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Mathematical modeling of a nonlinear two-phase flow in a porous medium and the inflow of volatile oil to a well taking into account inertial effects

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Abstract

This paper discusses a semi-analytic solution for the volatile oil influx into the well on the base of Forchheimer flow law. The solution is developed employing the concept of binary model for the two-phase petroleum hydrocarbon system in view of phase transformations and interphase mass transfer. Algorithms are developed for calculating the volatile oil reservoir key performance indicators by applying the material balance equations, which take into account the compaction behavior of rocks. A computer simulator for the volatile oil reservoir is modeled, proceeding from these algorithms. The inertial effects on the development process of a volatile oil reservoir, the rocks of which are exposed to elastic deformation, are studied by this simulator. In regard thereto, the reservoir development process is simulated in two variants in conformity with the constant depression: in the first case, it is assumed that the filtration occurs according to Darcy's law, while in the second one, the process is considered on the base of Forchheimer equation. A comparison of the results of these options made it possible to demonstrate the nature of the inertial effects on the volatile oil reservoir key performance indicators.

Keywords. Volatile oil, Inertial effects, Nonlinear flow, Compressibility, Deformation.

2010 Mathematics Subject Classification. 65L05, 34K06, 34K28.

1. Introduction

As is known, the violation of [28] law, i.e. nonlinear dependence of the fluid's flow velocity on the pressure gradient in porous reservoirs can occur for two reasons. The first reason involves the rheological properties of the fluid. Such fluids for which the linear dependence of the flow velocity on the pressure gradient is violated are called non-Newtonian fluids. The second reason, why nonlinear effects are manifested, which has been the subject matter of numerous studies [10, 22, 25, 32, 35], is due to the high velocity flow , and not the fluid itself. This is due to inertial forces that appear at high speeds of fluid flow through porous channels. In such cases inertial forces may no longer be neglected as compared to viscous forces. Such high velocity flows can be described by [19]. Until recently, it was believed that nonlinear effects in porous media may manifest themselves only during gas filtration. Therefore, the major scope of research in the field of inertial effects covered gas reservoirs, since it was believed that such high flow rates, leading to non-linear effects, existed only with the flow of gas [1]. This resulted in numerous studies devoted to gas reservoirs [13, 24]. So, in [27] a new diffusivity flow equation has been derived to describe fluid flow in porous media including non-Darcy behaviors based on the fundamental Forchheimer's equation. A wide range of fluid flow and porous media characteristics has been tested, and predictions of the numerical model [14] showed very consistent results in all ranges. However, the real properties (phase transformations and mass exchange between phases) of the hydrocarbon system were not taken into account in this case.

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The mathematical model of non-Darcy flow is presented in [30]. Then, an ideal model of the five-spot well pattern was established on the basis of practical field and laboratory experiment data. The non-Darcy simulation and Darcy simulation were conducted under the same reservoir condition. The simulation results of pressure gradient distribution, cumulative oil production, remaining oil distribution for Darcy flow, and non-Darcy flow were ultimately provided. Research shows that flow in low permeability reservoir belongs to the curve segment which indicates the rationality of the non-Darcy simulation software. The inertial effects on the filtration of the gas/condensate system have also been studied extensively. So, it was experimentally defined that in the near-wellbore section of gas/condensate wells, a high-pressure gradient induces both large condensate saturation and high gas velocities which lead to significant deviations from Darcy's law for gas permeability [31].

