategies. Here, the normalised forward sensitivity index method (elasticity index) and it is defined as the ratio of the relative change of R_0 concerning relative variation in a parameter as follows

$$\Upsilon_\ell^{R_0} = \frac{\ell}{R_0} \times \frac{\partial R_0}{\partial \ell},$$

Since the basic reproduction number for crimes associated with Kidnapping and Terrorism is R_0. The sensitivity analysis of R_0 to each parameter will be evaluated.

$$\frac{\partial R_0}{\partial \beta_2} = \frac{\Lambda\rho}{(b\eta + \mu)(\alpha_1 + \alpha_2 + \mu)} > 0, \quad \frac{\partial R_0}{\partial \Lambda} = \frac{\beta_2\rho}{(b\eta + \mu)(\alpha_1 + \alpha_2 + \mu)} > 0, \tag{2.24}$$
$$\frac{\partial R_0}{\partial \rho} = \frac{\beta_2\Lambda}{(b\eta + \mu)(\alpha_1 + \alpha_2 + \mu)} > 0, \quad \frac{\partial R_0}{\partial \alpha_1} = -\frac{\beta_2\Lambda\rho}{(b\eta + \mu)(\alpha_1 + \alpha_2 + \mu)^2} < 0, \tag{2.25}$$
$$\frac{\partial R_0}{\partial \alpha_2} = -\frac{\beta_2\Lambda\rho}{(b\eta + \mu)(\alpha_1 + \alpha_2 + \mu)^2} < 0, \quad \frac{\partial R_0}{\partial \eta} = -\frac{\beta_2\Lambda\rho b}{(b\eta + \mu)^2(\alpha_1 + \alpha_2 + \mu)} < 0, \tag{2.26}$$
$$\frac{\partial R_0}{\partial \mu} = -\frac{\beta_2\Lambda\rho(b\eta + \alpha_1 + \alpha_2 + 2\mu)}{(b\eta + \mu)^2(\alpha_1 + \alpha_2 + \mu)^2} < 0, \quad \frac{\partial R_0}{\partial b} = -\frac{\beta_2\Lambda\rho\eta}{(b\eta + \mu)^2(\alpha_1 + \alpha_2 + \mu)} < 0, \tag{2.27}$$
$$\frac{\partial R_0}{\partial \beta_1} = \frac{\Lambda\rho}{(a\eta + \mu)(\alpha_1 + \alpha_2 + \mu)} > 0, \quad \frac{\partial R_0}{\partial a} = -\frac{\beta_1\Lambda\rho\eta}{(b\eta + \mu)^2(\alpha_1 + \alpha_2 + \mu)} < 0. \tag{2.28}$$

(2.29)

thus,

$$\frac{\partial R_0}{\partial \beta_2} = \frac{1.734939759\Lambda}{\alpha_1+\alpha_2+0.00875}, \quad \frac{\partial R_0}{\partial \Lambda} = \frac{0.0006939759036}{\alpha_1+\alpha_2+0.00875}, \tag{2.30}$$

$$\frac{\partial R_0}{\partial \rho} = \frac{0.01927710843\Lambda}{\alpha_1+\alpha_2+0.00875}, \quad \frac{\partial R_0}{\partial b} = -\frac{0.002006677312\Lambda}{\alpha_1+\alpha_2+0.00875}, \tag{2.31}$$

$$\frac{\partial R_0}{\partial \alpha_2} = -2.797584897\Lambda, \quad \frac{\partial R_0}{\partial \alpha_1} = -2.797584897\Lambda, \tag{2.32}$$

$$\frac{\partial R_0}{\partial \eta} = -\frac{0.006688924372\Lambda}{\alpha_1+\alpha_2+0.00875}, \quad \frac{\partial R_0}{\partial \mu} = \frac{(-0.03344462186\alpha_1 - 0.03344462186\alpha_2 - 0.0009866163449)\,\Lambda}{(\alpha_1+\alpha_2+0.00875)^2}, \tag{2.33}$$

$$\frac{\partial R_0}{\partial \beta_1} = \frac{0.9290322581\Lambda}{\alpha_1+\alpha_2+0.00875}, \quad \frac{\partial R_0}{\partial a} = -\frac{0.0001798126951\Lambda}{\alpha_1+\alpha_2+0.00875}. \tag{2.34}$$

Using the explicit expression of the basic reproduction number R_0 with the parameters listed above, the estimated values of the sensitivities indices are listed in the Table 3 as;

TABLE 3. Sensitivity indices of for model.

