Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. 2, No. 1, 2014, pp. 26-36

Exact solutions of distinct physical structures to the fractional potential Kadomtsev–Petviashvili equation

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Abstract In this paper, Exp-function and $\left(\frac{G'}{G}\right)$-expansion methods are presented to derive traveling wave solutions for a class of nonlinear space-time fractional differential equations. As a results, some new exact traveling wave solutions are obtained.

Keywords. Exact solution; modified Riemann-Liouville derivative; solitons; space-time fractional potential Kadomtsev-Petviashvili (pKP) equation.

2010 Mathematics Subject Classification. 26A33, 34A08, 35R11, 83C15.

1. Introduction

Recently, theory and applications of fractional differential equations (FDEs) has been the focus of many studies due to their frequent appearance in various applications in mathematics, physics, biology, engineering, signal processing, systems identification, control theory, finance and fractional dynamics, and has attracted much attention of more and more scholars. The fractional differential equations (FDEs) have been investigated by many researchers [1, 2, 3]. Recent investigations show that the dynamic of many physical processes is described accurately by using fractional differential equations containing different types of fractional derivatives. The most popular derivatives of fractional order are the Caputo derivative, the Riemann-Liouville derivative and Grünwald-Letnikov derivative. A few years ago, Jumarie presented a different definition of the fractional derivative being a little modification of the Riemann-Liouville derivative.

Since fractional differential equations are used to describe a large variety of physical phenomena, finding exact solutions to FDEs is an important subject and a hot topic. Many powerful and efficient methods have been proposed so far including the fractional $\left(\frac{G'}{G}\right)$-expansion method [4, 5, 6, 7], the fractional exp-function method [8, 9, 10], the fractional first integral method [12, 13], the fractional sub-equation method [14, 15, 16], the fractional functional variable method [17], the fractioanal modified trial equation method [18, 19, 20], the fractional simplest equation method [21] and so on. Using these methods, solutions with various forms for given fractional differential equation have been established.

The organization of this paper is as follows. In section 2, we give the basic definitions and analysis of methods, then in section 3, we give applications. Some conclusions are given in last section.

2. Basic Definitions and Analysis of Methods

The Jumarie's modified Riemann–Liouville derivative [22] of order α is defined by the following expression:

$$D_t^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\xi)^{-\alpha}(f(\xi)-f(0))d\xi & , \quad 0<\alpha<1 \\ (f^{(n)}(t))^{(\alpha-n)} & , \quad n\le\alpha<n+1, \quad n\ge 1. \end{cases} \tag{2.1}$$

Now, some important properties of the fractional modified Riemann–Liouville derivative were summarized [23]

$$D_t^\alpha t^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)}t^{r-\alpha} \quad , \tag{2.2}$$

$$D_t^\alpha (f(t)g(t)) = g(t)D_t^\alpha f(t) + f(t)D_t^\alpha g(t), \tag{2.3}$$

$$D_t^\alpha f[g(t)] = f_g^{'}[g(t)]D_t^\alpha g(t) = D_g^\alpha [g(t)](g'(t))^\alpha. \tag{2.4}$$

The above equations play an important role in fractional calculus in the following applications.

Firstly we consider the following general nonlinear FDE of the type

$$P(u, D_t^\alpha u, D_x^\beta u, D_t^\alpha D_t^\alpha u, D_t^\alpha D_x^\beta u, D_x^\beta D_x^\beta u, ...) = 0, \quad 0<\alpha,\beta\le 1 \tag{2.5}$$

where u is an unknown function. Moreover, P is a polynomial of u and its partial fractional derivatives, in which the highest order derivatives and the nonlinear terms are involved.

Li and He [24, 25] proposed a fractional complex transform to convert fractional differential equations into ordinary differential equations (ODEs). So all analytical methods devoted to the advanced calculus can be easily applied to the fractional calculus. The traveling wave variable

$$u(x,t) = U(\xi), \tag{2.6}$$

$$\xi = \frac{kx^\beta}{\Gamma(1+\beta)} + \frac{\tau t^\alpha}{\Gamma(1+\alpha)}, \tag{2.7}$$

where τ and k are nonzero arbitrary constants, we can rewrite Eq. (2.5) in the following nonlinear ODE;

$$Q(U, U^{'}, U'', U''', ...) = 0, \tag{2.8}$$

where the prime denotes the derivation with respect to ξ.