Further, the research in this field has shown that at high flow rates, when the value of Re is above a certain value at which the flow becomes turbulent, the actual permeability of the porous medium deviates from the absolute permeability. Thus, due to the assumption of a constant permeability, the Forchheimer model has some limitations in describing the non-Darcy flow in porous media. In 2004, Barree and Conway proposed a new non-Darcy flow model, which didn't rely on the assumption of a constant Forchheimer factor and could describe the entire range of relationships between flow velocity and pressure gradient from low to high flow velocity through porous media. Therefore, the non-Darcy model [5] has attracted more and more attention in recent years [20, 29]. So, careful analysis of references has shown that insufficient attention was paid to the study of the nonlinear flow of volatile oils in porous media [18]. One of the characteristics of volatile oils in comparison with black oil is great mobility due to their low viscosity [12]. This means that a violation of the Darcy filtration law can occur at high depression in the bottom hole zone, due to the high flow velocity [16]. On the other hand, the rate of filtration of volatile oils in a porous medium under real reservoir conditions is significantly lower compared to the flow of hydrocarbon gases. However, volatile oils have much more density and viscosity. Therefore, bearing in mind the number $Re = \frac{\rho v \sqrt{k}}{\mu}$ (where $\mu, \rho,\ v$ and k are the dynamic viscosity, density, velocity, and permeability, respectively), the study of the influence of inertial forces on the volatile oils filtration process is of interest. This requires the creation of a mathematical model of the volatile oils filtration process in a porous medium based on the Forchheimer equation. The rheological properties of liquids and rocks, phase transformations and mass transfer between the phases of the hydrocarbon system must also be taken into account. Within the framework of such requirements, we succeeded in solving the problem and obtained the required formulae to predict the fluid influx into the well from the reservoir. An algorithm for calculation of the main indicators of volatile oil deposits development has been developed which served as a basis for the software developed by us. The computer studies are carried out and the impact of inertial effects is evaluated on the reservoir performance.

2. Mathematical Model of Unsteady-State Non-Linear Flow of Volatile Oils Through the Compacting Porous Media

The volatile oil is a hydrocarbon liquid of a very low specific gravity and a high content of dissolved gas. Such a hydrocarbon system differs from the black-oil due to the evaporation property of lighter components from the liquid phase when the reservoir pressure is lower than the vapor pressure. This leads to the formation of a two-phase hydrocarbon system in the reservoir. Mass transfer of components takes place between these phases. This makes mathematical modeling more difficult. There are basically two approaches to hydrodynamic modeling of volatile oil reservoirs: black-oil and compositional simulation models. Obviously, while black-oil simulation is easier and more accessible to the user, the results are not as accurate as in compositional simulation. However, compositional simulation is not always applicable in practice [18, 34].

The proposed approach for modeling the complex hydrocarbon systems in [8] is formally similar to a black-oil simulation and called the Binary modeling. Binary modeling is relatively simple and at the same time accurate in comparison with the Black-oil modeling. Therefore, the problem posed in this work is solved on the basis of the binary modeling. It is known that the binary model represents the complex hydrocarbon systems including volatile oil, as consisting of two mutual soluble pseudo-components and two phases, between which the mass transfer occurs. In this case, the motion of pseudo components in both phases is described by nonlinear partial differential equations [6, 23, 33], the analytical solution of which requires a special approach and some assumptions. The averaging method is often applied for the linearization of the equation of motion. To bring the equation to the classical heat equation, a pseudo-pressure function is introduced in the form of $H = \int \varphi(p, \rho)\, dp + const$. So, the resulting linear equation can easily be solved [15]. Further, a reverse transition of the pseudo-pressure to the true pressure will be needed. For this purpose, the approximation of the integrand by a logarithmic function of the form $\varphi = a^* \ln(p) \; - b^*$ is used, where the coefficients a^* and b^* are found from the boundary values of the function φ.

The solution of the equation of motion together with the filtration law allows us to obtain a formula for determining the fluid inflow into the well under the current reservoir pressure. And the material balance equations of the hydrocarbon system are used to determine the reservoir parameters' variation over time, such as pressure and oil saturation [3, 4, 21]. These two solutions complement each other and together they allow us to calculate the dynamics of key development parameters. This numerical model has been used in a number of our research studies to investigate the process of non-stationary filtration of two-phase hydrocarbon systems in deformable reservoirs. However, in these studies, it was assumed that the fluids filtration in a porous medium occurs according to Darcy's law.

