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An interval chaos insight to iterative decomposition method for Rossler differential equation by considering stable uncertain coefficients

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Abstract

Generally, in most applications of engineering, the parameters of the mathematical models are considered deterministic. Although, in practice, there are always some uncertainties in the model parameters; these uncertainties may be made wrong representation of the mathematical model of the system. These uncertainties can be generated from different reasons like measurement error, inhomogeneity of the process, chaotic behavior of systems, etc. This problem leads researchers to study these uncertainties and propose solutions for this problem. The iterative analysis is a method that can be utilized to solve these kinds of problems. In this paper, a new combined method based on interval chaotic and iterative decomposition method is proposed. The validation of the proposed method is performed on a chaotic Rossler system in stable Intervals. The simulation results are applied on 2 practical case studies and the results are compared with the interval Chebyshev method and Runge–Kutta method of order four (RK4) method. The final results showed that the proposed method has a good performance in finding the confidence interval for the Rossler models with interval uncertainties; the results also showed that the proposed method can handle the wrapping effect in a better manner to sharpen the range of non-monotonic interval.

Keywords. Chaos theory, Chaotic systems, Interval decomposition method, Rossler differential equations.

2010 Mathematics Subject Classification. 65L05, 34K06, 34K28.

1. Introduction

Generally, during the mathematical modeling of Chaotic Differential Equations (CDEs), the corresponding parameters have been considered exact values. However, the parameters of these chaotic systems have some uncertainties. These uncertainties can be generated from different reasons like neglecting some nonlinear terms on the model, simplifications and etc. These uncertainties lead the researcher to solve problems in the wrong way and consequently, the final result will be wrong. Uncertainties can be modeled by probabilistic variables, interval variables, etc. But the most proper method is to use the interval arithmetic [1, 4, 5, 12]. In the interval arithmetic, uncertainties stand throughout definite lower and upper bounds [9, 13, 19]. In other words, although the uncertainty quantity is unknown, but an interval can be defined for them. Chaotic systems like Rossler, Lorenz, and Chen include a wide range of applications like systems modeling, control, etc [1, 7, 8, 11]. There are different techniques that are introduced to solve these types of systems.

In recent decades, Iterative Interval Decomposition Methods (IIDM) have been shown as an effective, easy, and accurate methods to solve a great deal of Nonlinear, Chaos deterministic, or stochastic systems by approximation. They have also rapid convergence to achieve accurate solutions [6, 13, 17].

It is clear that the IIDM is a proper method for solving the CDEs. Now, what happened if these systems have some uncertainties? In this study, an improved decomposition method is introduced to achieve a proper and robust solution. The main idea is to find a proper interval bound that keeps the system stable even if the parameters are

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changed in the considered interval uncertainty [1, 12]. We also benefit from the orthogonal polynomial to simplify the complicated source terms to achieve a more compressed solution rather than the Taylor series [9, 16].

In this paper, the system which is of interest to us is again the Rossler system. As is well-known, the Rossler Differential Equations (RDEs) do not admit a closed form solution and moreover, they can exhibit both chaotic and non-chaotic behavior for distinct parameter values. This system has many applications, such as physical systems and genome reconstruction. Proper interval chaos has been subject to extensive theoretical study, and there are several representations and many characterizations of them [2]. Also, we give a new interval Iterative method for RDE that can be seen as a generalization of previous representations [2, 7, 14].

