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Generalization of Katugampola fractional kinetic equation involving incomplete H-function

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Abstract

In this article, Katugampola fractional kinetic equation (KE) has been expressed in terms of polynomials along with incomplete H-function, incomplete Meijer's G-function, incomplete Fox-Wright function, and incomplete generalized hypergeometric function, weighing the novel significance of the fractional KE that appear in a variety of scientific and engineering scenarios. τ-Laplace transform is used to solve the Kathugampola fractional KE. The obtained solutions have been presented with some real values and the simulation was done via MATLAB. Furthermore, the numerical and graphical interpretations are also mentioned to illustrate the main results. Each of the obtained conclusions is of a general nature and is capable of generating the solutions to several fractional KE.

Keywords. Fractional kinetic equation, Incomplete H-functions, Mellin-Barnes type contour.

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1. Introduction

To evaluate fractional-order differ-integral equations, fractional calculus is a useful mathematical technique. It has been established and advanced in engineering and scientific areas [22, 24, 41]. Applications of fractional differential equations have significantly enhanced biology, physics, chemistry, mathematical modeling [20], applied science, and engineering [2, 4, 7]. Additionally, the application of fractional calculus to real-world issues such as modeling phenomena using fractals [19], solving wave-like equations [21, 23], biological populations [5], random walk processes [40], fractal brownian motion, continuous time random walk [30], control theory and in the theory of time scales [1]. In order to study the rate of change of a star's chemical composition for each order, KE create a set of differential equations in the form of reaction rates for destruction and production, respectively.

The generalization and expansion of kinetic fractional equations [9, 16], that contain numerous fractional operators, has increased interest in applied mathematics as well as physics [3], chemistry, biology, engineering [38, 39], heat transfer, dynamical systems [33], and control systems [6, 11]. This has allowed for the mathematical modeling of a variety of physical phenomena like diffusion in porous media and kinetics in viscoelastic media [12, 31]. Kinetic fractional equations, which can take many different forms, have been extensively used in recent decades to describe and address a broad range of significant astrophysics and physics (see, e.g. [8, 25, 26, 32]) problems. Additionally, the Laplace transform, the Mellin transform, the Sumudu transform, the Fourier transform, the pathway-type transform, Prabhakar-type operators, and alternative methods are the most commonly used strategies for solving fractional kinetic equations. In particular, the Laplace transform method, which has been present in this paper is one of the effective way of finding analytical solutions of fractional kinetic equations. When solving fractional differential equations and special functions are utilized together with their applications. Because kinetic fractional equations are efficient and

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helpful in astrophysical calculations, they are related to every issue in a broad variety of mathematical physics and areas.

The use of the incomplete special functions in this new fractional generalization of the KE adds an additional aspect to the research. Katugampola fractional KE [29] has been expressed in terms of polynomial along with incomplete H-function, incomplete Meijer's G-function, incomplete Fox-Wright function and incomplete generalized hypergeometric function. Using the τ-Laplace transformation method, the solution to these fractional KE is obtained. Severalspecial instances are also be discussed briefly.

The fractional KE given by Haubold and Mathai [14] is as follows (see [8, 10]):

$$\frac{d\mathring{N}}{d\mathfrak{t}} = -\delta(\mathring{N}_{\mathfrak{t}}) + \mathfrak{p}(\mathring{N}_{\mathfrak{t}}), \tag{1.1}$$

where $\mathring{N} = \mathring{N}(\mathfrak{t})$ represents the rate of change of reaction, $\delta(\mathring{N}_{\mathfrak{t}})$ represents the destruction rate and $\mathfrak{p}(\mathring{N}_{\mathfrak{t}})$ represents the growth rate. Moreover, $\mathring{N}_{\mathfrak{t}}$ is stated as $\mathring{N}_{\mathfrak{t}}(\mathfrak{t}^*) = \mathring{N}(\mathfrak{t} - \mathfrak{t}^*)$ for $\mathfrak{t}^* > 0$. We also obtained some appropriate case of (1.1), when the quantity of homogeneities or spatial variation of $\mathring{N}(\mathfrak{t})$ is ignored, is provided by the subsequent equation:

$$\frac{d\mathring{N}_k}{d\mathfrak{t}} = -m_k \mathring{N}_k(\mathfrak{t}), \tag{1.2}$$

where $\mathring{N}_k(\mathfrak{t} = 0) = \mathring{N}_0$ express the varity of species density k at $t = 0$, $m_k > 0$ time. Ignoring the index k and integrating equation (1.2), we get:

$$\mathring{N}(\mathfrak{t}) - \mathring{N}_0 = -m \, {}_0\mathsf{D}_{\mathfrak{t}}^{-1} \, \mathring{N}(\mathfrak{t}), \tag{1.3}$$

where ${}_0\mathsf{D}_{\mathfrak{t}}^{-1}$ is the particular instance of the Riemann-Liouville (R-L) fractional integral operator ${}_0\mathsf{D}_{\mathfrak{t}}^{-\mu}$ and it is described as:

$${}_0\mathsf{D}_{\mathfrak{t}}^{-\mu} g(\mathfrak{t}) = \frac{1}{\Gamma(\mu)} \int_0^{\mathfrak{t}} (\mathfrak{t} - w)^{\mu-1} g(w) \, dw, \qquad \mathfrak{t}, \, \Re(\mu) > 0. \tag{1.4}$$

Houbold and Mathai [14] established the fractional generalization of the classical KE (1.3) and is defined as:

$$\mathring{N}(\mathfrak{t}) - \mathring{N}_0 = -m^{\mu} \, {}_0\mathsf{D}_{\mathfrak{t}}^{-\mu} \, \mathring{N}(\mathfrak{t}), \qquad m \in \mathbb{R}^+, \tag{1.5}$$

have the following as the solution to Equation (1.5):

$$\mathring{N}(\mathfrak{t}) = \mathring{N}_0 \sum_{s=0}^{\infty} \frac{(-1)^s}{\Gamma(s\mu + 1)} (m\mathfrak{t})^{\mu s} = \mathring{N}_0 E_{\mu}(-m^{\mu} \mathfrak{t}^{\mu}), \qquad \mu > 0, \tag{1.6}$$

where $E_{\mu}(z)$ is Mittag Leffler function [13, 28], that is described as:

$$E_{\mu}(z) = \sum_{r=0}^{\infty} \frac{z^r}{\Gamma(\mu r + 1)}, \qquad \mu > 0. \tag{1.7}$$

The ordinary and generalized Mittag-Leffler functions interpolate between a purely exponential law and power-like behavior of phenomena governed by ordinary kinetic equations and their fractional counterparts.

Moreover, Saxena and Kalla [34] thought of the following fractional KE as:

$$\mathring{N}(\mathfrak{t}) - \mathring{N}_0 g(\mathfrak{t}) = -m^{\mu} \, {}_0\mathsf{D}_{\mathfrak{t}}^{-\mu} \, \mathring{N}(\mathfrak{t}), \tag{1.8}$$

where $g(\mathfrak{t}) \in \mathfrak{L}(0, \infty)$, $\mathring{N}(\mathfrak{t})$ represents the species' population density at time $\mathfrak{t}$ and $\mathring{N}_0 = \mathring{N}(0)$ is the the species' population density at time $\mathfrak{t} = 0$.