According to exp-function method, which was developed by He and Wu [26], we assume that the wave solution can be expressed in the following form

$$U(\xi) = \frac{\sum_{n=-c}^{d} a_n \exp\left[n\xi\right]}{\sum_{m=-p}^{q} b_m \exp\left[m\xi\right]} \tag{2.9}$$

where p, q, c and d are positive integers which are known to be further determined, a_n and b_m are unknown constants. We can rewrite Eq. (2.9) in the following equivalent form.

$$U(\xi) = \frac{a_{-c} \exp\left[-c\xi\right] + ... + a_d \exp\left[d\xi\right]}{b_{-p} \exp\left[-p\xi\right] + ... + b_q \exp\left[q\xi\right]} \tag{2.10}$$

This equivalent formulation plays an important and fundamental part for finding the analytic solution of problems. To determine the value of c and p, we balance the linear term of highest order of equation Eq. (2.10) with the highest order nonlinear term. Similarly, to determine the value of d and q, we balance the linear term of lowest order of Eq. (2.10) with lowest order nonlinear term [27, 28].

Secondly suppose the solution of equation (2.8) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows:

$$U(\xi) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G}\right)^i, \quad a_m \neq 0, \tag{2.11}$$

where a_i $(i = 0, 1, 2, ..., m)$ are constants, while $G(\xi)$ satisfies the following second order linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{2.12}$$

where λ and μ are constants. Then the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in equation (2.8). By substituting equation (2.11) into equation (2.8) and using equation (2.12), we collect all terms with the same order of $\left(\frac{G'}{G}\right)$. Then by equating each coefficient of the resulting polynomial to zero, we obtain a set of algebraic equations for a_i $(i = 0, 1, 2, ..., m)$, λ, μ, τ and k. By solving the equations system, substituting a_i $(i = 0, 1, 2, ..., m)$, λ, μ, τ, k and the general solutions of equation (2.12) into equation (2.11), we can get a variety of exact solutions of equation (2.5) [29, 30].

3. Applications

We consider the space-time fractional potential Kadomtsev–Petviashvili (pKP) equation [31] in the form:

$$\frac{1}{4}D_x^{4\alpha}u+\frac{3}{2}D_x^{\alpha}uD_x^{2\alpha}u+\frac{3}{4}D_y^{2\alpha}u+D_t^{\alpha}(D_x^{\alpha}u)=0,\quad t>0,\ 0<\alpha\leq 1, \tag{3.1}$$

where α is a parameter describing the order of the fractional space-time derivative. When $\alpha=1$ in Eq. (3.1) is the fractional differential equation reduces to the KP type equation [32-37].

For our purpose, we introduce the following transformations;

$$u(x,y,t)=U(\xi) \tag{3.2}$$

$$\xi=\frac{kx^{\alpha}}{\Gamma(1+\alpha)}+\frac{ny^{\alpha}}{\Gamma(1+\alpha)}+\frac{ct^{\alpha}}{\Gamma(1+\alpha)}, \tag{3.3}$$

where $k,\ n$ and c are nonzero constants.

Substituting (3.3) into (3.1), reduces (3.1) into an ODE

$$\frac{k^4}{4}U''''+\frac{3k^3}{2}U'U''+\frac{3n^2}{4}U''+ckU''=0, \tag{3.4}$$

where "U'" $=\frac{dU}{d\xi}$.

Integrating equation (3.4) with respect to ξ yields

$$\frac{k^4}{4}U'''+\frac{3k^3}{4}(U')^2+(\frac{3n^2}{4}+ck)U'+\xi_0=0, \tag{3.5}$$

where ξ_0 is a constant of integration.