Parameter (ℓ)	**Sign**	$\Upsilon_\ell^{R_0}$ **for** R_0^{Kp}	**Parameter (ℓ)**	**Sign**	$\Upsilon_\ell^{R_0}$ **for** R_0^{Tr}
Λ	+	1.0000	Λ	+	1.0000
μ	−	0.9772	μ	−	0.7814
α_1	−	0.0635	α_1	−	0.0635
α_2	−	0.3810	α_2	−	0.3810
β_2	+	1.0000	β_1	+	1.0000
η	−	0.5783	η	−	0.7742
b	−	0.5783	a	−	0.7742
ρ	+	1.0000	ρ	+	1.0000

The most sensitive parameters include the proportion of recruitment per capita (Λ), the contact rate of susceptible individuals with terrorist and kidnapper groups (β_1, β_2), transmission probability rates per contact (ρ), and the proportion of informant to terrorist and kidnapper groups (a, b). Additionally, the proportion of UAV technology used to monitor criminal hideouts (η) is also highly sensitive. The important parameter is also the rate at which informants move to join terrorist and kidnapper with (a, b) and UAVs with (η) are used to track criminal activity across the border. Parallel to this, the least sensitive parameters in this paper are the natural death rate (μ) for each population as well as the rates at which terrorists and kidnappers move from C_1, C_2 with (a) and (b) to provide information.
Furthermore, in order to check the recruitment pools and the criminal hideouts, the sensitivity analysis was established to combat security challenges in Nigeria by increasing the negative index parameters (μ, a, b) and decreasing the positive index parameters $(\beta_1, \beta_2, \rho, \eta, \Lambda)$.

3. NUMERICAL ANALYSIS

Numerical analyses were carried out for the spread of crime in Nigeria. The system equation (2.1) was solved by using Maple20 software. The initial values for the population are susceptible group accounts for $S(t) = 10000$, terrorist group account for $T_r = 100$, kidnapper group $K_p = 100$ while the terrorist informant and kidnapper informant account for both $C_1, C_2 = 50$ each respectively. The complete list of the parameter values used in the numerical simulations is tabulated in Table 4.

Table 4. Parameter settings for numerical simulations.

Parameters	**Values**	**Sources**
β_1	0.000001	[25]
β_2	0.000001	”
a	0.86	”
b	0.14	”
α_1	0.008	”
α_2	0.0005	[25]
λ	0.001	Assume
d_1	0.083	Assume
d_2	0.083	Assumed
Λ	0.005	[25]
η	0.06	Estimated
ρ	0.0036	Assume
μ	200	[25]

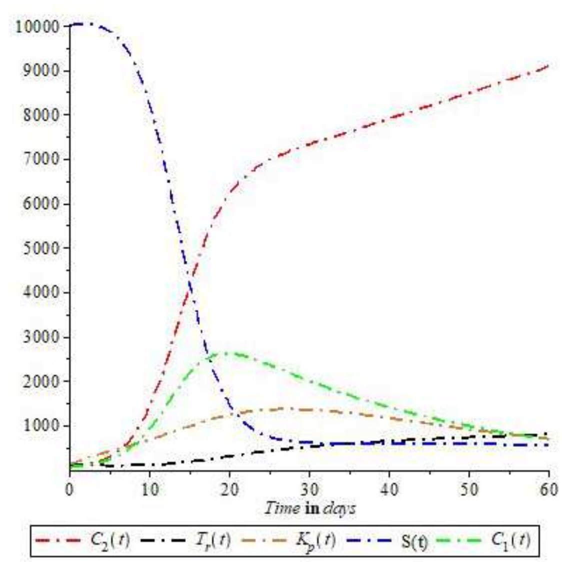

Figure 2. Graph of terrorist, kidnapper and informants with different rates in time.

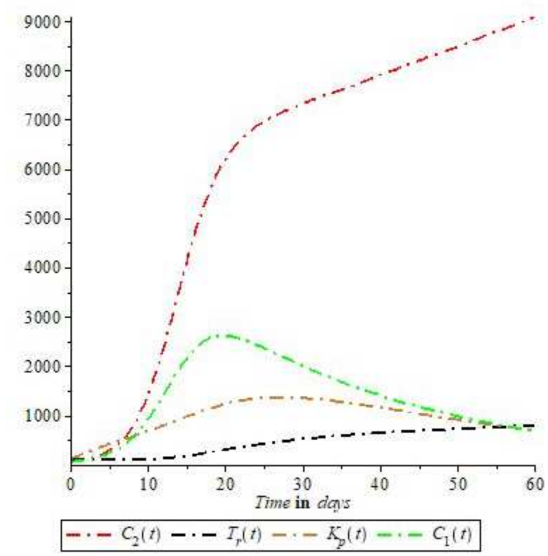

FIGURE 3. Graph of terrorist individuals with UAV control in time.