The following is the laplace transformation of the R-L fractional integration of $g(\mathfrak{t})$ stated in (1.4) equation:

$$L[{}_0\mathsf{D}_{\mathfrak{t}}^{-\mu} g(\mathfrak{t}) : \mathtt{u}] = \mathtt{u}^{-\mu} G(\mathtt{u}), \qquad \mathfrak{t} > 0, \Re(\mu), \, \Re(\mathtt{u}) > 0. \tag{1.9}$$

The Laplace transform of $g(\mathfrak{t})$ is denoted by the function $G(\mathfrak{u})$ and specified by:

$$G(\mathfrak{u}) = L[g(\mathfrak{t}), \mathfrak{u}] = \int_0^\infty e^{-\mathfrak{u}\mathfrak{t}} g(\mathfrak{t}) d\mathfrak{t}, \qquad \mathfrak{t},\ \Re(\mathfrak{u}) > 0. \tag{1.10}$$

The following is the description of the lower and upper incomplete gamma functions $\gamma(r,t)$ and $\Gamma(r,t)$ respectively:

$$\gamma(r,t) = \int_0^t \mathfrak{u}^{r-1}\, e^{-\mathfrak{u}}\, d\mathfrak{u}, \qquad \big(t \geq 0; \Re(r) > 0\big), \tag{1.11}$$

and

$$\Gamma(r,t) = \int_t^\infty \mathfrak{u}^{r-1}\, e^{-\mathfrak{u}}\, d\mathfrak{u}, \qquad \big(\Re(r) > 0;\ t \geq 0\big). \tag{1.12}$$

The following relation is satisfied by the incomplete gamma functions:

$$\gamma(r,t) + \Gamma(r,t) = \Gamma(r), \qquad \big(\Re(r) > 0\big). \tag{1.13}$$

Using the previously mentioned incomplete gamma functions in Equation (1.11) and (1.12), Srivastava et al. [37] in the recent past presented a pair of Mellin-Barnes contour integral representations of the incomplete H-function $\gamma_{\mathrm{u,v}}^{\mathrm{r,s}}$ and $\Gamma_{\mathrm{u,v}}^{\mathrm{r,s}}$ described as follows [15]:

$$\begin{aligned}
\gamma_{\mathrm{u,v}}^{\mathrm{r,s}}(z) &= \gamma_{\mathrm{u,v}}^{\mathrm{r,s}}\left[z \,\middle|\, \begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ (\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array}\right] \\
&= \gamma_{\mathrm{u,v}}^{\mathrm{r,s}}\left[z \,\middle|\, \begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), (\mathfrak{f}_2, \mathfrak{F}_2), \cdots, (\mathfrak{f}_\mathrm{u}, \mathfrak{F}_\mathrm{u}) \\ (\mathfrak{e}_1, \mathfrak{E}_1), (\mathfrak{e}_2, \mathfrak{E}_2), \cdots, (\mathfrak{e}_\mathrm{v}, \mathfrak{E}_\mathrm{v}) \end{array}\right] \\
&= \frac{1}{2\pi i} \int_{\mathcal{L}} \psi(l,x)\, z^{-l}\, dl,
\end{aligned} \tag{1.14}$$

where,

$$\psi(l,x) = \frac{\gamma(1-\mathfrak{f}_1-\mathfrak{F}_1 l, x) \prod_{j=1}^{\mathrm{r}} \Gamma(\mathfrak{e}_j + \mathfrak{E}_j l) \prod_{j=2}^{\mathrm{s}} \Gamma(1-\mathfrak{f}_j-\mathfrak{F}_j l)}{\prod_{j=\mathrm{r}+1}^{\mathrm{v}} \Gamma(1-\mathfrak{e}_j-\mathfrak{E}_j l) \prod_{j=\mathrm{s}+1}^{\mathrm{u}} \Gamma(\mathfrak{f}_j + \mathfrak{F}_j l)}, \tag{1.15}$$

and

$$\begin{aligned}
\Gamma_{\mathrm{u,v}}^{\mathrm{r,s}}(z) &= \Gamma_{\mathrm{u,v}}^{\mathrm{r,s}}\left[z \,\middle|\, \begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ (\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array}\right] \\
&= \Gamma_{\mathrm{u,v}}^{\mathrm{r,s}}\left[z \,\middle|\, \begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), (\mathfrak{f}_2, \mathfrak{F}_2), \cdots, (\mathfrak{f}_\mathrm{u}, \mathfrak{F}_\mathrm{u}) \\ (\mathfrak{e}_1, \mathfrak{E}_1), (\mathfrak{e}_2, \mathfrak{E}_2), \cdots, (\mathfrak{e}_\mathrm{v}, \mathfrak{E}_\mathrm{v}) \end{array}\right] \\
&= \frac{1}{2\pi i} \int_{\mathcal{L}} \Psi(l,x)\, z^{-l}\, dl,
\end{aligned} \tag{1.16}$$

where,

$$\Psi(l,x) = \frac{\Gamma(1-\mathfrak{f}_1-\mathfrak{F}_1 l, x) \prod_{j=1}^{\mathrm{r}} \Gamma(\mathfrak{e}_j + \mathfrak{E}_j l) \prod_{j=2}^{\mathrm{s}} \Gamma(1-\mathfrak{f}_j-\mathfrak{F}_j l)}{\prod_{j=\mathrm{r}+1}^{\mathrm{v}} \Gamma(1-\mathfrak{e}_j-\mathfrak{E}_j l) \prod_{j=\mathrm{s}+1}^{\mathrm{u}} \Gamma(\mathfrak{f}_j + \mathfrak{F}_j l)}. \tag{1.17}$$

The incomplete H-functions $\gamma_{p,\,q}^{m,\,n}(z)$ and $\Gamma_{p,\,q}^{m,\,n}(z)$ in (1.14) and (1.16) exists $\forall x \geq 0$ within the similar contour and circumstances as described in Mathai and Saxena [27]. The denotations (1.14) and (1.16) readily yield the decomposition formula:

$$\gamma_{p,\,q}^{m,\,n}(z) + \Gamma_{p,\,q}^{m,\,n}(z) = H_{p,\,q}^{m,\,n}(z), \tag{1.18}$$

for the familiar H-function.

The Srivastava [35] investigated a broader class of polynomials, which is summarised as follows:

$$S_V^U[t] = \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\, t^{\mathfrak{O}}, \tag{1.19}$$

where $U \in \mathbb{Z}^+$ and $A_{V,\,\mathfrak{O}}$ are real or complex numbers arbitrary constants. The notations $[k]$ indicate the floor function and $(\kappa)_\mu$ indicate the pochhammer symbol described by:

$$(\kappa)_0 = 1 \qquad and \qquad (\kappa)_\mu = \frac{\Gamma(\kappa+\mu)}{\Gamma(\kappa)}, \qquad \mu \in \mathbb{C},$$

in the form of the gamma function.

In this article, to solve the Katugampola fractional KE, we will consider the τ-Laplace transform [1, 17] that involves incomplete H-function, incomplete extended hypergeometric function, and incomplete Fox-wright functions described within the same group of circumstances defined in [36, 37].

2. Mathematical Preliminaries

Definition 2.1. The R-L integral operator is generalized into a distinct form by the Katugampola fractional operator, which was given by U. N. Katugampola [18] such as for $r \in \mathbb{C}$, then:

$$[{}^{\tau}\mathbb{I}_{b+}^{r}\mathfrak{f}](y) = \frac{\tau^{1-r}}{\Gamma(r)} \int_b^y \frac{u^{\tau-1} g(u)}{(y^\tau - u^\tau)^{(1-r)}} du, \quad (\Re(r), \tau > 0). \tag{2.1}$$

The left-sided fractional integral is the name given to the above integral.

$$[{}^{\tau}\mathbb{I}_{c-}^{r}\mathfrak{f}](y) = \frac{\tau^{1-r}}{\Gamma(r)} \int_y^c \frac{u^{\tau-1} g(u)}{(u^\tau - y^\tau)^{(1-r)}} du, \quad (\Re(r), \tau > 0). \tag{2.2}$$

The right-sided fractional integral is the name given to the above integral.

