Research Paper
Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. 13, No. 1, 2025, pp. 271-281
DOI:10.22034/cmde.2024.57337.2396

Extended hyperbolic function method for the model having cubic-quintic-septimal nonlinearity in weak nonlocal media

Hamood Ur Rehman[1], Ifrah Iqbal[1] , Mostafa Eslami[2,*], Mohammad Mirzazadeh[3], Sajjad A. Jedi Abduridha[4], and Mir Sajjad Hashemi[5]

[1]Department of Mathematics, University of Okara, Okara, Pakistan.

[2]Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar, Iran.

[3]Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, Rudsar-Vajargah, Iran.

[4]Department of Mathematics, Faculty of Basic Education, University of Kufa,Najaf, Iraq.

[5]Department of Mathematics, Basic Science Faculty, University of Bonab, Bonab, Iran.

Abstract

Optical solitons are self-trapped light beams that maintain their shape and transverse dimension during propagation. This paper investigates the propagation of solitons in an optical material with a weak nonlocal media, modeled by a cubic-quintic-septimal nonlinearity. The dynamics of solitons in optical waveguides are described by the cubic nonlinear Schrödinger equation and its extensions. This equation model applies to both the spatial propagation of beams and the temporal propagation of pulses in a medium exhibiting cubic nonlinearity. The novelty of the paper lies in the application of the extended hyperbolic function method to derive soliton solutions in optical materials with weak nonlocal media in the form of the periodic, bright, kink, and singular type solitons. The obtained solutions provide explicit expressions for the behavior of optical waves in media. These results shed light on the dynamics of nonlinear waves in optical materials and contribute to a better understanding of soliton propagation. The findings contribute to a more comprehensive understanding of the role of nonlocal nonlinearity and time constants in soliton solutions. Our findings provide a better understanding of the dynamics of the nonlinear waves in optical media and have many application for the field of optical communication and signal processing. The role of nonlocal nonlinearity and time constant on soliton solutions is also discussed with the help of graphs.

Keywords. Optical solitons, Nonlinear Schrödinger equation (NLSE), Solitons, Nonlocal nonlinearity.

2010 Mathematics Subject Classification. 65L05, 34K06, 34K28.

1. Introduction

Optical solitons have attracted significant attention due to their potential applications in various fields of optics and photonics [5, 24, 31, 47]. Various models have been formulated to explore this phenomenon, encompassing equations such as the Biswas-Arshed equation [32], perturbed NLSE [15], Klein-Gordon equation [27], bi-harmonic coupled NLSE [18], (2+1)-dimensional shallow water wave model [6], complex Ginzburg Landau-equation [7], fractional Jaulent Miodek [43], generalized Kadomestev-Petviashili equation [13], (2+1)-dimensional variable-coefficient Sawada-Kotera system [25], (2+1)-dimensional Date-Jimbo-Kashiwara-Miwa equation [20], generalized nonlinear fractional integro-differential equation [3], (4+ 1)-dimensional Fokas and (2+ 1)-dimensional breaking soliton equations [26], Kraenkel-Manna-Merle (KMM) equation [49], and KP hierarchy equations [19]. The investigation of solitons has yielded numerous exciting discoveries and advancements, including the development of new techniques for generating and controlling the localized light beams [22, 23, 34]. Moreover, researchers have explored the properties and behaviors of solitons in different types of nonlinear media, opening up new avenues for potential applications [1]. For instance, solitons in photo-refractive materials have been shown to exhibit interesting dynamics and have potential uses in image

Received: 01 July 2023 ; Accepted: 27 March 2024.
∗ Corresponding author. Email: mostafa.eslami@umz.ac.ir .

processing and holography [11, 12]. As researchers continue to explore the possibilities offered by optical solitons, we can expect to see further progress and advancement in this exciting field.

The dynamics of solitons in optical theory are governed by the cubic NLSE and its extension [2, 4, 35, 37, 46, 53]. This equation is valid for both spatially propagating beams and temporally pulses in media with cubic nonlinearity [8]. Interestingly, solitons can arise in a wide range of nonlinear systems, from Kerr-type [45] to liquid crystal [30], as well as a nonlocal cubic response in media [9].

Various studies have explored the stable propagation of solitons in media with modification of cubic-quintic nonlinearities [14, 50]. The septimal nonlinearity including cubic and quintic terms have important role in the formation of solitons. Recently, bright solitons were observed in metal colloids having quintic-septimal nonlinearity, modeled by NLSE with dissipative terms [38]. The management of nonlinear system, achieved by adjusting the strength of nonlinear terms can enable a controlled interplay between different nonlinear terms, leading to the enhancement or suppression of specific high-order nonlinearities [39, 40]. This technique has been experimentally demonstrated in various media such as metal colloids with a suppressed cubic nonlinearity and a septimal media induced by destructive interplay between cubic and quintic nonlinearities [40].