Firstly we begin the pKP equation to solve by using the exp-function method. We can determine values of d and q by balancing the order of U''' and $(U')^2$ in Eq.(3.5), we get

$$U'''=\frac{d_1\exp[(7q+d)\xi]+...}{d_2\exp[8q\xi]+...}, \tag{3.6}$$

and

$$(U')^2=\frac{d_3\exp[2(d+q)\xi]+...}{d_4\exp[4q\xi]+...}, \tag{3.7}$$

where d_i are determined coefficients only for simplicity. Balancing highest order of Exp-function in Eqs.(3.6) and (3.7) we obtain

$$7q+d=2d+6q, \tag{3.8}$$

which leads to the result

$$q=d. \tag{3.9}$$

In the same way, to determine values of c and p, we balance the linear term of the lowest order in Eq.(3.5),

$$U''' = \frac{... + c_1 \exp[-(7p + c)\xi]}{... + c_2 \exp[-8p\xi]}, \tag{3.10}$$

and

$$(U')^2 = \frac{... + c_3 \exp[-2(c + p)\xi]}{... + c_4 \exp[-4p\xi]}, \tag{3.11}$$

where c_i are determined coefficients only for simplicity. From (3.10) and (3.11), we have

$$-7p - c = -2c - 6p, \tag{3.12}$$

and this gives

$$p = c. \tag{3.13}$$

For simplicity, we set $p = c = 1$ and $q = d = 1$, so Eq.(2.6) reduces to

$$U(\xi) = \frac{a_1 \exp(\xi) + a_0 + a_{-1} \exp(-\xi)}{b_1 \exp(\xi) + b_0 + b_{-1} \exp(-\xi)}. \tag{3.14}$$

Substituting Eq.(3.14) into Eq.(3.5), and by the help of Maple, we have

$$\begin{aligned} \frac{1}{A}[&R_4 \exp(4\xi) + R_3 \exp(3\xi) + R_2 \exp(2\xi) + R_1 \exp(\xi) + R_0 \\ &+R_{-1} \exp(-\xi) + R_{-2} \exp(-2\xi) + R_{-3} \exp(-3\xi) + R_{-4} \exp(-4\xi)] = 0, \end{aligned} \tag{3.15}$$