Definition 2.2. Let $\mathfrak{g} : [0, \infty) \to \mathbb{R}$ be a real-valued function that is piecewise continuous and is of τ- exponential order $exp\left(d\frac{t^\tau}{\tau}\right)$, where d is a non-negative constant, then its τ-Laplace transform exists for $w > d$ and is defined as:

$$\mathbf{L}_\tau\{\mathfrak{g}(t); w\} = \int_0^\infty exp\left(d\frac{t^\tau}{\tau}\right) \frac{\mathfrak{g}(t)}{t(1-\tau)} dt, \quad (\tau > 0). \tag{2.3}$$

There are numerous physical implementations that depend on the convolution of the functions of $\mathfrak{g}(t)$ and $\mathfrak{h}(t)$, that are expressed for $t > 0$. The following integral gives the τ-Laplace convolution of functions $\mathfrak{g}(t)$ and $\mathfrak{h}(t)$:

$$\mathfrak{g}(t) *_\tau \mathfrak{h}(t) = \int_0^\infty \mathfrak{g}\{(t^\tau - \rho^\tau)^{\frac{1}{\tau}}\} \frac{\mathfrak{h}(\rho)}{\rho^{1-\tau}} d\rho, \qquad (\tau > 0), \tag{2.4}$$

which remains exists if the function $\mathfrak{g}(t)$ and $\mathfrak{h}(t)$ are at least piece-wise continuous. The fact that the τ-laplace transform of a convolution of two functions is the product of their transforms is one of the most crucial characteristics procured by the convolution in connection with the τ-Laplace transform (see, e.g [1, 17]).

τ-Laplace Convoluation Theorem: If $\mathfrak{g}(t)$ and $\mathfrak{h}(t)$ are two piecewise continuous functions on $[0, \infty)$ and have exponential order d, when $t \to \infty$, then

$$\mathbf{L}\{\mathfrak{g}(t) *_\tau \mathfrak{h}(t); w\} = \mathbf{L}\{\mathfrak{g}(t); w\}.\mathbf{L}\{\mathfrak{h}(t); w\}, \quad (\Re(w) > 0). \tag{2.5}$$

We define the τ-Laplace transform for Katugampola fractional integral:

$$\begin{aligned}\mathbf{L}_\tau\{{}^{\tau}\mathbb{I}_0^r\mathfrak{g}(t);w\} &= \frac{\tau^{1-r}}{\Gamma(r)}\mathbf{L}_\tau\{t^{\tau(r-1)} *_\tau \mathfrak{g}(t);w\}\\ &= \frac{\tau^{1-r}}{\Gamma(r)}\mathbf{L}_\tau\{t^{\tau(r-1)};w\}.\mathbf{L}_\tau\{\mathfrak{g}(t);w\}\\ &= w^{-r}\mathbf{L}_\tau\mathfrak{g}(t).\end{aligned} \tag{2.6}$$

by using the identity

$$\mathbf{L}_\tau\{t^u;w\} = \tau^{\frac{u}{\tau}}\frac{\Gamma\left(1+\frac{u}{\tau}\right)}{w^{\left(1+\frac{u}{\tau}\right)}}, \qquad (u \in \mathbb{R},\ w>0). \tag{2.7}$$

$$\Longleftrightarrow\ \mathbf{L}_\tau^{-1}\left(\frac{1}{w^{\left(1+\frac{u}{\tau}\right)}}\right) = \frac{1}{\tau^{\frac{u}{\tau}}\Gamma\left(1+\frac{u}{\tau}\right)}t^u, \tag{2.8}$$

in which $\mathbf{L}_\tau^{-1}$ considered as the τ-inverse Laplace transform.

3. Fractional Kinetic Equations: Generalized Solution

Theorem 3.1. *For all $\varrho, \varpi, \mathrm{a}, \mathrm{b}, \varphi > 0$, $U \in \mathbb{Z}^+$, $r \in \mathbb{C}$, and $A_{V,\mathfrak{O}}$ are arbitrary real or complex constant, then the equation*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} S_V^U[\mathrm{a}t^{\varrho}]\,\Gamma_{\mathrm{u,v}}^{\mathrm{r,s}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}_0^{\xi}\,\mathring{N}(t), \tag{3.1}$$

has solution:

$$\begin{aligned}\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1}\sum_{i=0}^{\infty}\left(\left(-\frac{d\,t^{\tau}}{\tau}\right)^{\xi}\right)^{i}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\,(\mathrm{a}\,t^{\varrho})^{\mathfrak{O}}\\ \times\,\Gamma_{\mathrm{u+1,\ v+1}}^{\mathrm{r,\ s+1}}\left[\mathrm{b}t^{\varpi}\ \middle|\ \begin{matrix}(\mathfrak{f}_1,\mathfrak{F}_1,x),\left(1-\left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}\right),\frac{\varpi}{\tau}\right),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\mathrm{u}}\\ \left(\frac{-\varphi-\varrho\mathfrak{O}+1-\xi i\tau}{\tau},\frac{\varpi}{\tau}\right),(\mathfrak{e}_j,\mathfrak{E}_j)_{1,\mathrm{v}}\end{matrix}\right].\end{aligned} \tag{3.2}$$

Proof. Applying τ-Laplace transform both side of Equation (3.1), we get:

$$\mathbf{L}_\tau\{\mathring{N}(t);w\} + \mathbf{L}_\tau\{d^{\xi}\ {}^{\tau}\mathbb{I}_0^{\xi}\,\mathring{N}(t);w\} = \mathbf{L}_\tau\{\mathring{N}_0 t^{\varphi-1}S_V^U[\mathrm{a}t^{\varrho}]\,\Gamma_{\mathrm{u,v}}^{\mathrm{r,s}}[\mathrm{b}t^{\varpi}];w\}. \tag{3.3}$$

Use Equation (2.7) in Equation (3.3) and replace Srivastava polynomial and incomplete H-function with Equations (1.19) and (1.16) respectively, we get:

$$\begin{aligned}\mathring{N}_\tau(w)(1+d^{\xi}w^{-\xi}) = \mathring{N}_0\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\,(a)^{\mathfrak{O}}\,\frac{1}{2\pi i}\int_{\mathcal{L}}\Psi(l,x)\,(b)^{-l}\\ \times\,\tau^{\frac{\varphi+\varrho\mathfrak{O}-\varpi l-1}{\tau}}\ \frac{\Gamma\left(1+\frac{\varphi+\varrho\mathfrak{O}-\varpi l-1}{\tau}\right)}{w^{\left(1+\frac{\varphi+\varrho\mathfrak{O}-\varpi l-1}{\tau}\right)}}dl.\end{aligned} \tag{3.4}$$

After some adjustment of terms and use of $(1+x)^{-1} = \sum_{r=0}^{\infty}(-1)^r x^r$ in Equation (3.4), we may write:

$$\begin{aligned}\mathring{N}_\tau(w) = \mathring{N}_0\tau^{\frac{\varphi-1}{\tau}}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\,(a\tau^{\frac{\varrho}{\tau}})^{\mathfrak{O}}\,\frac{1}{2\pi i}\int_{\mathcal{L}}\Psi(l,x)\,(b\tau^{\frac{\varpi}{\tau}})^{-l}\\ \times\,\Gamma\left(1+\frac{\varphi+\varrho\mathfrak{O}-\varpi l-1}{\tau}\right)\sum_{i=0}^{\infty}(-d^{\xi})^{i}\frac{1}{w^{\left(1+\frac{\varphi+\varrho\mathfrak{O}-\varpi l+\xi i\tau-1}{\tau}\right)}}dl.\end{aligned} \tag{3.5}$$