The propagation of solitons in septimal media is a topic that requires further examination. The highly unstable behavior is expected from the seventh-order nonlinearity, hence, the use of higher-order nonlinearities can lead the collapse of beam. Therefore, there is a need to investigate the cubic-quintic-septimal nonlinear model to supplement past studies and additional possibilities for experiments. This study focuses on analyzing the stable propagation of solitons in media that demonstrate nonlinearities up to the seventh order and is described as [28, 41, 48, 54]

$$\iota \frac{\partial \phi}{\partial t} + \beta \frac{\partial^2 \phi}{\partial x^2} + \delta |\phi|^2 \phi + \mu |\phi|^4 \phi + \lambda |\phi|^6 \phi + \nu \phi \frac{\partial^2 |\phi|^2}{\partial x^2} = 0, \tag{1.1}$$

where ϕ, t and x represent amplitude, propagation distance , and transverse spatial coordinate. The equation includes several coefficients, including β which represents the diffraction coefficient whereas δ, μ, and λ denote the coefficient of cubic, quintic, and sepitmal nonlinearity respectively. Additionally, the term ν is linked to weak nonlocal nonlinearity. In the literature, it has been noted that (1.1) expands upon the equation typically used for analyzing the propagation of beams in cubic-quintic-septic media [42]. This is achieved by adding the effects of nonlocal nonlinearity, represented by the term $\phi \frac{\partial^2 |\phi|^2}{\partial x^2}$.

The governed equation stands as a vital optical model because it emerges under specific conditions, typically in scenarios devoid of the fiber loss. They have found widespread applications in nonlinear optics, plasma physics, biomedicine, and ocean dynamics. In practical optical fibers, the introduction of loss explains the exponential decay of optical pulse power as it propagates through the fiber. This loss disrupts the delicate equilibrium between nonlinearity and dispersion, making the idealized scenario unattainable. This is the reason why the majority of optical fibers are non-uniform in nature. However, it's only when the dispersive and nonlinear effects balance each other that solitons with consistent pulse shapes can form. These equations are commonly employed to describe the transmission characteristics of optical solitons due to their attributes of high bit rates, long-distance propagation, and high capacity. In long-distance transmission, the waveform, amplitude, and velocity of optical solitons remain remarkably stable, making them essential for characterizing the dynamic features of optical pulses. As the primary solutions to the proposed equation, optical solitons are extensively applied in the realm of optical fiber communication. The origin of the nonlinear Kerr law can be traced to the nonlinear responses of light waves within optical fibers, arising from the non-harmonic motion of electrons bound in molecules under the influence of external electric fields. Several studies have delved into the analysis of the Equation (1.1), each employing distinct methods to explore its soliton solutions. In [28], the authors utilized the ansatz technique and identified bright and kink-type solitons. Another approach was taken by [48], where dark solitary waves were exclusively identified using a specialized ansatz.

In contrast to the existing literature, our paper employs the application of the EHFM to scrutinize the Equation (1.1). By applying this method, the set of explicit solutions are obtained that describe the propagation of beams in the medium. This approach has been shown to be effective in solving a wide range of nonlinear partial differential equations (NLPDEs) [10, 17, 33, 52]. These solutions encompass a wide spectrum, including kink, bright, singular, and periodic-singular solitons. The versatility of EHFM showcased in our work is a significant advancement, providing a richer understanding of the dynamics governed by (1.1). It is important to acknowledge that our analysis is conducted within specific parameter ranges. The use of certain conditions in our methods may occasionally pose challenges in finding solutions to the equations.

The remaining paper is structured as follows: Section 2 presents the concrete steps of the method. The method is applied to obtain the solution of proposed model for the construction of light beams in materials with nonlocal nonlinearity in section 3. Section 4 presents the graphical representation of the obtained results and discusses the effects of different parameters on the solutions. Last section summarizes the key findings of the study.

2. Extended hyperbolic function method

In this section, the steps of EHFM [16, 36, 44, 51] are briefly presented.
Assume, the NLPDE

$$G(\phi, \phi_t, \phi_x, \phi_{tt}, \phi_{tx}, ...) = 0, \tag{2.1}$$

where G is polynomial along its derivatives. Now, apply the following transformation on (2.1)

$$\phi(x,t) = \Upsilon(\epsilon)e^{i(kt-\mho x+\theta)}, \quad \epsilon = x - ct, \tag{2.2}$$

where k and $\mho$ and θ are unknown constants while $\Upsilon(\epsilon)$ is the analytic function. This transformation reduces the (2.1) into the following equation written as

$$Y(\Upsilon, \Upsilon', \Upsilon'', ...) = 0. \tag{2.3}$$

Suppose, the solution of (2.3) is

$$\Upsilon(\epsilon) = \sum_{i=0}^{m} \alpha_i \rho^i(\epsilon), \tag{2.4}$$

where $\alpha_i \neq 0$ and m is determined with the aid of homogeneous balancing principle. Now, $\rho(\epsilon)$ indulges two types of equations.
Type 1:

$$\rho'(\epsilon) = \rho(\epsilon)\sqrt{\hbar_1 + \hbar_2\rho^2(\epsilon)}, \quad \hbar_1, \hbar_2 \in R. \tag{2.5}$$

The (2.5) gives the following solutions:
Case 1: When $\hbar_1 > 0,\ \hbar_2 > 0$,

$$\rho_1(\epsilon) = -\sqrt{\frac{\hbar_1}{\hbar_2}} csch\sqrt{\hbar_1}(\epsilon).$$

Case 2: When $\hbar_1 < 0,\ \hbar_2 > 0$,

$$\rho_2(\epsilon) = \sqrt{\frac{-\hbar_1}{\hbar_2}} sec\sqrt{-\hbar_1}(\epsilon).$$

Case 3: When $\hbar_1 > 0,\ \hbar_2 < 0$,

$$\rho_3(\epsilon) = \sqrt{\frac{\hbar_1}{-\hbar_2}} sech\sqrt{\hbar_1}(\epsilon).$$