Using the results of [8], the equations describing the unsteady-state radial flow of volatile oils in the drainage area of the well are written as follows:

$$\frac{1}{r}\frac{\partial p}{\partial r}\left[r\varphi(p,s)\frac{\partial p}{\partial r}\right] = -\frac{\partial}{\partial t}\left[f(p,s)\right], \tag{2.1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\varphi_g(p,s)\frac{\partial p}{\partial r}\right] = -\frac{\partial}{\partial t}\left[f_g(p,s)\right], \tag{2.2}$$

where

$$\varphi(p,s) = \left[\frac{k_{ro}(s)}{\mu_o(p)B_o(p)} + \frac{k_{rg}(s)p\beta c(p)}{\mu_g(s)z(p)p_{at}}\right]k(p),$$

$$\varphi_g(p,s) = \left[\frac{k_{rg}(s)p\beta[1-c(p)\overline{\gamma}(p)]}{\mu_g(p)z(p)p_{at}} + \frac{k_{ro}(s)S(p)}{\mu_o(p)B_o(p)}\right]k(p),$$

$$f(p,s) = \left[\frac{s}{B_o(p)} + (1-s)\frac{p\beta c(p)}{z(p)p_{at}}\right]\phi(p),$$

$$f_g(p,s) = \left[\frac{(1-s)\,p\beta\left[1-c\,(p)\,\overline{\gamma}\,(p)\right]}{z\,(p)\,p_{at}} + s\frac{S\,(p)}{B_o\,(p)}\right]\phi(p),$$

where $k_{ro}(s)$, $k_{rg}(s)$ are oil-phase and gas-phase relative permeability, respectively. s is the reservoir oil saturation. β is the temperature correction factor. c is the vaporous hydrocarbons content of the gas phase; μ_o and μ_g are oil and gas-phase viscosity, respectively. B_o is the oil volume factor; z is the gas deviation factor (gas compressibility factor). S is the gas solubility in the liquid phase; $\overline{\gamma} = \frac{\gamma_o(p)}{\gamma_g(p)}$ is the ratio of the specific weight of liquid phase and the specific weight of the gas phase at the reservoir pressure p, p_{at} is the atmospheric pressure. k and ϕ are formation effective permeability and porosity, respectively. r is the radial coordinate and t is time.

Equation (2.1) describes the flow of liquid hydrocarbons and dissolved gas in them, and (2.2) describes the motion of gas and lighter oil components' vapor. To determine the well performance under the reservoir depletion conditions, a solution of equation (2.1) is required under the following boundary conditions:

$$r = r_e, p = p_e(t), r = r_w, p = p_w(t) \;\; and \;\; t = 0, p = p_0. \tag{2.3}$$

In this case, we will use the averaging method for linearizing the equation. We introduce the pseudo-pressure function H, in the following form:

$$H = \int \varphi(p,s)dp + const, \tag{2.4}$$

where r_e and r_w are reservoir or drainage area and wellbore radius, respectively; p_0, p_w, p_e are the initial reservoir pressure, the bottom hole pressure, and the pressure at the external boundary, respectively.

If the reservoir pressure is averaged over the coordinate, the right-hand side of equation (2.1) will depend only on time. Taking this into account, we equate the right-hand side of (2.1) to some function $\Phi(t)$ which is determined by the additional boundary condition below.

In this case, taking (2.4) into account, we rewrite (2.1) as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial H}{\partial r}\right\} = -\Phi(t). \tag{2.5}$$

Equation (2.5) describes the liquid phase flow in a porous media under the Darcy-law. The solution of this equation under the following boundary conditions

$$r = r_e, H = H_e(t), r = r_w, H = H_w(t),$$

is possible to obtain an expression for the oil flow rate of the well as follows [8]:

$$q = \frac{2\pi h(H_e - H_w)}{ln\frac{r_e}{r_w} - \frac{1}{2}}, \tag{2.6}$$

where h is the formation thickness.

Now, consider the case when filtration obeys the Forchheimer law, which for our case has the following form:

$$\frac{dH}{dr} = v + \frac{k}{\mu}bv^2, \tag{2.7}$$

where v is the flow velocity, b - non-Darcy flow coefficient is determined experimentally. It can be calculated by the following formula [7]:

$b = 0.005\frac{\rho(p)}{k(p)^{0.5}\phi(p)^{5.5}}$, where ϕ is the reservoir porosity.

Equation (2.7) with respect to v is quadratic and has the following solution:

$$v = -\frac{1}{2b_1} + \frac{1}{2b_1}\sqrt{1 + 4b_1\frac{\partial H}{\partial r}}, \tag{2.8}$$

where $b_1 = \frac{k(p)}{\mu(p)}b$.