Apply τ-inverse Laplace transform both side of Equation (3.5) and use Equation (2.8), we have:

$$\mathring{N}_\tau(t) = \mathring{N}_0\, t^{\varphi-1} \sum_{i=0}^{\infty} \left(-\left(\frac{d\, t^\tau}{\tau} \right)^\xi \right)^i \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\, (at^\varrho)^{\mathfrak{O}} \times \frac{1}{2\pi i} \int_{\mathcal{L}} \Psi(l,x)\, \frac{\Gamma(\frac{\tau+\varphi+\varrho\mathfrak{O}-\varpi l-1}{\tau})}{\Gamma\left(1+\frac{\varphi-\varpi l+\xi i\tau+\varrho\mathfrak{O}-1}{\tau}\right)} (b\,t^\varpi)^{-l} dl. \tag{3.6}$$

After some adjustment of terms, we obtain the intended outcomes. □

Theorem 3.2. *For all $\varrho, \varpi, \mathrm{a}, \mathrm{b}, \varphi > 0$, $U \in \mathbb{Z}^+$, $r \in \mathbb{C}$, and $A_{V,\mathfrak{O}}$ are arbitrary real or complex constant, then the equation*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} S_V^U[\mathrm{a}t^\varrho]\, \gamma^{\mathrm{r,s}}_{\mathrm{u,v}}[\mathrm{b}t^\varpi] = -d^\xi\, {}^\tau\mathbb{I}_0^\xi\, \mathring{N}(t), \tag{3.7}$$

has solution:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^\tau}{\tau} \right)^\xi \right)^i \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\, (\mathrm{a}\, t^\varrho)^{\mathfrak{O}} \times \gamma^{\mathrm{r,\, s+1}}_{\mathrm{u+1,\, v+1}} \left[\mathrm{b}t^\varpi \,\middle|\, \begin{matrix} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(1-(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}), \frac{\varpi}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{-\varphi-\varrho\mathfrak{O}+1-\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{matrix} \right]. \tag{3.8}$$

Proof. The proof is the immediate consequences of the definitions (1.16), (1.19), and parallel to the Theorem 3.1. Consequently, we exclude the proof. □

(i) Incomplete Meijer G-Function: If we take the particular values in Equation (1.16), that is $\mathfrak{F}_j = 1 (j = 1, 2, ..., u), \mathfrak{E}_j (j = 1, , 2, ..., v)$ and using the relation, namely:

$$\Gamma^{\mathrm{r,s}}_{\mathrm{u,v}} \left[z \,\middle|\, \begin{matrix} (\mathfrak{f}_1, 1, x), (\mathfrak{f}_j, 1)_{2,\mathrm{u}} \\ (\mathfrak{e}_j, 1)_{1,\mathrm{v}} \end{matrix} \right] = {}^{(\Gamma)}G^{\mathrm{r,s}}_{\mathrm{u,v}} \left[z \,\middle|\, \begin{matrix} (\mathfrak{f}_1, x), (\mathfrak{f}_j)_{2,\mathrm{u}} \\ (\mathfrak{e}_j)_{1,\mathrm{v}} \end{matrix} \right] = \frac{1}{2\pi i} \int_{\mathcal{L}} \overline{\Psi}(l,x)\, z^{-l}\, dl, \tag{3.9}$$

where,

$$\overline{\Psi}(l,x) = \frac{\Gamma(1-\mathfrak{f}_1-l,x) \prod_{j=1}^{\mathrm{r}} \Gamma(\mathfrak{e}_j+l) \prod_{j=2}^{\mathrm{s}} \Gamma(1-\mathfrak{f}_j-l)}{\prod_{j=\mathrm{r}+1}^{\mathrm{v}} \Gamma(1-\mathfrak{e}_j-l) \prod_{j=\mathrm{s}+1}^{\mathrm{u}} \Gamma(\mathfrak{f}_j+l)}, \tag{3.10}$$

in Equation (3.1) and (3.2), we obtain the following corollaries:

Corollary 3.3. *For all $\varrho, \varpi, \mathrm{a}, \mathrm{b}, \varphi > 0$, $U \in \mathbb{Z}^+$, $r \in \mathbb{C}$, and $A_{V,\mathfrak{O}}$ are arbitrary real or complex constant, then the equation*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} S_V^U[\mathrm{a}t^\varrho]\, {}^{(\Gamma)}G^{\mathrm{r,s}}_{\mathrm{u,v}}[\mathrm{b}t^\varpi] = -d^\xi\, {}^\tau\mathbb{I}_0^\xi\, \mathring{N}(t) \tag{3.11}$$

has solution:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^\tau}{\tau} \right)^\xi \right)^i \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\, (\mathrm{a}\, t^\varrho)^{\mathfrak{O}} \times {}^{(\Gamma)}G^{\mathrm{r,\, s+1}}_{\mathrm{u+1,\, v+1}} \left[\mathrm{b}t^\varpi \,\middle|\, \begin{matrix} (\mathfrak{f}_1, x), \left(1-(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}), \frac{\varpi}{\tau}\right), (\mathfrak{f}_j)_{2,\mathrm{u}} \\ \left(\frac{-\varphi-\varrho\mathfrak{O}+1-\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_j)_{1,\mathrm{v}} \end{matrix} \right]. \tag{3.12}$$

Corollary 3.4. *For all $\varrho, \varpi, \mathrm{a}, \mathrm{b}, \varphi > 0$, $U \in \mathbb{Z}^+$, $r \in \mathbb{C}$, and $A_{V,\mathfrak{O}}$ are arbitrary real or complex constant, then the equation*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} S_V^U[\mathrm{a}t^{\varrho}]\, {}^{(\gamma)}G^{\mathrm{r,s}}_{\mathrm{u,v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t) \tag{3.13}$$

has solution:

$$\begin{aligned}\mathring{N}(t) &= \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau} \right)^{\xi} \right)^{i} \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\, (\mathrm{a}\, t^{\varrho})^{\mathfrak{O}} \\ &\times\ {}^{(\gamma)}G^{\mathrm{r,\, s+1}}_{\mathrm{u+1,\, v+1}} \left[\mathrm{b}t^{\varpi} \left| \begin{array}{c} (\mathfrak{f}_1, x), \left(1-(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}), \frac{\varpi}{\tau}\right), (\mathfrak{f}_j)_{2,\mathrm{u}} \\ \left(\frac{-\varphi-\varrho\mathfrak{O}+1-\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_j)_{1,\mathrm{v}} \end{array} \right. \right].\end{aligned} \tag{3.14}$$

(ii) Incomplete Fox-Wright Function: If we take the substitution $\mathrm{b} = -\mathrm{b}, \mathrm{r} = 1, \mathrm{s} = \mathrm{u}, \mathrm{v} = \mathrm{v}+1, \mathfrak{f}_j \to (1-\mathfrak{f}_j)\, (j = 1, 2, ..., \mathrm{u})$, and $\mathfrak{e}_j \to (1-\mathfrak{e}_j)\, (j = 1, 2, ..., \mathrm{v})$ in Equation (3.1) and (3.2), using the following relation [37]:

$$\Gamma^{1,\mathrm{u}}_{\mathrm{u,v+1}} \left[-z \left| \begin{array}{c} (1-\mathfrak{f}_1, \mathfrak{F}_1, x), (1-\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ (0,1), (1-\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} \right. \right] = {}_{\mathrm{u}}\Psi^{(\Gamma)}_{\mathrm{v}} \left[\begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ (\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} ; z \right], \tag{3.15}$$

and using the definition:

$${}_{\mathrm{u}}\Psi^{(\Gamma)}_{\mathrm{v}} \left[\begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ (\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} ; z \right] = \sum_{l=0}^{\infty} \frac{\Gamma(\mathfrak{f}_1 + \mathfrak{F}_1 l, x) \prod_{j=2}^{\mathrm{u}} \Gamma(\mathfrak{f}_j + \mathfrak{F}_j l)}{\prod_{j=1}^{\mathrm{v}} \Gamma(\mathfrak{e}_j + \mathfrak{E}_j l)} \frac{z^l}{l!}, \tag{3.16}$$

and

$${}_{\mathrm{u}}\Psi^{(\gamma)}_{\mathrm{v}} \left[\begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ (\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} ; z \right] = \sum_{l=0}^{\infty} \frac{\gamma(\mathfrak{f}_1 + \mathfrak{F}_1 l, x) \prod_{j=2}^{\mathrm{u}} \Gamma(\mathfrak{f}_j + \mathfrak{F}_j l)}{\prod_{j=1}^{\mathrm{v}} \Gamma(\mathfrak{e}_j + \mathfrak{E}_j l)} \frac{z^l}{l!}, \tag{3.17}$$

then we get the following corollaries:

Corollary 3.5. *For all $\varrho, \varpi, \mathrm{a}, \mathrm{b}, \varphi > 0$, $U \in \mathbb{Z}^+$, $r \in \mathbb{C}$, and $A_{V,\mathfrak{O}}$ are arbitrary real or complex constant, then the equation*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} S_V^U[\mathrm{a}t^{\varrho}]\, {}_{\mathrm{u}}\Psi^{(\Gamma)}_{\mathrm{v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{3.18}$$

has solution:

$$\begin{aligned}\mathring{N}(t) &= \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau} \right)^{\xi} \right)^{i} \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\, (\mathrm{a}\, t^{\varrho})^{\mathfrak{O}} \\ &\times\ {}_{\mathrm{u+1}}\Psi^{(\Gamma)}_{\mathrm{v+1}} \left[\begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1+\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_i, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} ; \mathrm{b}t^{\varpi} \right].\end{aligned} \tag{3.19}$$

Corollary 3.6. *For all $\varrho, \varpi, \mathrm{a}, \mathrm{b}, \varphi > 0$, $U \in \mathbb{Z}^+$, $r \in \mathbb{C}$, and $A_{V,\mathfrak{O}}$ are arbitrary real or complex constant, then the equation*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} S_V^U[\mathrm{a}t^{\varrho}]\, {}_{\mathrm{u}}\Psi^{(\gamma)}_{\mathrm{v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{3.20}$$

has solution:

$$\begin{aligned}\mathring{N}(t) &= \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau} \right)^{\xi} \right)^{i} \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\, (\mathrm{a}\, t^{\varrho})^{\mathfrak{O}} \\ &\times\ {}_{\mathrm{u+1}}\Psi^{(\gamma)}_{\mathrm{v+1}} \left[\begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1+\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_i, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} ; \mathrm{b}t^{\varpi} \right].\end{aligned} \tag{3.21}$$

(iii) Incomplete Generalized Hypergeometric Function: If we take the particular substitution $\mathfrak{F}_j = 1\,(j = 1, 2, ..., \mathrm{u})$ and $\mathfrak{E}_j = 1\,(j = 1, 2, ..., \mathrm{v})$ in Corollary 3.5 and Corollary 3.6, making use of the relation [37]:

$$\Gamma^{1,\mathrm{u}}_{\mathrm{u},\mathring{v}+1}\left[-z\left|\begin{array}{c}(1-\mathfrak{f}_1,1,x),(1-\mathfrak{f}_j,1)_{2,\mathrm{u}}\\(0,1),(1-\mathfrak{e}_j,1)_{1,\mathrm{v}}\end{array}\right.\right] = \mathfrak{C}^{\mathrm{u}}_{\mathrm{v}}\ {}_{\mathrm{u}}\Gamma_{\mathrm{v}}\left[\begin{array}{c}(\mathfrak{f}_1,x),(\mathfrak{f}_j)_{2,\mathrm{u}}\\(\mathfrak{e}_j)_{1,\mathrm{v}}\end{array};z\right]$$
$$= {}_{\mathrm{u}}\Psi^{(\Gamma)}_{\mathrm{v}}\left[\begin{array}{c}(\mathfrak{f}_1,1,x),(\mathfrak{f}_j,1)_{2,\mathrm{u}}\\(\mathfrak{e}_j,1)_{1,\mathrm{v}}\end{array};z\right], \tag{3.22}$$

where $\mathfrak{C}^{\mathrm{u}}_{\mathrm{v}}$ is defined by:

$$\mathfrak{C}^{\mathrm{u}}_{\mathrm{v}} = \frac{\prod_{j=1}^{\mathrm{u}}\Gamma(\mathfrak{f}_j)}{\prod_{j=1}^{\mathrm{v}}\Gamma(\mathfrak{e}_j)}, \tag{3.23}$$

and using the definition:

$${}_{\mathrm{u}}\Gamma_{\mathrm{v}}\left[\begin{array}{c}(\mathfrak{f}_1,x),(\mathfrak{f}_j)_{2,\mathrm{u}}\\(\mathfrak{e}_j)_{1,\mathrm{v}}\end{array};z\right] = \frac{\prod_{j=1}^{\mathrm{v}}\Gamma(\mathfrak{e}_j)}{\prod_{j=1}^{\mathrm{u}}\Gamma(\mathfrak{f}_j)}\sum_{l=0}^{\infty}\frac{\Gamma(\mathfrak{f}_1+l,x)\prod_{j=2}^{\mathrm{u}}\Gamma(\mathfrak{f}_j+l)}{\prod_{j=1}^{\mathrm{v}}\Gamma(\mathfrak{e}_j+l)}\frac{z^l}{l!}$$
$$= \sum_{l=0}^{\infty}\frac{(\mathfrak{f}_1,x)_l,(\mathfrak{f}_2)_l...(\mathfrak{f}_{\mathrm{u}})_l}{(\mathfrak{e}_1)_l,...(\mathfrak{e}_{\mathrm{v}})_l}.\frac{z^l}{l!}, \tag{3.24}$$

and

$${}_{\mathrm{u}}\gamma_{\mathrm{v}}\left[\begin{array}{c}(\mathfrak{f}_1,x),(\mathfrak{f}_j)_{2,\mathrm{u}}\\(\mathfrak{e}_j)_{1,\mathrm{v}}\end{array};z\right] = \frac{\prod_{j=1}^{\mathrm{v}}\Gamma(\mathfrak{e}_j)}{\prod_{j=1}^{\mathrm{u}}\Gamma(\mathfrak{f}_j)}\sum_{l=0}^{\infty}\frac{\gamma(\mathfrak{f}_1+l,x)\prod_{j=2}^{\mathrm{u}}\Gamma(\mathfrak{f}_j+l)}{\prod_{j=1}^{\mathrm{v}}\Gamma(\mathfrak{e}_j+l)}\frac{z^l}{l!}$$
$$= \sum_{l=0}^{\infty}\frac{(\mathfrak{f}_1,x)_l,(\mathfrak{f}_2)_l...(\mathfrak{f}_{\mathrm{u}})_l}{(\mathfrak{e}_1)_l,...(\mathfrak{e}_{\mathrm{v}})_l}\frac{z^l}{l!}, \tag{3.25}$$

then we get the following corollaries:

Corollary 3.7. *For all $\varrho, \varpi, \mathrm{a}, \mathrm{b}, \varphi > 0$, $U \in \mathbb{Z}^+$, $r \in \mathbb{C}$, and $A_{V,\mathfrak{O}}$ are arbitrary real or complex constant, then the equation*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} S^U_V[\mathrm{a}t^{\varrho}]\,{}_{\mathrm{u}}\Gamma_{\mathrm{v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}^{\xi}_0\,\mathring{N}(t), \tag{3.26}$$

has solution:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1}\sum_{i=0}^{\infty}\left(\left(-\frac{d\,t^{\tau}}{\tau}\right)^{\xi}\right)^{i}\sum_{\mathfrak{O}=0}^{[V/U]}\frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!}A_{V,\mathfrak{O}}\,(\mathrm{a}\,t^{\varrho})^{\mathfrak{O}}$$
$$\times\ \mathfrak{C}^{\mathrm{u}+1}_{\mathrm{v}+1}\ {}_{\mathrm{u}+1}\Gamma_{\mathrm{v}+1}\left[\begin{array}{c}(\mathfrak{f}_1,\mathfrak{F}_1,x),\left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}\right),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\mathrm{u}}\\\left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1+\xi i\tau}{\tau}\right),(\mathfrak{e}_i,\mathfrak{E}_j)_{1,\mathrm{v}}\end{array};\mathrm{b}t^{\varpi}\right], \tag{3.27}$$

where $\mathfrak{C}^{\mathrm{u}}_{\mathrm{v}}$ is defined in Equation (3.23) and $\mathfrak{C}^{\mathrm{u}+1}_{\mathrm{v}+1} = \mathfrak{C}^{\mathrm{u}}_{\mathrm{v}}\frac{\Gamma\left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}\right)}{\Gamma\left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1+\xi i\tau}{\tau}\right)}$.