2. Attractive Interval Arithmetics

When a mathematical model of an engineering system is built, there are always some simplifications; although simplification reduces the system complication, but it makes some natural uncertainties in the model. In other words, some uncertain coefficients are appeared in the model [11, 17]. Hence, utilizing normal methods for modeling or solving these types of systems cause some problems. However, an uncertainty coefficient has an unknown quantity weight on the edges, but it is bounded and can be considered in an interval. Iterative Interval arithmetic provides a set of methods to keep track of these uncertainties during the computations [4, 13]. The interval set for an interval number can be described as,

$$\mathbb{R}_\Lambda = \{\Lambda = [\underline{\lambda}, \overline{\lambda}] |\ \underline{\lambda} = \inf \Lambda,\ \bar{\lambda} \sup \Lambda \in \mathbb{R},\ \underline{\lambda} \leq \overline{\lambda}\}. \tag{2.1}$$

The midpoint value, the width of the interval number, and the radius of an interval can be defined as:

$$\lambda_C = \frac{1}{2}\left(\overline{\lambda} + \underline{\lambda}\right), \qquad \lambda_M = \frac{\lambda_W}{2}, \qquad \lambda_W = \overline{\lambda} - \underline{\lambda}. \tag{2.2}$$

The basic interval arithmetic operations are described so that the interval guarantees the reliability of interval results. The main interval arithmetic operations between two interval numbers λ and γ are given as follows:

$$\Lambda \pm \Gamma = [\underline{\lambda} \pm \underline{\gamma}, \overline{\lambda} \pm \overline{\gamma}],$$

$$\Lambda \times \Gamma = [min\{\underline{\lambda}\underline{\gamma}, \overline{\lambda}\underline{\gamma}, \underline{\lambda}\overline{\gamma}, \overline{\lambda}\overline{\gamma}\}, \max\{\underline{\lambda}\underline{\gamma}, \overline{\lambda}\underline{\gamma}, \underline{\lambda}\overline{\gamma}, \overline{\lambda}\overline{\gamma}\}],$$

$$\Lambda/\Gamma = \Lambda \times \frac{1}{\Gamma}, \qquad \frac{1}{\Gamma} = \left\{\frac{1}{\gamma} | \gamma \in \Gamma\right\}, if \quad 0 \notin \Gamma,$$

The interval function F is an inclusion function f if $\forall \Lambda = \subseteq \mathbb{R},\ f(\Lambda) \subset F(\Lambda)$. The main objective of this study is to find an interval function F from f to achieve an interval form of our method.

3. IIDM For Chaotic Quadratic Systems

In this paper, we study a special class of sensetive quadratic systems with uncertainty parameters

$$\Sigma_\Lambda = \begin{cases} \dot{X}(t) & = AX(t) + BX(t)X^T(t) + R(t), \qquad t \in \mathbb{R}^+, \\ \\ Y(t) & = CX(t), \end{cases} \tag{3.1}$$

so, parametric linearization form we have

$$\Sigma_\Lambda = \begin{cases} \dot{X}(t) & = V(X)X(t) + R(t), \qquad V(X) = A + F(X, B), \\ \\ Y(t) & = CX(t), \end{cases} \tag{3.2}$$

where $X(t) = (x_1(t); \cdots; x_n(t))^T \in \mathbb{R}^n$, $R(t) = (r_1(t), \cdots, r_m(t))^T$ and $Y(t) = (y_1(t), \cdots, y_p(t))^T \in \mathbb{R}^p$ are respectively the state, the input and the output vectors. For $B \in \mathbb{R}^{n \times p}$ and $C \in \mathbb{R}^{p \times n}$ are matrices which elements are either fixed to zero or assumed free non-zero parameters and also we assume $R(t)$ an interval, where $\alpha \leq b_{ij}(t) \leq \beta, \quad \alpha, \beta \in \mathbb{R}$

and this is important for slove the RDEs [1, 2, 7, 14].