Proceeding from the argument that in the equation (2.5) $v = \frac{\partial H}{\partial r}$ in the Darcy law, then, taking into account (2.8), we rewrite equation (2.5) for the case of the nonlinear flow as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[-\frac{1}{2b_1} + \frac{1}{2b_1}\sqrt{1 + 4b_1\frac{\partial H}{\partial r}}\right]\right\} = -\Phi(t). \tag{2.9}$$

Equation (2.9) can easily be solved with respect to the pseudo-pressure H under the following boundary conditions:

$$r = r_e, H = H_e(t), r = r_w, H = H_w(t). \tag{2.10}$$

The unknown function $\Phi(t)$ is determined by the additional boundary condition:

$$\left.\frac{\partial H}{\partial r}\right|_{r=r_e} = 0.$$

Thus, the problems (2.1) and (2.3) are reduced to the problems (2.9) and (2.10), the solution of which allows us to obtain an expression for determining the oil production rate by $r = r_w$ for filtration according to the Forchheimer law as follows:

$$q_o = 2\pi h\left(\frac{-A+\sqrt{A^2+4B(H_e-H_w)}}{2B}\right), \tag{2.11}$$

where $A = ln\frac{r_e}{r_w} - \frac{1}{2}\frac{r_e^2-r_w^2}{r_e^2}$; $B = \frac{k(p)}{\mu(p)}b\left(\frac{1}{r_w} - \frac{1}{r_e} - 2\frac{r_e-r_w}{r_e^2} + \frac{r_e^3-r_w^3}{3r_e^4}\right)$.

Expression (2.11) allows us to determine the inflow rate to the well during filtration by a Forchheimer law. In the special case, when the flow occurs at low velocities and it obeys the Darcy law (i.e. $b = 0$), then (2.11) should be equated with the analogous expression obtained according to Darcy's law (2.6). To confirm this, we show that

$$\lim_{b\to 0} q = \frac{2\pi h(H_e - H_w)}{ln\frac{r_e}{r_w} - \frac{1}{2}}.$$

So, to uncover the indeterminate form $\frac{0}{0}$ using the L'Hospital's Rule:

$$\begin{aligned}\lim_{b\to 0} q &= 2\pi h \lim_{b\to 0}\frac{-A+\sqrt{A^2+4B(H_e-H_w)}}{2B}\\ &= 2\pi h\lim_{b\to 0}\frac{\left[-A+\sqrt{A^2+4B(H_e-H_w)}\right]'}{[2B]'}\\ &= \frac{1}{A}(H_e-H_w)\\ &= \frac{2\pi h(H_e-H_w)}{ln\frac{r_e}{r_w} - \frac{1}{2}\frac{(r_e^2-r_w^2)}{r_e^2}}.\end{aligned}$$

Since $r_w^2 \ll r_e^2$ then $\frac{r_w^2}{r_e^2} \approx 0$ and we finally get:

$$\lim_{b\to 0} q = \frac{2\pi h(H_e - H_w)}{ln\frac{r_e}{r_w} - \frac{1}{2}}.$$

This is an expression for determining the production rate under the Darcy law (2.6). Thus, we prove the adequacy of expression (2.11).

For practical use (2.11), a transition from pseudo-depression $(H_e - H_w)$ to true depression $(p_e - p_w)$ by (2.4) is required. For this purpose, the integrand φ was investigated and it was established that it is well approximated by the logarithmic function in the form:

$$\varphi = a^* ln(\ p) - b^*. \tag{2.12}$$

Taking this approximation into account, we integrate (2.4) within $[p_w, p_e]$ and obtain the expression of $(H_e - H_w)$ as follows:

$$H_e - H_w = a^*\left[p_e ln p_e - p_e - p_w ln p_w + p_w\right] - b^*(p_e - p_w), \tag{2.13}$$

where the relations for calculating the coefficients a^* and b^* are obtained from (2.4) and (2.11), taking into account the corresponding boundary values of φ :

$$a^* = \frac{\varphi_e - \varphi_w}{ln\frac{p_e}{p_w}}, b^* = \frac{\varphi_e - \varphi_w}{ln\frac{p_e}{p_w}} ln p_e - \varphi_e, \tag{2.14}$$

where φ_e, φ_w are the values of φ for contour and bottom hole pressures p_e and p_w, respectively.