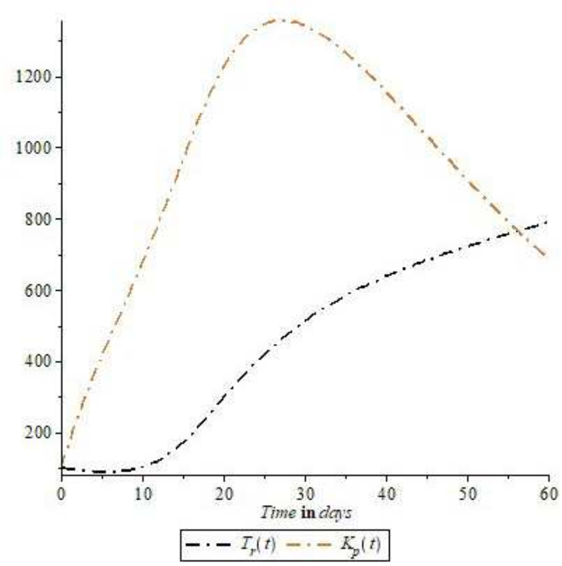

FIGURE 4. Graph of terrorist and kidnapper with different rates in time.

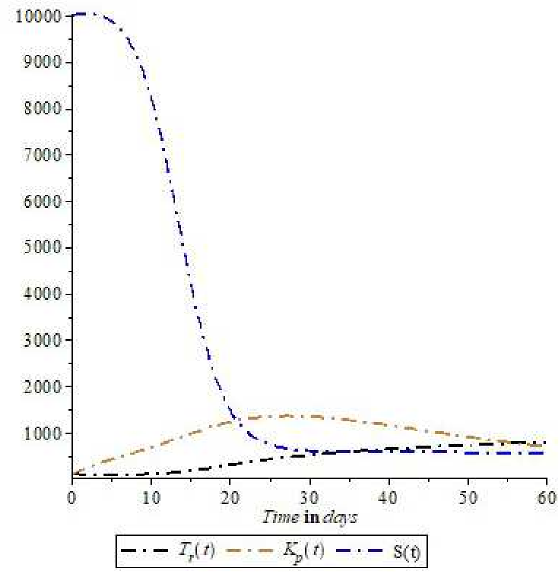

FIGURE 5. Graph of terrorist, kidnapper and susceptible with different rates in time.

4. Results and Discussion

The results of the numerical simulation of model system (1) discuss the control strategies of UAVs to reduce the menace in a population. Considering the simulation results on graphs, the effects are given by β_1, β_2 with different rates on the susceptible, terrorist, kidnapper, terrorist informant and kidnapper informant shown in Figure 1. Also, from Figure 2, the scenario shows the effect of using technology (UAV) to monitor the object along the border and reduce the number of terrorist informants together with that of kidnapper individuals at rate η in time. Similarly, in Figure 3, changing η, a, b on the number of both informant individuals by keeping the other parameters constant reduced the threat in time. It can be deduced from the Graph that, as the value of η increases, the number of informant individuals increases. In Figure 4, the simulation shows the effect of using UAV deployed to monitor the activities of criminals at rates η, α_1 and α_2 in time. It can also be deduced from the Graph that the use of technology at 50% increases and identify the number of criminal individuals and decreases their number in a population.
Finally, from figures 2, 3, 4, and 5, the scenario shows the significant effect of using technology to counter criminal threats. Thus, counter-insurgency must concentrate on reducing the number of individuals that join the informant using Technology strategies to reduce the transition rates within the population.

5. conclusion

A model of security challenges in Nigeria using technological intelligence was formulated and subdivided into five (5) compartments. The sensitivity analysis strategy and UAV technology are incorporated as controls to combat the transmission dynamics of the criminal. The results of the numerical simulations are presented on graphs, and integrated control strategies should be taken to fight against terrorists, kidnappers and other criminal individuals. Finally, this necessitates efforts to carry out research to address security challenges in Nigeria.