In the iterative decomposition method, the unknown function, i.e. $X(t)$ is decomposed into an infinite series: $X(t) = \sum_{i=0}^{\infty} X_i(t)$ where $X_0, X_1, \cdots$ are evaluated recursively. It is important to know that if the function has nonlinearity $N(X(t))$, it should be solved by the following equation:

$$N(X(t)) = \sum\nolimits_{n=0}^{\infty} E_n, \tag{3.3}$$

where, $E_n = E_n(X_0(t), X_1(t), \dots, X_n(t))$ are the polynomials:

$$E_n = \frac{1}{n!}\frac{d^n}{d\alpha^n} N\left(\sum\nolimits_{i=0}^{\infty} \alpha^i X_i(t)\right)\Big|_{\alpha=0} \qquad n = 0, 1, 2, \dots$$

Now we will improve iterative decomposition method to CDEs, consider an quadratic differential equation 3.1 as follows:

$$LX(t) = AX(t) + BX(t)X^T(t) + U(t),$$

here, $X^T BX$ describes the quadratic nonlinear operator, L defines the highest invertible derivative, A is the linear differential operator less order than L and U represents the source term R and control variable. By applying the inverse term "L^{-1}" into the expression $LX = AX + X^T BX + U$, we have

$$X = \Theta_0 + G + L^{-1}(AX) + L^{-1}(BXX^T), \tag{3.4}$$

where the function G describes the integration of the source term and control term and Θ_0 is the given conditions, since RDEs are controllable, then we can use iterative method for Eq 3.4. By considering the last equation , the recurrence relation of x can be simplified as follows:

$$\begin{cases} X_0 = \Theta_0 + G, \\ \\ X_{k+1} = L^{-1}(AX_k) + L^{-1}(BX_k X_k^T), \quad k \geq 0. \end{cases} \tag{3.5}$$

If the series 3.5 converges to the considered purpose, then

$$X(t) = \lim_{M\to\infty} \tilde{X}_M(t), \tag{3.6}$$

where, $\tilde{X}_M(t) = \sum_{i=0}^{M} X_i(t)$ [13].
In [9, 16], a new improved version of the decomposition method is introduced using an orthogonal approximation method. The illustrated method has overcome to the Taylor series in accuracy to expand the source term function. The advantage of the modified approach is verified through several illustrative examples. Since, in this paper, we expand the source term in series:

$$R(t) \approx \sum_{i=0}^{N} a_i T_i(t), \tag{3.7}$$

where $T_i(t)$ represents the first kind of polynomial and can be evaluated as follows:

$$T_0(t) = 1,$$

$$T_1(t) = t,$$

$$T_{k+1}(t) = 2t T_k(t) - T_{k-1}(t), \quad k \geq 1$$

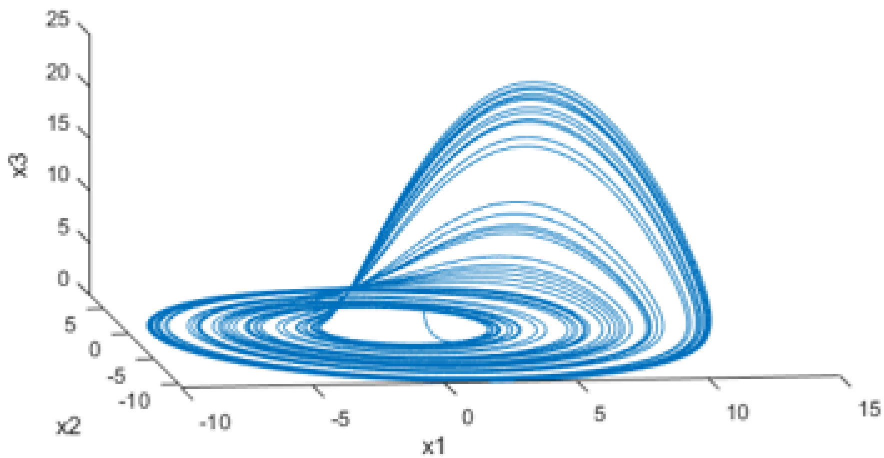

FIGURE 1. Chaotic Orbits of Rossler System

Since, by Eq 3.7 we have:

$$\begin{cases} X_0 = \Theta + L^{-1}\left(\sum_{i=0}^{N} a_i\, T_i(t)\right), \\ \\ X_{k+1} = L^{-1}(AX_k) + L^{-1}(BX_K X_K^T), \quad k \geq 0. \end{cases} \tag{3.8}$$

4. IIDM For CDE With Uncertainty

Definition 4.1. An "interval chaos" is a chaotic nonlinear system that is extreme sensitivity to small perturbations in its interval initial conditions by a variation on parameters.