So, to calculate the value of the instantaneous production rate of the well 1.10, taking into account (2.13) and (2.14), we rewrite it in the following final form:

$$q_o = \frac{2\pi h}{2B}\left[-A + \sqrt{A^2 + 4BC}\right], \tag{2.15}$$

$$C = \frac{\varphi_e - \varphi_w}{ln\frac{p_e}{p_w}}[\frac{lnp_e^{p_e}}{lnp_w^{p_w}} - p_e + p_w(\frac{\varphi_e - \varphi_w}{ln\frac{p_e}{p_w}}lnp_e - \varphi_e)(p_e - p_w)].$$

Formula (2.15) allows us to calculate the instantaneous oil production rate at a specific value of the reservoir pressure p_e, and the corresponding saturation s.

To simulate the reservoir development process, it is necessary to supplement the solution obtained above with equations describing the change in reservoir pressure and saturation in time. The algorithm for calculating reservoir pressure and oil saturation values at any time is proposed below.

3. Algorithm For Calculating the Reservoir Key Indicators

The application of (2.15) requires the determination of reservoir pressure and oil saturation values at any time. Below we get an algorithm for calculating the values of reservoir pressure and saturation. To this end, we will use the material balance equations for the liquid and gas phases of the hydrocarbon system [11]:

$$q_o = -\frac{d}{dt}\left[\frac{s}{B_o(p)} + (1-s)\frac{p\beta c(p)}{z(p)\,p_{at}}\right]\Omega, \tag{3.1}$$

$$q_g = -\frac{d}{dt}\left[\frac{(1-s)\,p\beta}{z(p)\,p_{at}}\left[1 - c(p)\,\overline{\gamma}\right] + s\frac{S(p)}{B_o(p)}\right]\Omega. \tag{3.2}$$

From the system of equations (3.1) and (3.2) we obtain a system of differential equations describing changes in pressure and oil saturation in the reservoir:

$$\frac{dp}{dt} = -\frac{\frac{q_o}{\Omega_0\overline{\Omega}}(\alpha_4 + G\alpha_2) - (\alpha_2\alpha_3 + \alpha_1\alpha_4)\frac{1}{\overline{\Omega}}\frac{d\overline{\Omega}}{dt}}{(\alpha_5+\alpha_6)\alpha_4 + (\alpha_7+\alpha_8)\alpha_2}, \tag{3.3}$$

$$\frac{ds}{dt} = -\frac{\frac{q_o G}{\Omega_0\overline{\Omega}} + (\alpha_7+\alpha_8)\frac{dp}{dt} + \alpha_3\frac{1}{\overline{\Omega}}\frac{d\overline{\Omega}}{dt}}{\alpha_4}, \tag{3.4}$$

where the oil production rate (g_o) is calculated by the formula (2.15), total pore volume

$$\Omega(p,t) = ({r_e}^2 - {r_w}^2)h\varphi(p), \qquad \overline{\Omega} = \frac{\Omega}{\Omega_0},$$

$$\alpha_1 = (1-s)\frac{p\beta c(p)}{z(p)p_{at}} - s\frac{1}{B_o(p)}, \qquad \alpha_2 = \frac{p\beta c(p)}{z(p)p_{at}} - \frac{1}{B_o(p)},$$

$$\alpha_3 = s\frac{S(p)}{B_o(p)} - (1-s)\frac{p\beta}{z(p)p_{at}}[1 - c(p)\overline{\gamma}(p)], \qquad \alpha_4 = \frac{S(p)}{B_o(p)} - \frac{p\beta}{z(p)p_{at}}[1 - c(p)\overline{\gamma}(p)],$$

$$\alpha_5 = (1-s)\left\{\frac{p\beta c(p)}{z(p)p_{at}}\right\}', \qquad \alpha_6 = s\left[\frac{1}{B_o(p)}\right]',$$

$$\alpha_7 = s\left[\frac{S(p)}{B_o(p)}\right]', \qquad \alpha_8 = (1-s)\left[\frac{p\beta}{z(p)p_{at}}[1 - c(p)\overline{\gamma}(p)]\right]',$$