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References

[1] G. Adamu and M. O. Ibrahim, *Modelling and Analysis of Counter-Terrorism: Via Drone*, Math. Comput Sci., *3*(3) (2022), 57–75.

[2] G. Adamu and M. O. Ibrahim,. *Elzaki transform method for solving deterministic modelling of terrorism*, Iranian J. Numer. Anal. Optim., *12*(2) (2022), 334–354.

[3] G. Adamu and M. O. Ibrahim, *Mathematical modelling of dynamics behaviour of terrorism and control*, Caspian J. Math. Sci., *9*(1) (2020), 68–85.

[4] T. S. Akinyetun, *Banditry in Nigeria: Insights from situational action and situational crime prevention theories*, conflict trends, *2021*(4) (2021), 32–38.

[5] A. Farrow, *Drone Warfare as a Military Instrument of Counterterrorism Strategy*, Air. Space. Power. J., *28*(4) (2016), 4–23.

[6] Global Terrorism Index, *Measuring the Impact of Terrorism*, Inst. Econ. Peace, 2021.

[7] Global Terrorism Index, *Measuring the Impact of Terrorism*, Inst. Econ. Peace, 2022.

[8] Global Terrorism Index, *Measuring the Impact of Terrorism*, Inst. Econ. Peace, 2023.

[9] S. A. Kambai, M. O. Durojaye, and R. T. Abah, *A mathematical modelling of the impact of proliferation of arms and weapons on Nigeria and control*, Int. J. Math. Analy. Model., *6*(1) (2023), 153–172.

[10] A. B. Okrinya and J. I. Consul, *Mathematical Modelling of Kidnapping with Rehabilitation*, J. Sci. Engi. Res., *5*(8) (2018), 195–20.

[11] J. P. Lassalle, *The stability of dynamical systems*, Socie. Indust. Appl. Math (SIAM), 1976.

[12] O. E. Jonathan, A. J. Olusola, T. C. A. Bernadin, and T. M. Inoussa, *Impacts of crime on socio-economic development*, Mediterranean J. Social Sci., *12*(5) (2021), 71.

[13] M. Y. Li and J. S. Muldowney, *A geometric approach to global-stability problems*, SIAM J. Math. Anal., *27*(4) (1996), 1070-1083.

[14] M. Y. Li and J. S. Muldowney, *On RA Smith's autonomous convergence theorem*, Rocky Mt. J. Math., *25*(1) (1995), 365-378.

[15] B. Mataru, O. J. Abonyo, and D. Malonza, *Mathematical Model for Crimes in Developing Countries with Some Control Strategies*, J. Appl. Math., (2023), 1–14.

[16] D. R. Merkin, *Introduction to the Theory of Stability*, Springer. Sci. & Business Media, New York., (2012).

[17] F. Mohammad and U. A. M. Roslan, *Analysis on the crime model using dynamical approach*, AIP confer. proce., *1870*(1) (2017), 040067.

[18] C. Okoye, O. C. Collins, and G. C. E. Mbah, *Mathematical approach to the analysis of terrorism dynamics*, Secur. J., *33*(3) (2020), 427–438.

[19] F. C. Onuoha, C. Nwangwu, and M. I. Ugwueze, *Counterinsurgency operations of the Nigerian military and Boko Haram insurgency: expounding the viscid manacle*, Secur. J., *33*(3) (2020), 401–426.

[20] S. M. Raimundo, H. M. Yang, F. A. Rubio, D. Greenhalgh, and E. Massad, *Modelling criminal careers of different levels of offence*, Appl. Math. Comput., 453 (2023), 128073.

[21] S. Ryan, *Drones, Aid and Education: The Three Ways to Counter Terrorism*, London, South Bank University, 2019.

[22] P. Van den Driessche and J. Watmough, *Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission*, Math. bios., *180*(1-2) (2002), 29–48.

[23] P. Van den Driessche, *Reproduction numbers of infectious disease models*, Infec. Dise. Modell., *2*(3) (2017), 288–303.

[24] J. I. Walsh, *The effectiveness of drone strikes in counterinsurgency and counter-terrorism campaigns*, Stra. Stud. Inst. and US Army War Coll. Press. (2013).

[25] Y. Wang and F Bu, *Modelling radicalization of terrorism under the influence of multiple ideologies*, American Inst. Math. Sci., *7*(3) (2022), 4833–4850.

[26] J. P. Yaacoub, H. Noura, O. Salman, and A. Chehab, *Security analysis of drones systems: Attacks, limitations, and recommendations*, Internet of Things., (11) (2020), 100218.