An important problem in the nonlinear system is known as stabilizing unstable periodic orbits (UPOs) in chaotic systems. An efficient scheme for stabilizing UPOs using small parameter perturbation has been proposed by Ott, Grebogy, and York (OGY) [2, 11]. A scalar time delays constant which is the period of the UPOs must be stabilized to use the time-delayed state as a tracking UPOs embedded in chaotic attractors.

To generalize this concept, we use more than one interval for each initial value and parameter of the system by Eq (2.2). That is, given a bound λ, we say that $UPOs$ has a $\lambda-$interval representation. Without loss of generality, we assume that different intervals do not share endpoints. Thus the interval UPOs are the state S with $I(S) < \varepsilon$.

This method is in the form of recursive feedback proportional to the difference between the state of the new chaotic system and this past on old state at times and where denotes as the time for the control parameter and is adjusted to match the period of the UPOs to be stabilized. In this paper, we propose adaptive stabilizing control of chaotic systems with time-delayed feedback control. We consider a general controlled continues time uncertain chaotic system $\dot{X}(t) = f(X, t, \Theta, U)$ where $U = K(X(t) - X(t - \tau))$ corresponds to the control inputs with gain matrix K and Θ is a vector of unknown constant parameters. Then we design a fixed point stabilization controller to stabilize the UPOs solution embedded in the chaotic attractor region [11, 20].

Let $\Omega \in \mathbb{R}^n$ be a chaotic bounded attractor. Suppose that X_M is an unstable periodic orbit solution embedded in a chaotic bounded attracted set of the system. We consider the iterative feedback control with a proper step $h > 0$ by following $U = K(X_M(t) - X_M(t - \tau))$ to be added to the nonlinear system to form of controlled uncertain chaotic nonlinear dynamical systems $\dot{X}(t) = f(X, t, \Theta, U)$ such that the controlled system orbit can track the X_k on the other

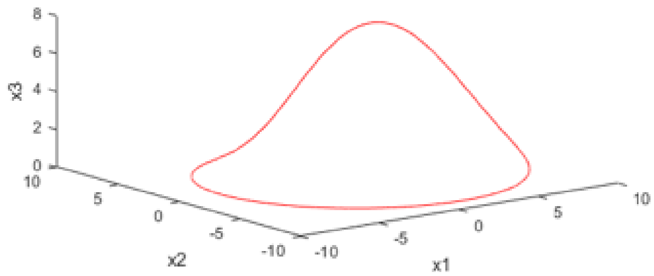

FIGURE 2. An Orbit of Rossler System

hand $X(t) = \lim_{M\to\infty} X_M(t)$. The goal is to find K with proper known step h that related numbers of iteration of M [20].

The following example illustrates the proposed sample of the special class the chaotic nonlinear quadratic system representation.

Figure 1 shows the chaotic behavior of the Rossler system. we apply the effectiveness of the proposed method to iterative stabilizing inherent UPOs in the Rossler system with uncertain parameters:

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ x_3 & 0 & -c \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} + U. \tag{4.1}$$

The uncontrolled $U = 0$ Rossler system exhibits a chaotic behavior if $a = b = 0.2$ and $c = 5.7$. We used the parametric linearization decomposition method to solve the systems with time step size $h = 0.001$. We let run until a periodic orbit of a predetermined length is located. The simulation of Rossler's system started at an arbitrary initial condition targeting a UPO of a length near. Figure2 shows the chaotic behavior of the Rossler system whit the initial condition, and the time response Rossler system states $\tau = 5.86$ [15].