" $'$ " means the derivative with respect to p. $\overline{\mu}(p)$ is the ratio of the viscosities of the liquid and gas phases. $\psi(s)$ is the ratio of the relative phase permeabilities of the gas and liquid phases. Ω_0 is the initial value of Ω. G is the gas/oil ratio. It was obtained from the equations of motion (2.1) and (2.2) taking into account the physical meaning of their left-hand sides [8]:

$$G = \frac{\frac{\overline{\mu}(p)B_o(p)p\beta}{z(p)p_{at}}[1 - c(p)\overline{\gamma}(p)] + \frac{S(p)}{\psi(s)}}{\frac{1}{\psi(s)} + \frac{\overline{\mu}(p)B_o(p)p\beta c(p)}{z(p)p_{at}}}. \tag{3.5}$$

In the equations (3.3) and (3.4) $\overline{\Omega}$ and consequently $\frac{d\overline{\Omega}}{dt}$, for elastic formations are determined by the following law [2]):

$$\overline{\Omega} = \frac{\Omega}{\Omega_0} = exp[\ a_m(p - p_0)],$$

and

$$\frac{d\overline{\Omega}}{dt} = a_m exp[\ a_m(p - p_0)]\frac{dp}{dt}, \tag{3.6}$$

where a_m is the rock compressibility factor. Taking into account (3.6), we rewrite (3.3) and (3.4) in the following form:

$$\frac{dp}{dt} = -\frac{\frac{q_o}{\Omega_0\overline{\Omega}}(\alpha_4 + G\alpha_2)}{(\alpha_5 + \alpha_6)\alpha_4 + (\alpha_7 + \alpha_8)\alpha_2 - (\alpha_2\alpha_3 + \alpha_1\alpha_4)\frac{a_m}{\overline{\Omega}}e^{a_m(p-p_0)}}, \tag{3.7}$$

$$\frac{ds}{dt} = -\frac{\frac{q_o G}{\Omega_0\overline{\Omega}} + (\alpha_7 + \alpha_8)\frac{dp}{dt} + \alpha_3\frac{a_m}{\overline{\Omega}}e^{[a_m(p-p_0)]}\frac{dp}{dt}}{\alpha_4}. \tag{3.8}$$

The system of ordinary differential equations (3.6)-(3.8) can be solved by the Runge-Kutta method [26] using the principle of the changing stationary states. Hence, within each time interval, all the coefficients are assumed to be constant. The system (3.6)-(3.8) allows determining the reservoir pressure, oil saturation, and pore volume at any time during the development of a volatile oil reservoir represented by compacting rocks.

For the computer application of the above calculation formulas, the algorithm presented in Figure 1 can be used.

4. Study of the inertial effects on the reservoir key performance indicators

Based on the above algorithm, the software is developed. This program allows you to change all the investigated parameters. Its user interface is shown in Figure 2. The change profiles of the development indices have been investigated using this software when the filtration obeys the non-Darcy law. For the validation of the model and to evaluate the nonlinear effects, calculations are carried out both for Darcy flow and for non-Darcy flow. The following initial data were used:

Initial formation pressure p_0=340 atm

The reservoir thickness $h = 20$ m

Well drainage area radius r_e= 1000 m

Well radius r_w= 0.1 m

The initial absolute permeability $k_0 = 0.15 * 10^{-12} m^2$;

The permeability change factor β_k=0.001 $1/atm$

Initial formation porosity $\phi_0 = 0.13$;

The porosity change factor a_m =0.0001 $1/atm$.

Computer calculations are performed for two variants, according to Darcy's linear law and Forchheimer's law. For all initial data, both variants are identical. Since the development is carried out at a depression of 50 atm, the calculations are therefore carried out until the pressure in the reservoir drops to 51 atm. The results of the calculations are shown in Figures 3-5.