Case 4: When $\hbar_1 < 0,\ \hbar_2 > 0$,

$$\rho_4(\epsilon) = \sqrt{\frac{-\hbar_1}{\hbar_2}} csc\sqrt{-\hbar_1}(\epsilon).$$

Case 5: When $\hbar_1 < 0,\ \ \hbar_2 > 0,$

$$\rho_5(\epsilon) = cos\sqrt{-\hbar_1}(\epsilon) + \imath sin\sqrt{-\hbar_1}(\epsilon).$$

Case 6: When $\hbar_1 = 0,\ \ \hbar_2 > 0,$

$$\rho_6(\epsilon) = \frac{1}{\sqrt{\hbar_2}(\epsilon)}.$$

Case 7: When $\hbar_1 = 0,\ \ \hbar_2 < 0,$

$$\rho_7(\epsilon) = \frac{1}{\sqrt{-\hbar_2}(\epsilon)}.$$

Type 2:

$$\rho'(\epsilon) = \hbar_1 + \hbar_2\rho^2(\epsilon), \quad \hbar_1, \hbar_2 \in R. \tag{2.6}$$

Case 1: When $\hbar_1\hbar_2 > 0,$

$$\rho_8(\epsilon) = sgn(\hbar_1)\sqrt{\frac{\hbar_1}{\hbar_2}}tan(\sqrt{\hbar_1\hbar_2}(\epsilon)).$$

Case 2: When $\hbar_1\hbar_2 > 0,$

$$\rho_9(\epsilon) = \text{-}sgn(\hbar_1)\sqrt{\frac{\hbar_1}{\hbar_2}}cot(\sqrt{\hbar_1\hbar_2}(\epsilon)).$$

Case 3: When $\hbar_1\hbar_2 < 0,$

$$\rho_{10}(\epsilon) = sgn(\hbar_1)\sqrt{\frac{-\hbar_1}{\hbar_2}}tanh(\sqrt{-\hbar_1\hbar_2}(\epsilon)).$$

Case 4: When $\hbar_1\hbar_2 < 0,$

$$\rho_{11}(\epsilon) = sgn(\hbar_1)\sqrt{\frac{-\hbar_1}{\hbar_2}}coth(\sqrt{-\hbar_1\hbar_2}(\epsilon)).$$

Case 5: When $\hbar_1 = 0,\ \ \hbar_2 > 0,$

$$\rho_{12}(\epsilon) = -\frac{1}{\hbar_2(\epsilon)}.$$

Case 6: When $\hbar_1 < 0,\ \ \hbar_2 = 0,$

$$\rho_{14}(\epsilon) = \hbar_1(\epsilon).$$

At the end, by inserting (2.4) into (2.3) along (2.5) and (2.6), the algebraic system of equations are acquired and values of constants are determined.

3. Application of EHFM

By using the (2.4) in (1.1), the following equation is obtained

$$\beta\Upsilon''(\epsilon) - \left(\beta\mho^2 - 2\nu\Upsilon'(\epsilon)^2 + k\right)\Upsilon(\epsilon) - i(2\beta\mho + c)\Upsilon'(\epsilon) + \delta\Upsilon(\epsilon)^3 + \mu\Upsilon(\epsilon)^5 + \lambda\Upsilon(\epsilon)^7 + 2\nu\Upsilon(\epsilon)^2\Upsilon''(\epsilon) = 0. \tag{3.1}$$

Splitting (3.1), the real part is

$$\beta\Upsilon''(\epsilon) - \left(\beta\mho^2 - 2\nu\Upsilon'(\epsilon)^2 + k\right)\Upsilon(\epsilon) + \delta\Upsilon(\epsilon)^3 + \mu\Upsilon(\epsilon)^5 + \lambda\Upsilon(\epsilon)^7 + 2\nu\Upsilon(\epsilon)^2\Upsilon''(\epsilon) = 0, \tag{3.2}$$

while the imaginary part is

$$-(2\beta\mho + c)\Upsilon'(\epsilon) = 0. \tag{3.3}$$

By solving the above equation, we get

$$c = -2\beta\mho. \tag{3.4}$$

By using balancing rule in (3.2), we acquire $m{=}\frac{1}{2}$. So, another transformation

$$\Upsilon(\epsilon) = \Theta(\epsilon)^{\frac{1}{2}}, \tag{3.5}$$

is applied on (3.2), which yields the following equation

$$-4\Theta(\epsilon)^2\left(\beta\mho^2 + k - \nu\Theta''(\epsilon)\right) + 2\beta\Theta(\epsilon)\Theta''(\epsilon) - \beta\Theta'(\epsilon)^2 + 4\delta\Theta(\epsilon)^3 + 4\lambda\Theta(\epsilon)^5 + 4\mu\Theta(\epsilon)^4 = 0. \tag{3.6}$$

By the implementation of balancing rule on above equation, we get $m = 1$. Hence, we assume the solution of (3.6) is

$$\Theta(\epsilon) = \alpha_0 + \alpha_1\rho(\epsilon), \quad \alpha_1 \neq 0. \tag{3.7}$$