5. Numerical illustrative examples

To demonstrate the effectiveness of the proposed method, we give two different examples of Rossler nonlinear differentials. The iterative Scheame 3.5 or 3.8 was coded in MATLAB and we employ the MATLAB's built-in fourth-order Runge-Kutta procedure RK4. We have set the parameters a and b at 0.2 with $c = 2.3$ (for non-chaotic) and $c = 5.7$ (for chaotic). The initial conditions used are $x_1(0) = 2.0$, $x_2(0) = 3.0$ and $x_3(0) = 2.0$ for both computations[7].

5.1. **Case study 1.** First, we consider the non-chaotic solutions of the system 4.1 for $0 \le t \le 20$ when a and b at 0.2 with $c = 2.3$

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ x_3 & 0 & -2.3 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix}, \tag{5.1}$$

where

$$\Sigma_\Lambda = \begin{cases} \dot{X}(t) & = V(X)X(t) + U(t), \\ Y(t) & = CX(t), \end{cases} \tag{5.2}$$

when

Table 1. **(Non-chaotic case)** Absolute errors between various MIIDM and RK4 solutions $\Delta X = ||X_{RK4} - X_{MIIDM}||$ with $h = 0.001$.

Time	Δx_1	Δx_2	Δx_3	$MaxError$
0.0	0.000	0.000	0.000	0.000
4	4.111E-06	2.481E-06	1.249E-07	10E-07
8	2.135E-05	6.754E-06	2.451E-07	10E-07
12	6.247E-06	2.411E-06	2.059E-06	10E-06
16	8.084E-06	7.631E-06	4.612E-06	10E-06
20	1.675E-05	1.235E-05	1.892E-07	10E-05

$$\begin{cases} V(X) = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ x_3 & 0 & -2.3 \end{pmatrix}, \quad R = \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix}, \\ \\ x_1(0) \in [2.0]\,, \quad x_2(0) \in [\beta] = [2.99, 3.01]\,, \quad x_3(0) \in [2.0]\,, \end{cases} \tag{5.3}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix}. \tag{5.4}$$

The problem above shows a nonlinear differential equation where uncertainty in the initial condition $[\beta]$ is also uncertain and appeared the only thing we know is that it stands in an interval.
The purpose of the solution is to find a region that includes all different values within the represented interval. According to the formula 3.2,

$$LX = V(X)X + R. \tag{5.5}$$

By applying the inverse operator $L^{-1} = \int\limits_0^t \{.\}\, dt$ into the main equation 5.5 with 5.3,

$$X(t) = \begin{pmatrix} 2 - \int_0^t \sum_{n=0}^{\infty}(x_{2n}(t) + x_{3n}(t))dt \\ [2.99, 3.001] + \int_0^t \sum_{n=0}^{\infty}(x_{1n}(t) + 0.2x_{2n}(t))dt \\ 2 + 0.2t + \int_0^t \sum_{n=0}^{\infty}(x_{1n}(t) - 2.3)x_{3n}(t)dt \end{pmatrix} \tag{5.6}$$

and finally the recurrence relation 3.5 and initial values below can be utilized to achieve the $X(t)$.

$$x_{10}(t) = 2, \quad x_{20}(t) = [2.99, 3.01], \quad x_{30}(t) = 2 + 0.2t$$

By calculating the problem in the time interval between 0 and 20 and the same step size h = 0.001, The absolute values were obtained to determine its performance against RK4 in Table 1 [7] . As we proceed with comparing with 2-iterate of the Mean of the Iterative Interval Decomposition Method (MIIDM) with step (h = 0.001), the accuracy is strengthened by a maximum error of $|10^{-5}|$. Table 1 shows more details of this comparison. Ultimately, all the results present a clear message: IIDM is an excellent tool in solving the system with a non-chaotic behavior.

5.2. **Case study 2.** Now, consider the chaotic solutions of the Rossler system 4.1 for $0 \le t \le 20$ when a and b at 0.2 with $c = 5.7$. In this case, we have a problem with more complicated.