Figure 3 shows a plot of the change in reservoir pressure over time in cases where the flow obeys the Forchheimer law (solid line) and when it is assumed that the filtration occurs according to Darcy's linear law (dashed line). Just note, the reservoir pressure is higher in the case when the inertial effects are taken into account. And the difference between variants by the end of development reaches 43 percent. In addition, there is a more intense pressure drop at the start of the process in both variants. This period lasts about 2.5 years. Further, pressure reduction intensity decreases somewhat. This phenomenon is associated with the characteristic changes in production rates over time (see: in Figure 4(a)). Curves $q_o(t)$ show that at the beginning the development process is accompanied by an intensive drop in reservoir pressure in both variants. Attention is drawn to the fact that the initial production rate for Darcy filtering is almost twice as large as in the case when the Forchheimer equation was used as the filtration law. In this respect, it should be noted that this filtration model does not consider the first phase of filtration, within which the drainage radius continuously increases, at the boundary of which the pressure remains constant equal to the initial one and after a certain time reaches the contour where the pressure gradient is zero and the pressure begins to drop. It is known that the first phase lasts only a few hours, so this period is usually not considered. However, it should also be noted that during such a short time the well production rate reaches its maximum level from zero. Therefore, in the $q_o(t)$ plot the flow rate does not start from zero. Thus, it turns out that such a big difference between the initial production rates is due to the influence of inertial effects at the beginning, and specifically in the first phase of filtration, which is understandable.

If we look at such an important parameter as the oil recovery factor, we will see that, in the flow according to the Forchheimer law, this figure is significantly less than in the case of the Darcy law. So, if for 5.5 years in the case of the Darcy law, oil is extracted at 30% of the initial balance oil reserves, then under the Forchheimer law, to achieve the same level of production, it takes 6.9 years, which is 25.4% more. And for $\eta = 50\%$ development time is 12.5 and 15.3 years, respectively.

Comparative high reservoir pressures in the case when inertial effects are considered to provide higher values of oil saturation (Figure 5(b)) and relatively high gas factors (Figure 5(a)).

So, the results presented above confirmed that the influence of inertial effects on the development process of volatile oil deposits can be significant, despite the fact that the volatile oil has a high viscosity in comparison with gas.

5. Conclusion

A formula is obtained for the calculation of the production rate at the current reservoir pressure and oil saturation. To achieve this, an original mathematical approach was applied which allowed the modification of the Darcy's flow model for that of Forchheimer. A complete mathematical model has been created for predicting the main parameters of the development of a volatile oil reservoir, taking into account non-linear filtration effects and using the material balance equations. Based on this model, a computer program [23, 27] has been developed that allows the study of the influence of inertial forces on reservoir key performance indicators. A comparison of the results of computer simulation based on Darcy's and Forchheimer's laws showed that the influence of inertial effects on the development process is significant. Therefore, consideration of non-linearity of the filtration should be taken into account in modeling and designing the development of volatile oil reservoirs.

It is also established that the rock compressibility weakens the influence of inertial effects on the process through a reduction in production rate.

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7. Nomenclature

Parameters with the "o" and "g" index correspond to a liquid and gas phase, respectively;

p_0= initial reservoir pressure, *atm*

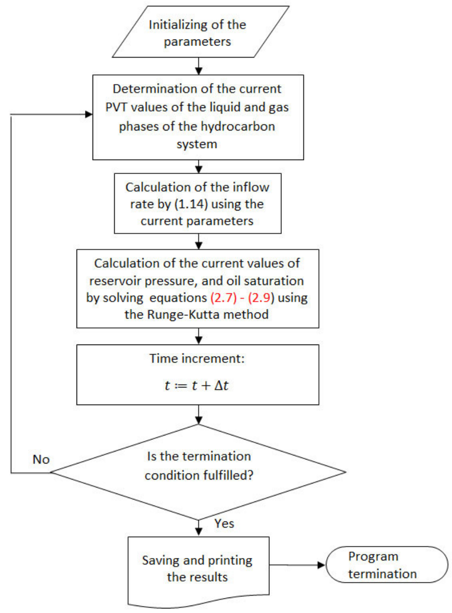


FIGURE 1. Simplified flowchart of the algorithm for calculation of the general development parameters.