Corollary 3.8. *For all $\varrho, \varpi, \mathrm{a}, \mathrm{b}, \varphi > 0$, $U \in \mathbb{Z}^+$, $r \in \mathbb{C}$, and $A_{V,R}$ are arbitrary real or complex constant, then the equation*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} S^U_V[\mathrm{a}t^{\varrho}]\,{}_{\mathrm{u}}\gamma_{\mathrm{v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}^{\xi}_0\,\mathring{N}(t), \tag{3.28}$$

has solution:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau} \right)^{\xi} \right)^{i} \sum_{\mathfrak{O}=0}^{[V/U]} \frac{(-V)_{U\mathfrak{O}}}{\mathfrak{O}!} A_{V,\mathfrak{O}}\, (\mathrm{a}\, t^{\varrho})^{\mathfrak{O}}$$
$$\times\, \mathfrak{C}_{\mathrm{v}+1}^{\mathrm{u}+1}\ {}_{\mathrm{u}+1}\gamma_{\mathrm{v}+1} \left[\begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1+\xi i \tau}{\tau}\right), (\mathfrak{e}_i, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} ; \mathrm{b} t^{\varpi} \right], \tag{3.29}$$

where $\mathfrak{C}_{\mathrm{v}}^{\mathrm{u}}$ *is defined in Equation* (3.23) *and* $\mathfrak{C}_{\mathrm{v}+1}^{\mathrm{u}+1} = \mathfrak{C}_{\mathrm{v}}^{\mathrm{u}} \frac{\Gamma\left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1}{\tau}\right)}{\Gamma\left(\frac{\tau+\varphi+\varrho\mathfrak{O}-1+\xi i\tau}{\tau}\right)}$.

4. Applications

A few implications and uses of the aforementioned findings are addressed in this section. By properly specializing the coefficient $A_{V,\mathfrak{O}}$ to produce a wide number of the existing polynomials, certain unique instances of the resultant discoveries can be developed. We look at the following instances to illustrate this:

Example 4.1. *Prove that the solution of*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1}\, \Gamma_{\mathrm{u,v}}^{\mathrm{r,s}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{4.1}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau} \right)^{\xi} \right)^{i} \Gamma_{\mathrm{u}+1,\ \mathrm{v}+1}^{\mathrm{r},\ \mathrm{s}+1} \left[\mathrm{b}t^{\varpi} \,\middle|\, \begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(1 - \left(\frac{\tau+\varphi-1}{\tau}\right), \frac{\varpi}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{-\varphi+1-\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} \right], \tag{4.2}$$

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1}\, {}^{(\Gamma)}G_{\mathrm{u,v}}^{\mathrm{r,s}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{4.3}$$

is obtained as

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau} \right)^{\xi} \right)^{i} {}^{(\Gamma)}G_{\mathrm{u}+1,\ \mathrm{v}+1}^{\mathrm{r},\ \mathrm{s}+1} \left[\mathrm{b}t^{\varpi} \,\middle|\, \begin{array}{c} (\mathfrak{f}_1, x), \left(1 - \left(\frac{\tau+\varphi-1}{\tau}\right), \frac{\varpi}{\tau}\right), (\mathfrak{f}_j)_{2,\mathrm{u}} \\ \left(\frac{-\varphi+1-\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_j)_{1,\mathrm{v}} \end{array} \right], \tag{4.4}$$

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1}\, {}_{\mathrm{u}}\Psi_{\mathrm{v}}^{(\Gamma)}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{4.5}$$

is obtained as

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau} \right)^{\xi} \right)^{i} {}_{\mathrm{u}+1}\Psi_{\mathrm{v}+1}^{(\Gamma)} \left[\begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(\frac{\tau+\varphi-1}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{\tau+\varphi+-1+\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_i, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} ; \mathrm{b}t^{\varpi} \right] \tag{4.6}$$

and

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1}\, {}_{\mathrm{u}}\Gamma_{\mathrm{v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{4.7}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau} \right)^{\xi} \right)^{i} \mathfrak{C}_{\mathrm{v}+1}^{\mathrm{u}+1}\ {}_{\mathrm{u}+1}\Gamma_{\mathrm{v}+1} \left[\begin{array}{c} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(\frac{\tau+\varphi-1}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{\tau+\varphi-1+\xi i\tau}{\tau}\right), (\mathfrak{e}_i, \mathfrak{E}_j)_{1,\mathrm{v}} \end{array} ; \mathrm{b}t^{\varpi} \right], \tag{4.8}$$

where $\mathfrak{C}_{\mathrm{v}}^{\mathrm{u}}$ *is defined in equation* (3.23) *and* $\mathfrak{C}_{\mathrm{v}+1}^{\mathrm{u}+1} = \mathfrak{C}_{\mathrm{v}}^{\mathrm{u}} \frac{\Gamma\left(\frac{\tau+\varphi-1}{\tau}\right)}{\Gamma\left(\frac{\tau+\varphi-1+\xi i\tau}{\tau}\right)}$.

Solution: If we set $U = 1, a = 1, \varrho = 0, A_{V,0} = 1$ and $A_{V,\mathfrak{O}} = 0\ \forall\ \mathfrak{O} \neq 0$ (i.e $S_V^U[\mathrm{a}t^{\varrho}] = 1$) in Equations (3.1), (3.11), (3.18), and (3.26). The assertions (4.1), (4.3), (4.5), and (4.7) of this example follow from the Theorem 3.1, Corollary 3.3, Corollary 3.5, and Corollary 3.7 respectively.

Example 4.2. *Prove that the solution of*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi+\frac{V}{2}-1} H_V\left(\frac{1}{2\sqrt{t}}\right) \Gamma^{\mathrm{r,s}}_{\mathrm{u,v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}^{\xi}_0\, \mathring{N}(t), \tag{4.9}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\,t^{\tau}}{\tau}\right)^{\xi}\right)^{i} \sum_{\mathfrak{D}=0}^{[V/2]} (-1)^{\mathfrak{D}} \frac{V!}{\mathfrak{D}!(V-2\mathfrak{D})!}(2t)^{\mathfrak{D}}$$

$$\times\, \Gamma^{\mathrm{r,\ s+1}}_{\mathrm{u+1,\ v+1}}\left[\mathrm{b}t^{\varpi}\ \middle|\ \begin{matrix} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(1-(\frac{\tau+\varphi+\mathfrak{D}-1}{\tau}), \frac{\varpi}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{-\varphi-\mathfrak{D}+1-\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{matrix}\right], \tag{4.10}$$