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ x_3 & 0 & -5.7 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix}, \tag{5.7}$$

where

TABLE 2. **(Chaotic case)** Absolute errors between various MIIDM and RK4 solutions $\Delta X = ||X_{RK4} - X_{MIIDM}||$ with $h = 0.001$.

Time	Δx_1	Δx_2	Δx_3	$MaxError$
0.0	0.000	0.000	0.000	0.000
4	4.081E-06	1.886E-06	2.270E-08	10E-06
8	5.049E-06	1.021E-05	7.475E-07	10E-05
12	7.343E-05	7.325E-05	3.475E-04	10E-04
16	1.652E-04	2.151E-07	3.763E-05	10E-04
20	1.167E-04	2.163E-05	1.232E-07	10E-04

$$\Sigma_\Lambda = \begin{cases} \dot{X}(t) & = V(X)X(t) + U(t), \\ Y(t) & = CX(t), \end{cases} \tag{5.8}$$

when

$$\begin{cases} A = 0, \quad V(X) = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ x_3 & 0 & -5.8 \end{pmatrix}, \quad R = \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix}, \\ \\ x_1(0) \in [1.99, 2.01], \quad x_2(0) \in \beta = [2.99, 3.01], \quad x_3(0) \in [1..99, 2.01], \end{cases} \tag{5.9}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{pmatrix} \times \begin{pmatrix} x_1(t) - x_1(t-\tau) \\ x_2(t) - x_2(t-\tau) \\ x_3(t) - x_3(t-\tau) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix}, \tag{5.10}$$

where $L = \dfrac{d}{dt}$, $AX = 0$, $U(t) = K(X(t) - X(t-\tau)) + R$, we get a controller U with error and trial $k_{11} = -0.1$, $k_{22} = -0.2$ and $k_{33} = -0.1$ stabilizing the system on UPO whit a known period τ and known parameters. From the main equation 3.8 with 5.7 and 5.10, we have the fixed point problem:

$$X = \begin{pmatrix} \alpha - \int_0^t \sum_{n=0}^{\infty} (0.1x_{1n}(t) - 0.1x_{1n}(t-\tau) + x_{2n}(t) + x_{3n}(t))dt \\ \beta - \int_0^t \sum_{n=0}^{\infty} (x_{1n}(t) - 0.2x_{2n}(t-\tau))dt \\ \gamma + 0.2t + \int_0^t \sum_{n=0}^{\infty} (x_{1n}(t)x_{3n}(t) - 5.8x_{3n}(t) - x_{3n}(t-\tau))dt \end{pmatrix} \tag{5.11}$$

Now using the decomposition method with initial conditions with disturbance $x_{10}(t) = \alpha$, $x_{20}(t) = \beta$, $x_{30}(t) = \gamma$, we get X, where

$$\alpha = [1.99, 2.01], \quad \beta = [2.99, 3.01], \quad \gamma = [1.09, 2.01].$$

Finally with using $\tilde{x}_{kM}(t) = \sum_{i=0}^{M} x_{ki}(t), \; k = 1, 2, 3$ the solution has been achieved[13].

In Table 2, we show a similar set of case studies as in the non-chaotic situation. The MIIDM was performed at 2 iteration steps with $h = 0.001$. The maximum error has now been decreased to 10^{-4} [7]. Observation shows that the accuracy between both time steps used is considered very precise. Furthermore, it is important to know that sometimes lower and upper bounds have crossed over with each other in the CDE. In this situation, we should consider the general bound in between them as the reliability region. We do note, however, that the results displayed in the chaotic case are less accurate compared to the non-chaotic case. This is due to the fact that its chaotic state has sensitive dependence on initial conditions.

6. Conclusions

The iterative interval decomposition method is introduced for solving Rossler equations with uncertainties. This approach provides a robust approximation of the solution. The main advantage of this approach over traditional numerical methods is that the proposed method is the first time which is used interval arithmetic to provide a robust result for Rossler CDE with uncertain coefficients. In addition, in necessary for increasing the system accuracy, Polynomials are utilized to expand the source term.

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