Type 1
Now, by putting (3.7) and (2.5) into (3.6), the system of equations is attained. By resolving this system, the obtained values of constants are written as below

$$\begin{aligned}
&\alpha_0 = 0,\\
&\alpha_1 = \sqrt{\frac{-2\hbar_2\nu}{\lambda}},\\
&\delta = -\hbar_1\nu,\\
&\mho = \frac{1}{2}\sqrt{\frac{-3\lambda k + 2\mu\hbar_1}{2\mu\nu}},\\
&\beta = \frac{8\mu\nu}{3\lambda}.
\end{aligned}$$

Now, by using these values of constants along solutions of (2.5) in (3.7) and by means of (3.5), the following solutions of (1.1) are obtained
Case 1: When $\hbar_1 > 0,\ \ \hbar_2 > 0,$

$$\phi_1(x,t) = \left(-\sqrt{\frac{-2\hbar_1\nu}{\lambda}}csch\sqrt{\hbar_1}(\epsilon)\right)^{\frac{1}{2}} e^{i(kt-\mho x+\theta)}.$$

Case 2: When $\hbar_1 < 0,\ \ \hbar_2 > 0,$

$$\phi_2(x,t) = \left(\sqrt{\frac{2\hbar_1\nu}{\lambda}}sec\sqrt{-\hbar_1}(\epsilon)\right)^{\frac{1}{2}} e^{i(kt-\mho x+\theta)}.$$

Case 3: When $\hbar_1 > 0,\ \ \hbar_2 < 0,$

$$\phi_3(x,t) = \left(\sqrt{\frac{2\hbar_1\nu}{\lambda}}sech\sqrt{\hbar_1}(\epsilon)\right)^{\frac{1}{2}} e^{i(kt-\mho x+\theta)}.$$

Case 4: When $\hbar_1 < 0,\ \ \hbar_2 > 0,$

$$\phi_4(x,t) = \left(\sqrt{\frac{2\hbar_1\nu}{\lambda}}csc\sqrt{-\hbar_1}(\epsilon)\right)^{\frac{1}{2}} e^{i(kt-\mho x+\theta)}.$$

Type 2:
Similary for type 2, by solving system of equations which is obtained by inserting (3.7) and (2.6) into (3.6), the

following values of constants are obtained

$$\alpha_0 = \alpha_0,$$
$$\alpha_1 = \sqrt{\frac{-\hbar_2\alpha_0}{\hbar_1}},$$
$$\lambda = \frac{2\mu\hbar_1\hbar_2}{\alpha_0^2},$$
$$\mu = -\frac{3\hbar_1\hbar_2\left(8\nu\alpha_0 - \beta\right)}{4\alpha_0^2},$$
$$\mho = \sqrt{\frac{\hbar_1\hbar_2\beta - k}{\beta}},$$
$$\delta = \frac{2\hbar_1\hbar_2(-\beta + 2\nu\alpha_0)}{\alpha_0}.$$

By using these values of constants, the solutions of (1.1) is obtained as like type 1.
Case 1: When $\hbar_1\hbar_2 > 0$,

$$\phi_5(x,t) = \left(\alpha_0 + \left(sgn(\hbar_1)\sqrt{-\alpha_0}tan(\sqrt{\hbar_1\hbar_2}(\epsilon))\right)\right)^{\frac{1}{2}} e^{i(kt-\mho x+\theta)}.$$

Case 2: When $\hbar_1\hbar_2 > 0$,

$$\phi_6(x,t) = \left(\alpha_0 - \left(sgn(\hbar_1)\sqrt{-\alpha_0}cot(\sqrt{\hbar_1\hbar_2}(\epsilon))\right)\right)^{\frac{1}{2}} e^{i(kt-\mho x+\theta)}.$$

Case 3: When $\hbar_1\hbar_2 < 0$,

$$\phi_7(x,t) = \left(\alpha_0 + \left(sgn(\hbar_1)\sqrt{\alpha_0}tanh(\sqrt{-\hbar_1\hbar_2}(\epsilon))\right)\right)^{\frac{1}{2}} e^{i(kt-\mho x+\theta)}.$$

Case 4: When $\hbar_1\hbar_2 < 0$,

$$\phi_8(x,t) = \left(\alpha_0 + \left(sgn(\hbar_1)\sqrt{\alpha_0}coth(\sqrt{-\hbar_1\hbar_2}(\epsilon))\right)\right)^{\frac{1}{2}} e^{i(kt-\mho x+\theta)}.$$

4. Results and Discussion

The primary objective of this study is to introduce comprehensive closed-form solutions for a wide range of NLPDEs. These closed-form solutions are intended to serve as efficient tools, acting as versatile solvers that can greatly benefit mathematicians, engineers, and physicists. The significance of these solutions lies in their ability to elucidate complex phenomena in various applied scientific disciplines. Specifically, in the field of optical fiber propagation, the dynamics of bright and dark solitons are intricately linked to the delicate balance between self-phase modulation and group velocity dispersive effects [29]. The nature of these solutions, whether they manifest as solitons, periodic patterns, or dissipative structures, depending on the specific values of physical parameters present in the dispersion and nonlinear coefficients. This paper is dedicated to the exploration and discovery of diverse solution types, including periodic structures, optical dark solitons, optical bright solitons, and singular optical solutions. This section gives graphical representation to visualize the behavior of soliton waves in the cubic-quintic-septimal equation having nonlocal non-linearity. By using the EHFM, the 3D graphs with projection and 2D graphs are generated to provide a clear and intuitive representation of the dynamics of solitons waves under different parameters values. The effect of changing parameters values on the behavior of solitons waves is demonstrated through 2D plots to show the dependence of wave's amplitude and speed on the parameters.