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi+\frac{V}{2}-1} H_V\left(\frac{1}{2\sqrt{t}}\right) {}^{(\Gamma)}G^{\mathrm{r,s}}_{\mathrm{u,v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}^{\xi}_0\, \mathring{N}(t), \tag{4.11}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\,t^{\tau}}{\tau}\right)^{\xi}\right)^{i} \sum_{\mathfrak{D}=0}^{[V/2]} (-1)^{\mathfrak{D}} \frac{V!}{\mathfrak{D}!(V-2\mathfrak{D})!}(2t)^{\mathfrak{D}}$$

$$\times\, {}^{(\Gamma)}G^{\mathrm{r,\ s+1}}_{\mathrm{u+1,\ v+1}}\left[\mathrm{b}t^{\varpi}\ \middle|\ \begin{matrix} (\mathfrak{f}_1, x), \left(1-(\frac{\tau+\varphi+\mathfrak{D}-1}{\tau}), \frac{\varpi}{\tau}\right), (\mathfrak{f}_j)_{2,\mathrm{u}} \\ \left(\frac{-\varphi-\mathfrak{D}+1-\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_j)_{1,\mathrm{v}} \end{matrix}\right], \tag{4.12}$$

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi+\frac{V}{2}-1} H_V\left(\frac{1}{2\sqrt{t}}\right) {}_{\mathrm{u}}\Psi^{(\Gamma)}_{\mathrm{v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}^{\xi}_0\, \mathring{N}(t), \tag{4.13}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\,t^{\tau}}{\tau}\right)^{\xi}\right)^{i} \sum_{\mathfrak{D}=0}^{[V/2]} (-1)^{\mathfrak{D}} \frac{V!}{\mathfrak{D}!(V-2\mathfrak{D})!}(2t)^{\mathfrak{D}}$$

$$\times\, {}_{\mathrm{u+1}}\Psi^{(\Gamma)}_{\mathrm{v+1}}\left[\begin{matrix} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(\frac{\tau+\varphi+\mathfrak{D}-1}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{\tau+\varphi+\mathfrak{D}-1+\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_i, \mathfrak{E}_j)_{1,\mathrm{v}} \end{matrix}\ ; \mathrm{b}t^{\varpi}\right]. \tag{4.14}$$

and

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi+\frac{V}{2}-1} H_V\left(\frac{1}{2\sqrt{t}}\right) {}_{\mathrm{u}}\Gamma_{\mathrm{v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\ {}^{\tau}\mathbb{I}^{\xi}_0\, \mathring{N}(t), \tag{4.15}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\,t^{\tau}}{\tau}\right)^{\xi}\right)^{i} \sum_{\mathfrak{D}=0}^{[V/2]} (-1)^{\mathfrak{D}} \frac{V!}{\mathfrak{D}!(V-2\mathfrak{D})!}(2t)^{\mathfrak{D}} \tag{4.16}$$

$$\times\, \mathfrak{C}^{\mathrm{u+1}}_{\mathrm{v+1}}\ {}_{\mathrm{u+1}}\Gamma_{\mathrm{v+1}}\left[\begin{matrix} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(\frac{\tau+\varphi+\mathfrak{D}-1}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{\tau+\varphi+\mathfrak{D}-1+\xi i\tau}{\tau}\right), (\mathfrak{e}_i, \mathfrak{E}_j)_{1,\mathrm{v}} \end{matrix}\ ; \mathrm{b}t^{\varpi}\right]. \tag{4.17}$$

where $\mathfrak{C}^{\mathrm{u}}_{\mathrm{v}}$ *is defined in Equation* (3.23) *and* $\mathfrak{C}^{\mathrm{u+1}}_{\mathrm{v+1}} = \mathfrak{C}^{\mathrm{u}}_{\mathrm{v}} \frac{\Gamma\left(\frac{\tau+\varphi+\mathfrak{D}-1}{\tau}\right)}{\Gamma\left(\frac{\tau+\varphi+\mathfrak{D}-1+\xi i\tau}{\tau}\right)}$.

Solution: If we set $a = 1, \varrho = 0, A_{V,\mathfrak{D}} = (-1)^{\mathfrak{D}}$ and $U = 2$ (i.e $S_V^2[t] \to t^{V/2} H_V\left(\frac{1}{2\sqrt{t}}\right)$, where $H_V(t)$ is Hermite polynomial) and making use of the connection, that is (see [35]):

$$H_V(t) = \sum_{\mathfrak{D}=0}^{[V/2]} (-1)^{\mathfrak{D}} \frac{V!}{\mathfrak{D}!(V-2\mathfrak{D})!} (2t)^{V-2\mathfrak{D}}, \tag{4.18}$$

in Equations (3.1), (3.11), (3.18), and (3.26). The assertions (4.9), (4.11), (4.13), and (4.15) of this example follow from the Theorem 3.1, Corollary 3.3, Corollary 3.5, and Corollary 3.7 respectively.

Example 4.3. *Prove that the solution of*

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} L_V^{\alpha}(at^{\varrho})\, \Gamma_{\mathrm{u,v}}^{\mathrm{r,s}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\; {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{4.19}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau}\right)^{\xi}\right)^{i} \sum_{\mathfrak{D}=0}^{V} \binom{V+\alpha}{V-\mathfrak{D}} \frac{(-at^{\varrho})^{\mathfrak{D}}}{\mathfrak{D}!}$$

$$\times\, \Gamma_{\mathrm{u+1,\, v+1}}^{\mathrm{r,\, s+1}} \left[\mathrm{b}t^{\varpi} \;\middle|\; \begin{matrix} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(1 - (\frac{\tau+\varphi+\varrho\mathfrak{D}-1}{\tau}), \frac{\varpi}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{-\varphi-\varrho\mathfrak{D}+1-\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_j, \mathfrak{E}_j)_{1,\mathrm{v}} \end{matrix} \right], \tag{4.20}$$

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} L_V^{\alpha}(at^{\varrho})\; {}^{(\Gamma)}G_{\mathrm{u,v}}^{\mathrm{r,s}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\; {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{4.21}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau}\right)^{\xi}\right)^{i} \sum_{\mathfrak{D}=0}^{V} \binom{V+\alpha}{V-\mathfrak{D}} \frac{(-at^{\varrho})^{\mathfrak{D}}}{\mathfrak{D}!}$$

$$\times\, {}^{(\Gamma)}G_{\mathrm{u+1,\, v+1}}^{\mathrm{r,\, s+1}} \left[\mathrm{b}t^{\varpi} \;\middle|\; \begin{matrix} (\mathfrak{f}_1, x), \left(1 - (\frac{\tau+\varphi+\varrho\mathfrak{D}-1}{\tau}), \frac{\varpi}{\tau}\right), (\mathfrak{f}_j)_{2,\mathrm{u}} \\ \left(\frac{-\varphi-\varrho\mathfrak{D}+1-\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_j)_{1,\mathrm{v}} \end{matrix} \right], \tag{4.22}$$