where

$$
\begin{aligned}
&A = (b_{-1}\exp(-\xi) + b_0 + b_1\exp(\xi))^4,\\
&R_4 = \xi_0 b_1^4,\\
&R_3 = kca_1b_1^2b_0 - \tfrac{3}{4}n^2a_0b_1^3 + 4\xi_0b_1^3b_0 - \tfrac{1}{4}k^4a_0b_1^3\\
&-kca_0b_1^3 + \tfrac{3}{4}n^2a_1b_1^2b_0 + \tfrac{1}{4}k^4a_1b_1^2b_0,\\
&R_2 = 2kca_1b_1b_0^2 + 2kca_1b_1^2b_{-1} - 2kca_0b_0b_1^2 - \tfrac{3}{2}k^3a_1b_0a_0b_1\\
&-\tfrac{3}{2}n^2a_{-1}b_1^3 + 6\xi_0b_1^2b_0^2 - 2k^4a_{-1}b_1^3 + \tfrac{3}{4}k^3a_1^2b_0^2 + \tfrac{3}{4}k^3a_0^2b_1^2\\
&+4\xi_0b_1^3b_{-1} + 2k^4a_1b_1^2b_{-1} + \tfrac{3}{2}n^2a_1b_1b_0^2 + \tfrac{3}{2}n^2a_1b_1^2b_{-1}\\
&-2kca_{-1}b_1^3 - \tfrac{3}{2}n^2a_0b_1^2b_0 - k^4a_1b_1b_0^2 + k^4a_0b_1^2b_0,\\
&R_1 = 6kca_1b_1b_0b_{-1} - \tfrac{1}{4}k^4a_0b_1b_0^2 + 3k^3a_1^2b_0b_{-1} - \tfrac{5}{4}k^4a_{-1}b_1^2b_0\\
&+3k^3a_{-1}b_1^2a_0 - \tfrac{3}{4}n^2a_0b_1b_0^2 + kca_1b_0^3 - \tfrac{3}{4}n^2a_0b_1^2b_{-1} + 12\xi_0b_1^2b_0b_{-1}\\
&+\tfrac{23}{4}k^4a_0b_1^2b_{-1} - \tfrac{15}{4}n^2a_{-1}b_1^2b_0 - 3k^3a_1a_0b_{-1}b_1 - kca_0b_1b_0^2\\
&-3k^3a_1b_0a_{-1}b_1 - kca_0b_1^2b_{-1} - \tfrac{9}{2}k^4a_1b_1b_0b_{-1} + \tfrac{9}{2}n^2a_1b_1b_0b_{-1}\\
&-5kca_{-1}b_1^2b_0 + \tfrac{3}{4}n^2a_1b_0^3 + \tfrac{1}{4}k^4a_1b_0^3 + 4\xi_0b_1b_0^3,\\
&R_0 = 4kca_1b_1b_{-1}^2 + 4kca_1b_0^2b_{-1} - 4kca_{-1}b_1^2b_{-1} - 4kca_{-1}b_1b_0^2\\
&-8k^4a_1b_1b_{-1}^2 + k^4a_1b_0^2b_{-1} + 8k^4a_{-1}b_1^2b_{-1} - k^4a_{-1}b_1b_0^2\\
&-\tfrac{3}{2}k^3a_1b_0^2a_{-1} + 6\xi_0b_1^2b_{-1}^2 + 3n^2a_1b_1b_{-1}^2 + 3n^2a_1b_0^2b_{-1} - 3n^2a_{-1}b_1^2b_{-1},\\
&R_{-1} = -\tfrac{9}{2}n^2a_{-1}b_1b_0b_{-1} - 3k^3a_{-1}b_1a_0b_{-1} + kca_0b_1b_{-1}^2\\
&+\tfrac{9}{2}k^4a_{-1}b_1b_0b_{-1} + kca_0b_{-1}b_0^2 + 5kca_1b_0b_{-1}^2 - 3k^3a_1b_{-1}a_{-1}b_0\\
&-\tfrac{3}{4}n^2a_{-1}b_0^3 + 4\xi_0b_0^3b_{-1} - \tfrac{1}{4}k^4a_{-1}b_0^3 + \tfrac{1}{4}k^4a_0b_{-1}b_0^2 + \tfrac{3}{4}n^2a_0b_{-1}b_0^2\\
&+\tfrac{15}{4}n^2a_1b_0b_{-1}^2 - kca_{-1}b_0^3 + 3k^3a_1b_{-1}^2a_0 + 3k^3a_{-1}^2b_1b_0 + 12\xi_0b_1b_0b_{-1}^2\\
&+\tfrac{3}{4}n^2a_0b_1b_{-1}^2 + \tfrac{5}{4}k^4a_1b_0b_{-1}^2 - \tfrac{23}{4}k^4a_0b_{-1}^2b_1 - 6kca_{-1}b_1b_0b_{-1},\\
&R_{-2} = -\tfrac{3}{2}k^3a_{-1}b_0a_0b_{-1} - 2kca_{-1}b_0^2b_{-1} + 2kca_0b_{-1}^2b_0 - 2kca_{-1}b_1b_{-1}^2 + 2k^4a_1b_{-1}^3\\
&+\tfrac{3}{4}k^3a_{-1}^2b_0^2 + \tfrac{3}{4}k^3a_0^2b_{-1}^2 + 4\xi_0b_1b_{-1}^3 + \tfrac{3}{2}n^2a_1b_{-1}^3 + 6\xi_0b_0^2b_{-1}^2 - \tfrac{3}{2}n^2a_{-1}b_0^2b_{-1},\\
&+\tfrac{3}{2}n^2a_0b_{-1}^2b_0 - 2k^4a_{-1}b_1b_{-1}^2 + k^4a_{-1}b_0^2b_{-1} - k^4a_0b_{-1}^2b_0 - \tfrac{3}{2}n^2a_{-1}b_1b_{-1}^2 + 2kca_1b_{-1}^3,\\
&R_{-3} = -kca_{-1}b_0b_{-1}^2 + \tfrac{1}{4}k^4a_0b_{-1}^3 + 4\xi_0b_0b_{-1}^3 + \tfrac{3}{4}n^2a_0b_{-1}^3\\
&-\tfrac{1}{4}k^4a_{-1}b_0b_{-1}^2 - \tfrac{3}{4}n^2a_{-1}b_0b_{-1}^2 + kca_0b_{-1}^3,\\
&R_{-4} = \xi_0b_{-1}^4.
\end{aligned}
\tag{3.16}
$$