Figure 1 shows the the graph of $\phi_2(x,t)$ under the values of the $c = 0.5$, $h_1 = -1$, $\theta = 1$, $\lambda = 0.1$, $k = -0.98$, $\mho = 0.8$, $\delta = 0.5$ and $h_2 = 1$. The clear periodicity can be observed in the graph. The graph of $\phi_3(x,t)$ exhibits a localized and stable waveform with a high intensity feature, known as "bright soliton" for the values of $c = 0.5$, $h_1 = 1$,

$\theta = 1$, $\lambda = 0.1$, $k = -0.98$ $\nu = 1$, $\mho = 0.8$, $\delta = 0.5$ and $h_2 = -1$. The cubic-quintic-septimal equation having nonlocal nonlinearity has additional coefficient that determines the nonlocal behavior of the equation. By changing the value of ν, it is observed that the soliton solutions can be adjusted to move up or down as shown in 2D graphs of Figures 1 and 2.

Figures 3 and 4 present the graphical representation for $\phi_7(x,t)$ and $\phi_8(x,t)$ respectively. The considered parameters are $c = 0.5$, $h_1 = -1$, $\theta = 1$, $\lambda = 0.1$, $k = -0.98$, $\mho = 0.8$,$\delta = 0.5$, $a_0 = 1$, and $h_2 = 1$. The $\phi_7(x,t)$ and $\phi_8(x,t)$ display the kink soliton and singular soliton which show sharp change and discontinuity in the wave form respectively. By changing the values of constant of time in the traveling wave transformation, the soliton wave move left or right. Hence, the following outcomes are drawn:

• The up and down movement of soliton wave refer to a change in amplitude of the wave. It is observed that by increasing the coefficient of nonlocal nonlinearity leads to an increase in amplitude while decreasing it leads to decrease in amplitude of soliton.

• On the other hand, when the wave moves left or right, it means there is a change in position of soliton. This is due to change of constant of time in traveling wave transformation. Increasing and decreasing of the value leads to rightward and leftward movement respectively.

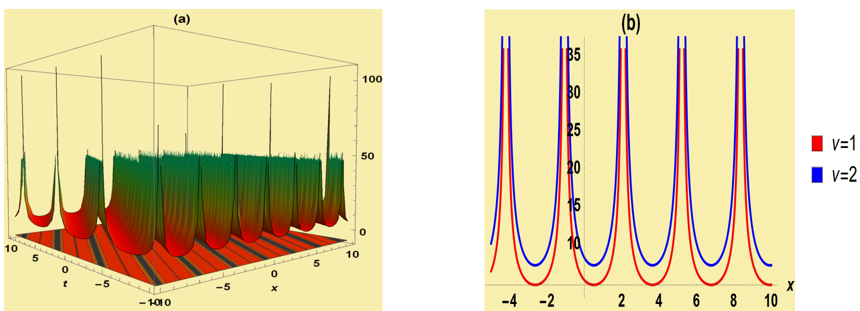

FIGURE 1. Structures of the solution of $\phi_2(x,t)$, describe periodic soliton solutions **(a)** 3D plot with projection within interval $-10 \leq x \leq 10$, and $-10 \leq t \leq 10$ and **(b)** 2D plot at different ν within interval $-5 \leq x \leq 10$ and $t = 1$.

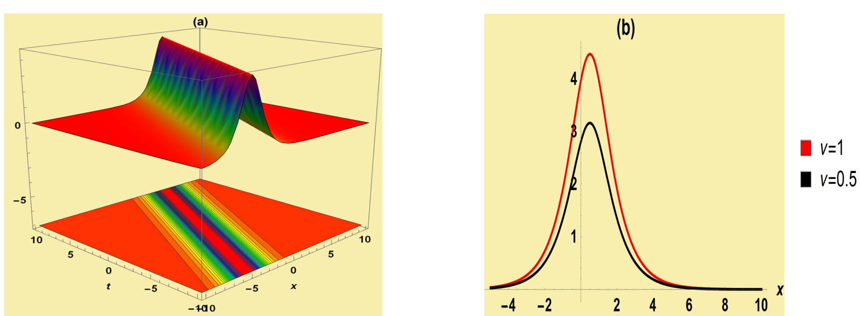

FIGURE 2. Structures of the solution of $\phi_3(x,t)$, describe bright soliton solutions **(a)** 3D plot with projection within $-10 \leq x \leq 10$, and $-10 \leq t \leq 10$ and **(b)** 2D plot at different ν within interval $-5 \leq x \leq 10$ and $t = 1$.

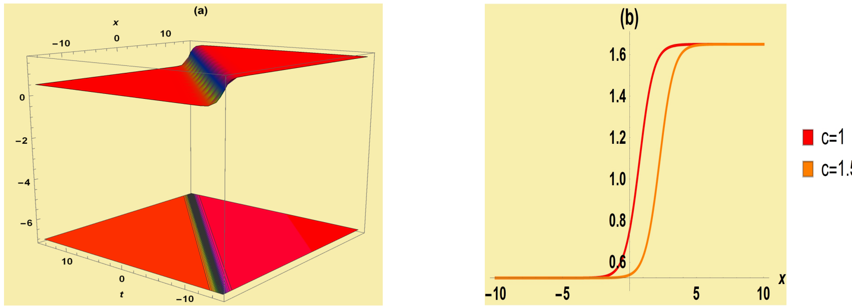

FIGURE 3. Structures of the solution of $\phi_7(x,t)$, describe kink soliton solutions **(a)** 3D plot with projection within $-10 \leq x \leq 10$, and $-10 \leq t \leq 10$ and **(b)** 2D plot at different c within interval $-10 \leq x \leq 10$ and $t = 1$.