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} L_V^{\alpha}(at^{\varrho})\, {}_{\mathrm{u}}\Psi_{\mathrm{v}}^{(\Gamma)}[\mathrm{b}t^{\varpi}] = -d^{\xi}\; {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{4.23}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau}\right)^{\xi}\right)^{i} \sum_{\mathfrak{D}=0}^{V} \binom{V+\alpha}{V-\mathfrak{D}} \frac{(-at^{\varrho})^{\mathfrak{D}}}{\mathfrak{D}!}$$

$$\times\, {}_{\mathrm{u+1}}\Psi_{\mathrm{v+1}}^{(\Gamma)} \left[\begin{matrix} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(\frac{\tau+\varphi+\varrho\mathfrak{D}-1}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{\tau+\varphi+\varrho\mathfrak{D}-1+\xi i\tau}{\tau}, \frac{\varpi}{\tau}\right), (\mathfrak{e}_i, \mathfrak{E}_j)_{1,\mathrm{v}} \end{matrix} ; \mathrm{b}t^{\varpi} \right]. \tag{4.24}$$

and

$$\mathring{N}(t) - \mathring{N}_0 t^{\varphi-1} L_V^{\alpha}(at^{\varrho})\, {}_{\mathrm{u}}\Gamma_{\mathrm{v}}[\mathrm{b}t^{\varpi}] = -d^{\xi}\; {}^{\tau}\mathbb{I}_0^{\xi}\, \mathring{N}(t), \tag{4.25}$$

is obtained as:

$$\mathring{N}(t) = \mathring{N}_0 t^{\varphi-1} \sum_{i=0}^{\infty} \left(\left(-\frac{d\, t^{\tau}}{\tau}\right)^{\xi}\right)^{i} \sum_{\mathfrak{D}=0}^{V} \binom{V+\alpha}{V-\mathfrak{D}} \frac{(-at^{\varrho})^{\mathfrak{D}}}{\mathfrak{D}!} \tag{4.26}$$

$$\times\, \mathfrak{C}_{\mathrm{v+1}}^{\mathrm{u+1}}\; {}_{\mathrm{u+1}}\Gamma_{\mathrm{v+1}} \left[\begin{matrix} (\mathfrak{f}_1, \mathfrak{F}_1, x), \left(\frac{\tau+\varphi+\varrho\mathfrak{D}-1}{\tau}\right), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\mathrm{u}} \\ \left(\frac{\tau+\varphi+\varrho\mathfrak{D}-1+\xi i\tau}{\tau}\right), (\mathfrak{e}_i, \mathfrak{E}_j)_{1,\mathrm{v}} \end{matrix} ; \mathrm{b}t^{\varpi} \right]. \tag{4.27}$$

where $\mathfrak{C}_{\mathrm{v}}^{\mathrm{u}}$ *is defined in Equation* (3.23) *and* $\mathfrak{C}_{\mathrm{v+1}}^{\mathrm{u+1}} = \mathfrak{C}_{\mathrm{v}}^{\mathrm{u}} \frac{\Gamma\left(\frac{\tau+\varphi+\varrho\mathfrak{D}-1}{\tau}\right)}{\Gamma\left(\frac{\tau+\varphi+\varrho\mathfrak{D}-1+\xi i\tau}{\tau}\right)}$.

Solution: If we set $A_{V,\mathfrak{D}} = \binom{V+\alpha}{V}\frac{1}{(\alpha+1)_{\mathfrak{D}}}$ and $U = 1$ (i.e $S_V^1[t] \to L_V^{(\alpha)}(t)$, where $L_V^{(\alpha)}(t)$ is Laguerre polynomial) and making use of the connection, that is (see [35]).

$$L_V^{\alpha}(t) = \sum_{\mathfrak{D}=0}^{V} \binom{V+\alpha}{V-\mathfrak{D}} \frac{(-t)^{\mathfrak{D}}}{\mathfrak{D}!}, \tag{4.28}$$

in Equations (3.1), (3.11), (3.18), and (3.26). The assertions (4.9), (4.11), (4.13), and (4.15) of this example follow from the Theorem 3.1, Corollary 3.3, Corollary 3.5, and Corollary 3.7 respectively.

Remark 4.4. Numerous additional outcomes can be obtained by applying the findings from results (3.7), (3.13), (3.20), and (3.28).

5. Graphical Results and Discussions

This section uses MATLAB to draw and simulate graphs of the numerical solution for fractional kinetic equation (3.1) for various values of different parameters, which are presented in Figures 1, 2, and 3. Figures show that, depending on the fractional parameters, the reaction rate $\mathring{N}(t)$ continuously declines with time t. A valid region of convergence is defined as $0 \leq t \leq 2$ in terms of time interval. It is also seen that the equivalent value of $\mathring{N}(t)$ first increases as the value of ξ grows, but eventually exhibits the reverse behavior.

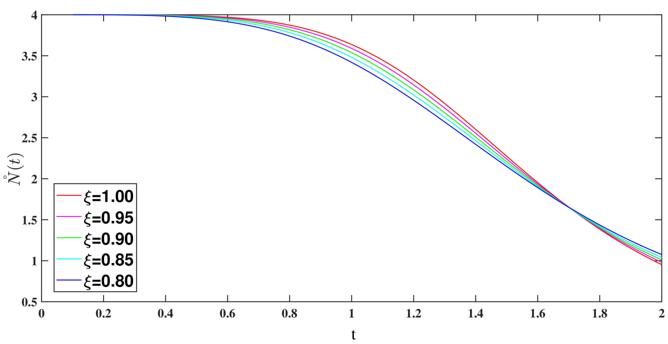

Figure 1. Plots of solution $\mathring{N}(t)$ for the fractional kinetic Equation (3.1) when $d = 0.5, \tau = 5$, and $\mathring{N}_0 = 4$.

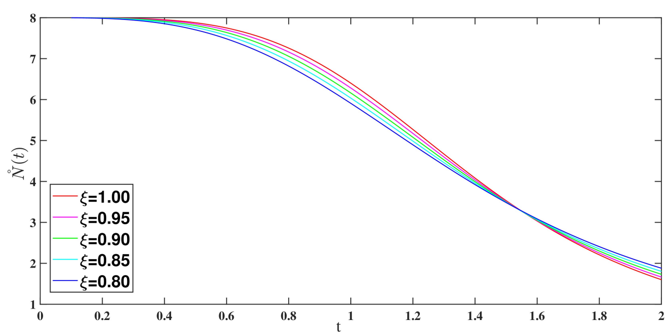

Figure 2. Plots of solution $\mathring{N}(t)$ for the fractional kinetic Equation (3.1) when $d = 1, \tau = 4$, and $\mathring{N}_0 = 8$.

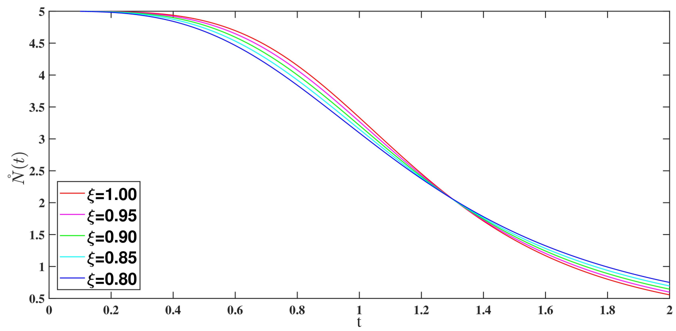

FIGURE 3. Plots of solution $\mathring{N}(t)$ for the fractional kinetic Equation (3.1) when $d = 0.5, \tau = 3$, and $\mathring{N}_0 = 5$.

6. Conclusion

This study attempts to establish a novel fractional generalization of the classical KE and analyze its solution using the τ-Laplace transformation method. A family of functions, incomplete H-function, Meijer's G-function, incomplete Fox-Wright function, and the incomplete generalized hypergeometric function along with the family of polynomials have all been studied in addition to several other novel fractional KE and their solutions. The major conclusions contained in Theorems 3.1, 3.2, and their corollaries are all of a general type.

In a similar manner, numerous fractional KE and their outcomes found in previous research can be bought as special instances of the primary findings. Also, Srivastava polynomial generalize various other polynomials like the Hermite polynomial, Jacobi polynomial, Laguerre polynomial, Gegenbauer polynomial, Legendre polynomial, Tchebycheff polynomial, Gould-Hopper polynomial and several other polynomials. Based on this study, we can conclude that it may be helpful in astrophysics to compute the change in the chemical composition in stars. The behavior of the obtained solutions is studied with the help of graphs. The main findings can therefore be used to create a variety of KE and their potential solutions by applying arbitrary constraints to the appropriate basic values. Future work will continue this investigation into the more generalized KE and the suggested solutions.

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