Solving this system of algebraic equations by using Maple, we get the following results

Case 1: Consider

$$
\begin{aligned}
&a_1 = a_1, && a_0 = \frac{b_0(a_1-2kb_1)}{b_1}, && a_{-1} = 0,\\
&b_1 = b_1, && b_0 = b_0, && b_{-1} = 0,\\
&c = -\frac{3n^2+k^4}{4k}, && k = k, && \xi_0 = 0,
\end{aligned}
\tag{3.17}
$$

where b_0 and $b_1 \neq 0$ are free parameters. Substituting these results into (3.14), we get the following exact solution

$$u_1(x,y,t) = \frac{a_1\exp(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}-\frac{(3n^2+k^4)t^\alpha}{4k\Gamma(1+\alpha)})+\frac{b_0(a_1-2kb_1)}{b_1}}{b_1\exp(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}-\frac{(3n^2+k^4)t^\alpha}{4k\Gamma(1+\alpha)})+b_0}. \tag{3.18}$$

Case 2: Consider

$$\begin{aligned} &a_1 = 0, && a_0 = \frac{b_0(a_{-1}+2kb_{-1})}{b_{-1}}, && a_{-1} = a_{-1},\\ &b_1 = 0, && b_0 = b_0, && b_{-1} = b_{-1},\\ &c = -\frac{3n^2+k^4}{4k}, && k = k, && \xi_0 = 0, \end{aligned} \tag{3.19}$$

where $b_{-1} \neq 0$ and a_{-1} are free parameters. Substituting these results into (3.14), we get the following exact solution

$$u_2(x,y,t) = \frac{\frac{b_0(a_{-1}+2kb_{-1})}{b_{-1}}+a_{-1}\exp(-(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}-\frac{(3n^2+k^4)t^\alpha}{4k\Gamma(1+\alpha)}))}{b_0+b_{-1}\exp(-(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}-\frac{(3n^2+k^4)t^\alpha}{4k\Gamma(1+\alpha)}))}. \tag{3.20}$$

If we take $b_1 = 1$, $b_0 = 1$, $a_1 = 1$ and $k = 1$ Eq. (3.18) u_1 becomes

$$u_1(x,y,t) = \frac{\cosh(\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}-\frac{(3n^2+1)t^\alpha}{4\Gamma(1+\alpha)})+\sinh(\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}-\frac{(3n^2+1)t^\alpha}{4\Gamma(1+\alpha)})-1}{\cosh(\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}-\frac{(3n^2+1)t^\alpha}{4\Gamma(1+\alpha)})+\sinh(\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}-\frac{(3n^2+1)t^\alpha}{4\Gamma(1+\alpha)})+1}, \tag{3.21}$$

Similarly, $b_{-1} = 1$, $b_0 = 1$, $a_{-1} = 1$ and $k = -1$ Eq. (3.20) u_2 becomes

$$u_2(x,y,t) = \frac{\cosh(\frac{-x^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}+\frac{(3n^2+1)t^\alpha}{4\Gamma(1+\alpha)})-\sinh(\frac{-x^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}+\frac{(3n^2+1)t^\alpha}{4\Gamma(1+\alpha)})-1}{\cosh(\frac{-x^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}+\frac{(3n^2+1)t^\alpha}{4\Gamma(1+\alpha)})-\sinh(\frac{-x^\alpha}{\Gamma(1+\alpha)}+\frac{ny^\alpha}{\Gamma(1+\alpha)}+\frac{(3n^2+1)t^\alpha}{4\Gamma(1+\alpha)})+1}, \tag{3.22}$$

which are the exact solutions of the space-time fractional pKP equation.