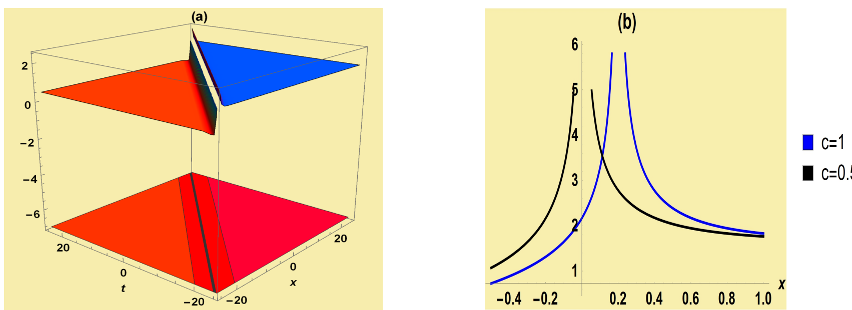

FIGURE 4. Structures of the solution of $\phi_8(x,t)$, describe singular soliton solutions **(a)** 3D plot with projection within $-20 \leq x \leq 20$, and $-20 \leq t \leq 20$ and **(b)** 2D plot at different c within interval $-5 \leq x \leq 10$ and $t = 1$.

5. Conclusion

In this study, we have studied the propagation of beams in materials using (1.1), which is the generalized equation of NLSE that accounts for the effects of nonlocal nonlinearity. To analyze this equation, EHFM is employed to obtain the solutions in the form of bright, periodic, singular, and kink soliton solutions. The obtained results provide a better understanding of the dynamics of light propagation in materials where nonlocal effects play a significant role meta-materials. So, the current work contributes to the development of more comprehensive models for the study of light-matter interactions and provides a direction for future research in this area. The utilization of both 3D graphs with projections and 2D graphs has significantly enhanced our comprehension of the dynamics governing soliton waves across various parameter values. Through these visual representations, we have gained a clear and intuitive insight into how soliton waves behave under distinct parameter settings. The pivotal role of these graphical representations lies in demonstrating the effects induced by changes in parameter values on the behavior of soliton waves. Specifically, the 2D plots effectively illustrate the interdependence between the wave's amplitude and speed with alterations in the parameters. In future investigations, the inclusion of fractional order parameters offers the potential for solving the equation presented in terms of various operators, such as the conformable derivative, M-truncated fractional derivative, beta derivative, and others.

Declarations

Ethical Approval: Not applicable.

Competing interests: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors' contributions: All authors have equal contribution in this work.

Funding: No funding received for this paper.

Availability of data and materials: No applicable.

References

[1] J. S. Aitchison, A. M. Weiner, Y. Silberberg, D. E. Leaird, M. K. Oliver, J. L. Jackel, and P. Smith, *Experimental observation of spatial soliton interactions*, Optics letters, *16*(1) (1991), 15–17.

[2] L. Akinyemi, A. Houwe, S. Abbagari, A. M. Wazwaz, H. M. Alshehri, and M. Osman, *Effects of the higher order dispersion on solitary waves and modulation instability in a monomode fiber*, Optik, *288* (2023), 171202.

[3] K. K. Ali, M. A. Abd El Salam, E. M. Mohamed, B. Samet, S. Kumar, and M. Osman, *Numerical solution for generalized nonlinear fractional integro-differential equations with linear functional arguments using chebyshev series*, Advances in Difference Equations, *2020*(1) (2020), 1–23.

[4] A. U. Awan, M. Tahir, and H. U. Rehman, *Singular and bright-singular combo optical solitons in birefringent fibers to the biswas-arshed equation*, Optik, *210* (2020), 164489.

[5] Z. Chen, M. Segev, and D. N. Christodoulides, *Optical spatial solitons: historical overview and recent advances*, Reports on Progress in Physics, *75*(8) (2012), 086401.

[6] Y. Chu, M. M. Khater, and Y. Hamed, *Diverse novel analytical and semi-analytical wave solutions of the generalized* $(2+1)$*-dimensional shallow water waves model*, AIP Advances, *11*(1) (2021), 015223.

[7] Y. Chu, M. A. Shallal, S. M. Mirhosseini-Alizamini, H. Rezazadeh, S. Javeed, and D. Baleanu, *Application of modified extended* tanh *technique for solving complex ginzburg-landau equation considering kerr law nonlinearity*, Computers, Materials & Continua, *66*(2) (2021), 1369–1377.

[8] C. B. Clausen, O. Bang, and Y. S. Kivshar, *Spatial solitons and induced kerr effects in quasi-phase-matched quadratic media*, Physical review letters, *78*(25) (1997), 4749.

[9] C. Conti, M. Peccianti, and G. Assanto, *Observation of optical spatial solitons in a highly nonlocal medium*, Physical review letters, *92*(11) (2004), 113902.

[10] H. Ding-Jiang and Z. Hong-Qing, *Extended hyperbola function method and its application to nonlinear equations*, Communications in Theoretical Physics, *41*(6) (2004), 801.