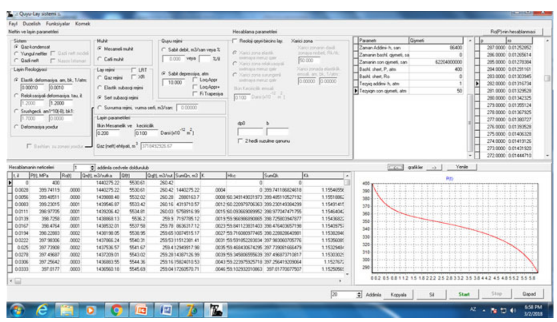


FIGURE 2. Computer simulation of the development process of the volatile oil reservoir.

p_w= = bottomhole pressure, atm
p_e= pressure at the external boundary, atm
p_{at}= atmospheric pressure, atm
r_e= reservoir or drainage area radius, m
r_w= wellbore radius, m
a_m= rock compressibility factor, $1/atm$
k= formation effective permeability, $10^{-12}m^2$
k_0 = initial permeability, $10^{-12}m^2$
k_{ro}= oil-phase relative permeability, dimensionless
k_{rg}= gas-phase relative permeability, dimensionless
s= oil saturation, dimensionless
v= velocity, m/s
Ω= oil-saturated porous volume, m^3

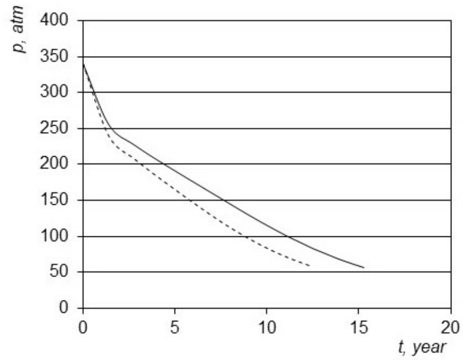
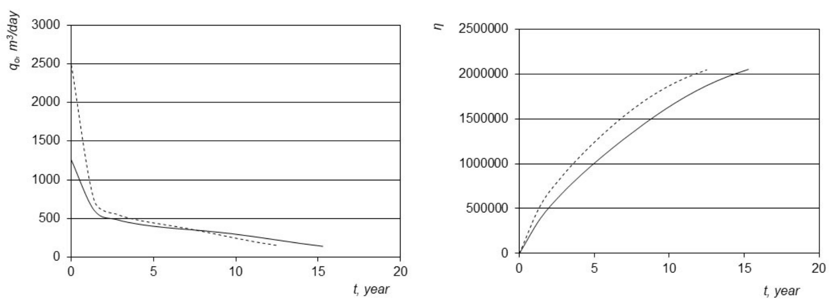
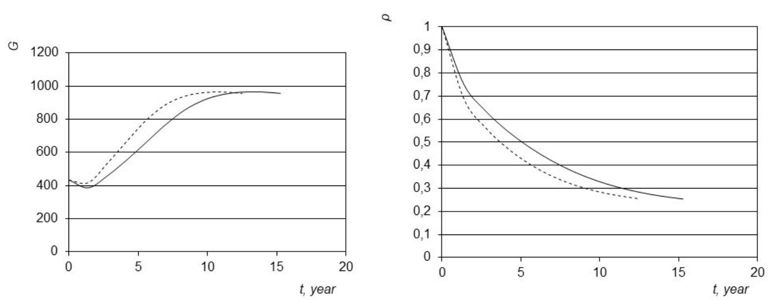


FIGURE 3. Reservoir pressure drop vs. time in the case when non-linear effects are considered and they are ignored.



(a) Change of oil production rate vs. time. (b) Change of cumulative oil production vs. time.

FIGURE 4.



(a) Change of gas/oil ratio vs. time. (b) Change of the oil saturation vs. time.

FIGURE 5.

Ω_0= initial porous volume, m^3
q_o= oil production rate, m^3/s
μ_o= oil viscosity, $atm \cdot s$
μ_g= gas-phase viscosity, $atm \cdot s$
ρ= oil density, kg/m^3

B_o= oil volume factor, dimensionless
S = solubility of gas in liquid phase, m^3/m^3
z= gas-law deviation factor (gas compressibility factor), dimensionless
β = temperature correction factor, dimensionless
c = vaporous hydrocarbons content in the gas phase, m^3/m^3
$\overline{\gamma} = \frac{\gamma_o(p)}{\gamma_g(p)}$ ratio of the specific weight of liquid phase and the specific weight of gas phase under reservoir pressure;
ϕ= formation porosity, dimensionless
ϕ_0 = initial formation porosity, dimensionless
t = time, s.

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