Remark 1: Comparing our results, Eqs. (3.21) and (3.22), with Borhanifar's results in [38], it can be seen that the results are different. And these solutions have not been reported other authors in the literature.

Now we study the pKP equation to solve by using the $\left(\frac{G'}{G}\right)$-expansion method. By using the ansatz (3.5), for the linear term of highest order U''' with the highest order and the nonlinear term $(U')^2$, balancing U''' with $(U')^2$ in Eq. (3.5) gives

$$m + 3 = 2m + 2, \tag{3.23}$$

so that

$$m = 1. \tag{3.24}$$

Suppose that the solutions of (3.5) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows:

$$U(\xi) = a_0 + a_1\left(\frac{G'}{G}\right), a_1 \neq 0. \tag{3.25}$$

By using Eq. (2.12) and Eq. (3.25) we have

$$U'(\xi) = -a_1\left(\frac{G'}{G}\right)^2 - a_1\lambda\left(\frac{G'}{G}\right) - a_1\mu, \tag{3.26}$$

and

$$\begin{aligned} U'''(\xi) = -6a_1\left(\tfrac{G'}{G}\right)^4 - 12a_1\lambda\left(\tfrac{G'}{G}\right)^3 - (8a_1\mu + 7a_1\lambda^2)\left(\tfrac{G'}{G}\right)^2 \\ -(8a_1\lambda\mu + a_1\lambda^3)\left(\tfrac{G'}{G}\right) - a_1\mu^2 - 2a_1\lambda^2\mu. \end{aligned} \tag{3.27}$$

By substituting Eqs. (3.25)-(3.27) into Eq. (3.5), collecting the coefficients of $\left(\frac{G'}{G}\right)^i$ $(i = 0, ..., 4)$ and setting them to zero, we obtain the equation system

$$\begin{aligned} &-\tfrac{3}{2}k^4a_1 + \tfrac{3}{4}k^3a_1^2 = 0, \\ &-3k^4a_1\lambda + \tfrac{3}{2}k^3a_1^2\lambda = 0, \\ &-2k^4a_1\mu - \tfrac{7}{4}k^4a_1\lambda^2 + \tfrac{3}{2}k^3a_1^2\mu + \tfrac{3}{4}k^3a_1^2\lambda^2 - kca_1 - \tfrac{3}{4}n^2a_1 = 0, \\ &-\tfrac{1}{4}k^4a_1\lambda^3 - \tfrac{3}{4}n^2a_1\lambda + \tfrac{3}{2}k^3a_1^2\lambda\mu - 2k^4a_1\lambda\mu - kca_1\lambda = 0, \\ &-\tfrac{1}{2}k^4a_1\lambda^2\mu - \tfrac{3}{4}n^2a_1\mu + \tfrac{3}{4}k^3a_1^2\mu^2 - \tfrac{1}{4}k^4a_1\mu^2 - kca_1\mu + \xi_0. \end{aligned} \tag{3.28}$$

By solving this system with the aid of Maple, we obtain

$$\begin{aligned} a_0 = a_0, \quad a_1 = 2k, \quad c = \tfrac{4k^4\mu - k^4\lambda^2 - 3n^2}{4k} \\ k = k, \quad n = n, \quad \xi_0 = \tfrac{1}{2}k^5\lambda^2\mu - \tfrac{1}{2}k^5\mu^2, \end{aligned} \tag{3.29}$$

where λ and μ are arbitrary constants. By using Eq. (3.29), expression (3.35) can be written as

$$U(\xi) = a_0 + 2k\left(\frac{G'}{G}\right). \tag{3.30}$$

By substituting general solutions of Eq. (2.12) into Eq. (3.30) we have three types of travelling wave solutions of the space-time fractional potential Kadomtsev–Petviashvili (pKP) equation as follows:

When $\lambda^2 - 4\mu > 0$, we obtain the hyperbolic function traveling wave solution

$$U_1(\xi) = a_0 - k\lambda + k\sqrt{\lambda^2 - 4\mu}\left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right), \tag{3.31}$$

where $\xi = \frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ny^\alpha}{\Gamma(1+\alpha)} + \frac{(4k^4\mu - k^4\lambda^2 - 3n^2)t^\alpha}{4k\Gamma(1+\alpha)}$, and C_1, C_2 are arbitrary constants. In particular, if $C_1 \neq 0,\ C_2 = 0,\ \lambda > 0,\ \mu = 0$, then the traveling wave solution of (3.31) can be written as:

$$u_3(x,y,t) = a_0 - k\lambda + k\lambda \tanh\left\{\frac{\lambda}{2}\left(\frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ny^\alpha}{\Gamma(1+\alpha)} - \frac{(k^4\lambda^2 + 3n^2)t^\alpha}{4k\Gamma(1+\alpha)}\right)\right\}. \tag{3.32}$$

And assuming $C_1 = 0,\ C_2 \neq 0,\ \lambda > 0,\ \mu = 0$,then we obtain

$$u_4(x,y,t) = a_0 - k\lambda + k\lambda \coth\left\{\frac{\lambda}{2}\left(\frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ny^\alpha}{\Gamma(1+\alpha)} - \frac{(k^4\lambda^2 + 3n^2)t^\alpha}{4k\Gamma(1+\alpha)}\right)\right\}. \tag{3.33}$$

When $\lambda^2 - 4\mu < 0$, we obtain the trigonometric function traveling wave solution

$$U_2(\xi) = a_0 - k\lambda + ik\lambda\left(\frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right), \tag{3.34}$$

Also, if we assume $C_1 \neq 0,\ C_2 = 0,\ \lambda > 0,\ \mu = 0$, then

$$u_5(x,y,t) = a_0 - k\lambda + k\lambda \tanh\left\{\frac{\lambda}{2}\left(\frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ny^\alpha}{\Gamma(1+\alpha)} - \frac{(k^4\lambda^2 + 3n^2)t^\alpha}{4k\Gamma(1+\alpha)}\right)\right\}, \tag{3.35}$$

and when $C_1 = 0,\ C_2 \neq 0,\ \lambda > 0,\ \mu = 0$, the solution of Eq. (3.34) becomes

$$u_6(x,y,t) = a_0 - k\lambda + k\lambda \coth\left\{\frac{\lambda}{2}\left(\frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ny^\alpha}{\Gamma(1+\alpha)} - \frac{(k^4\lambda^2 + 3n^2)t^\alpha}{4k\Gamma(1+\alpha)}\right)\right\}. \tag{3.36}$$

So we obtain the solutions $u_3(x,y,t)$ and $u_4(x,y,t)$.

When $\lambda^2 - 4\mu = 0$, we obtain the rational function solution

$$u_7(x,y,t) = a_0 - k\lambda + \frac{2kC_2}{C_1 + C_2\left(\frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ny^\alpha}{\Gamma(1+\alpha)} + \frac{(4k^4\mu - k^4\lambda^2 - 3n^2)t^\alpha}{4k\Gamma(1+\alpha)}\right)}. \tag{3.37}$$

Remark 2: Compare our results, Eqs. (3.32), (3.33) and (3.37),with Budhiraja's and Zayed's solutions in [39, 40], it can be seen that the results are different.

4. Conclusions

In this paper, we have seen that three types of exact analytical solutions including the hyperbolic function solutions, trigonometric function solutions and rational solutions for the space-time fractional pKP equation are successfully found out by using the exp-function and (G'/G)-expansion methods. This study shows that the exp-function and (G'/G)-expansion methods are quite efficient and practically well suited for finding exact solutions of the pKP equation. The performance of these methods are reliable and effective and give the exact solitary wave solutions and periodic wave solutions. The availability of computer symbolic systems like Mathematica or Maple facilitates the tedious algebraic calculations. Thus, we deduce that the proposed method can be extended to solve many systems of nonlinear time-fractional partial differential equations.

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