[11] G. C. Duree Jr, J. L. Shultz, G. J. Salamo, M. Segev, A. Yariv, B. Crosignani, P. Di Porto, E. J. Sharp, and R. R. Neurgaonkar, *Observation of self-trapping of an optical beam due to the photo refractive effect*, Physical review letters, *71*(4) (1993), 533.

[12] G. Duree, M. Morin, G. Salamo, M. Segev, B. Crosignani, P. Di Porto, E. Sharp, andA. Yariv, *Dark photo refractive spatial solitons and photo refractive vortex solitons*, Physical review letters, *74* (11) (1995), 1978.

[13] M. R. A. Fahim, P. R. Kundu, M. E. Islam, M. A. Akbar, and M. Osman, *Wave profile analysis of a couple of* (3+1)*-dimensional nonlinear evolution equations by sine-gordon expansion approach*, Journal of Ocean Engineering and Science, *7*(3) (2022), 272–279.

[14] X. Gao and J. Zeng, *Two-dimensional matter-wave solitons and vortices in competing cubic-quintic nonlinear lattices*, Frontiers of Physics, *13*(1) (2018), 130501.

[15] M. S. Hashemi and M. Mirzazadeh, *Optical solitons of the perturbed nonlinear schrö dinger equation usinglie symmetry method*, Optik, *281* (2023), 170816.

[16] Z. Hong, H. Ji-Guang, W. Wei-Tao, and A. Hong-Yong, *Applications of extended hyperbolic function method for quintic discrete nonlinear schrödinger equation*, Communications in Theoretical Physics, *47*(3) (2007), 474.

[17] Y. Huang and Y. Shang, *The extended hyperbolic function method for generalized forms of nonlinear heat conduction and huxley equations*, Journal of Applied Mathematics, *2012* (2012).

[18] M. S. Iqbal, M. S. Hashemi, R. Naeem, M. A. Tarar, M. Farheen, and M. Inc, *Construction of solitary wave solutions of bi-harmonic coupled schrödinger system through* $\phi 6$*-methodology*, Optical and Quantum Electronics, *55*(5) (2023), 404.

[19] M. A. Iqbal, M. M. Miah, M. M. Rasid, H. M. Alshehri, and M. Osman, *An investigation of two integro-differential kp hierarchy equations to find out closed form solitons in mathematical physics*, Arab Journal of Basic and Applied Sciences, *30*(1) (2023), 535–545.

[20] H. F. Ismael, H. Bulut, C. Park, and M. Osman, *M-lump, n-soliton solutions, and the collision phenomena for the* $(2+1)$*-dimensional date-jimbo-kashiwara-miwa equation*, Results in Physics, *19* (2020), 103329.

[21] M. Jafari, A. Zaeim, and A. Tanhaeivash, *Symmetry group analysis and conservation laws of the potential modified kdv equation using the scaling method*, International Journal of Geometric Methods in Modern Physics, *19*(07) (2022), 2250098.

[22] M. Jafari, S. Mahdion, A. Akgül, and S. M. Eldin, *New conservation laws of the boussinesq and generalized kadomtsev–petviashvili equations via homotopy operator*, Results in Physics, *47* (2023), 106369.

[23] M. Jafari and R. Darvazebanzade, *Approximate symmetry group analysis and similarity reductions of the perturbed mkdv-ks equation*, Computational Methods for Differential Equations, *11*(1) (2023), 175–182.

[24] Y. S. Kivshar and G. P. Agrawal, *Optical solitons: from fibers to photonic crystals*, Academic press, 2003.

[25] D. Kumar, C. Park, N. Tamanna, G. C. Paul, and M. Osman, *Dynamics of two-mode sawada-kotera equation: Mathematical and graphical analysis of its dual-wave solutions*, Results in Physics, *19* (2020), 103581.

[26] S. Kumar, M. Niwas, M. Osman, and M. Abdou, *Abundant different types of exact soliton solution to the (4+ 1)-dimensional fokas and (2+ 1)-dimensional breaking soliton equations*, Communications in Theoretical Physics, *73*(10) (2021), 105007.

[27] S. Malik, M. S. Hashemi, S. Kumar, H. Rezazadeh, W. Mahmoud, and M. Osman, *Application of new kudryashov method to various nonlinear partial differential equations*, Optical and Quantum Electronics, *55*(1) (2023), 8.

[28] A. Messouber, H. Triki, Y. Liu, A. Biswas, Y. Yıldırım, A. A. Alghamdi, and Q. Zhou, *Chirped spatial solitons on a continuous-wave background in weak nonlocal media with polynomial law of nonlinearity*, Physics Letters A, *467* (2023), 128731.

[29] G. D. McDonald, C. C. Kuhn, K. S. Hardman, S. Bennetts, P. J. Everitt, P. A. Altin, J. E. Debs, J. D. Close, and N. P. Robins, *Bright solitonic matter-wave interferometer*, Physical review letters, *113*(1) (2014), 013002.

[30] M. Peccianti, C. Conti, G. Assanto, A. De Luca, and C. Umeton, *All-optical switching and logic gating with spatial solitons in liquid crystals*, Applied Physics Letters, *81*(18) (2002), 3335–3337.

[31] R. U. Rahman, M. M. M. Qousini, A. Alshehri, S. M. Eldin, K. El-Rashidy, and M. Osman, *Evaluation of the performance of fractional evolution equations based on fractional operators and sensitivity assessment*, Results in Physics, *49* (2023), 106537.

[32] H. U. Rehman, M. Tahir, M. Bibi, and Z. Ishfaq, *Optical solitons to the biswas–arshed model in birefringent fibers using couple of integration techniques*, Optik, *218* (2020), 164894.

[33] H. U. Rehman, M. A. Imran, N. Ullah, and A. Akgül, *Exact solutions of* $(2+1)$*-dimensional schrödinger's hyperbolic equation using different techniques*, Numerical Methods for Partial Differential Equations, (2020).

[34] H. U. Rehman, A. R. Seadawy, M. Younis, S. Yasin, S. T. Raza, and S. Althobaiti, *Monochromatic optical beam propagation of paraxial dynamical model in kerr media*, Results in Physics, *31* (2021), 105015.

[35] H. U. Rehman, N. Ullah, and M. A. Imran, *Optical solitons of biswas-arshed equation in birefringent fibers using extended direct algebraic method*, Optik, *226* (2021), 165378.

[36] H. U. Rehman, A. U. Awan, E. M. Tag-ElDin, S. E. Alhazmi, M. F. Yassen, and R. Haider, *Extended hyperbolic function method for the* $(2+1)$*-dimensional nonlinear soliton equation*, Results in Physics, *40* (2022), 105802.

[37] H. U. Rehman, R. Akber, A. M. Wazwaz, H. M. Alshehri, and M. Osman, *Analysis of brownian motion instochastic schrödinger wave equation using sardar sub-equation method*, Optik, *289* (2023), 171305.

[38] A. S. Reyna, K. C. Jorge, and C. B. de Araújo, *Two-dimensional solitons in a quintic-septimal medium*, Physical Review A, *90*(6) (2014), 063835.

[39] A. S. Reyna and C. B. de Araújo, *Nonlinearity management of photonic composites and observation of spatial-modulation instability due to quintic nonlinearity*, Physical Review A, *89* (6) (2014), 063803.

[40] A. S. Reyna and C. B. de Araújo, *Spatial phase modulation due to quintic and septic nonlinearities inmetal colloids*, Optics Express, *22*(19) (2014), 22456–22469.

[41] A. S. Reyna, B. A. Malomed, and C. B. de Araújo, *Stability conditions for one-dimensional optical solitons in cubic-quintic-septimal media*, Physical Review A, *92*(3) (2015), 033810.

[42] A. S. Reyna and C. B. de Araújo, *An optimization procedure for the design of all-optical switches basedon metal-dielectric nanocomposites*, Optics Express, *23*(6) (2015), 7659–7666.

[43] S. Sahoo, S. Saha Ray, M. A. M. Abdou, M. Inc, and Y. M. Chu, *New soliton solutions of fractional jaulent-miodek system with symmetry analysis*, Symmetry, *12*(6) (2020), 1001.

[44] Y. Shang, Y. Huang, and W. Yuan, *The extended hyperbolic functions method and new exact solutions to the zakharov equations*, Applied Mathematics and Computation, *200*(1) (2008), 110–122.

[45] M. Soljačić, S. Sears, and M. Segev, *Self-trapping of "necklace" beams in self-focusing kerr media*, Physical review letters, *81*(22) (1998), 4851.

[46] A. M. Sultan, D. Lu, M. Arshad, H. U. Rehman, and M. S. Saleem, *Soliton solutions of higher order dispersive cubic-quintic nonlinear schrödinger equation and its applications*, Chinese Journal of Physics, *67* (2020), 405–413.

[47] M. Tiemann, T. Halfmann, and T. Tschudi, *Photo refractive spatial solitons as wave guiding elements for optical telecommunication*, Optics Communications, *282*(17) (2009), 3612–3619.

[48] H. Triki, K. Porsezian, P. T. Dinda, and P. Grelu, *Dark spatial solitary waves in a cubic-quintic-septimal nonlinear medium*, Physical Review A, *95* (2) (2017), 023837.

[49] A. Tripathy, S. Sahoo, H. Rezazadeh, Z. Izgi, and M. Osman, *Dynamics of damped and undamped wave natures in ferromagnetic materials*, Optik, *281* (2023), 170817.

[50] N. Ullah, H. U. Rehman, M. Imran, and T. Abdeljawad, *Highly dispersive optical solitons with cubic lawand cubic-quintic-septic law nonlinearities*, Results in Physics, *17* (2020), 103021.

[51] N. Ullah, M. I. Asjad, T. Muhammad, and A. Akgül, *Analysis of power law non-linearity in solitonic solutions using extended hyperbolic function method*, (2022).

[52] S. W. Yao, N. Ullah, H. U. Rehman, M. S. Hashemi, M. Mirzazadeh, and M. Inc, *Dynamics on novel wave structures of non-linear schrödinger equation via extended hyperbolic function method*, Results in Physics, *48* (2023), 106448.

[53] M. Younis, M. Iftikhar, and H. U. Rehman, *Exact solutions to the nonlinear schrödinger and eckhaus equations by modified simple equation method*, Journal of Advanced Physics, *3*(1) (2014), 77–79.

[54] Q. Zhou, D. Yao, S. Ding, Y. Zhang, F. Chen, F. Chen, and X. Liu, *Spatial optical solitons in fifth order and seventh order weakly nonlocal nonlinear media*, Optik, *124*(22) (2013), 5683–5686.