Research Paper
Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. 10, No. 3, 2022, pp. 746-754
DOI:10.22034/cmde.2021.45176.1903

On exact solutions of the generalized Pochhammer-Chree equation

Asif Yokus[1,*], Karmina K. Ali[1,2], Resat Yilmazer[1], and Hasan Bulut[1]
[1]Department of Mathematics, Faculty of Science, Firat University, Elazig, Turkey.
[2]Department of Mathematics, Faculty of Science, University of Zakho, Zakho, Iraq.

Abstract

In the current study, we consider the generalized Pochhammer-Chree equation with a term of order n. Based on the $(1/G^{'})$-expansion method and with the aid of symbolic computation, we construct some distinct exact solutions for this nonlinear model. Various exact solutions are produced to the studied equation including singular solutions and periodic wave solutions. In addition, 2D, 3D, and contour plots are graphed for all obtaining solutions via choosing the suitable values for the involved parameters. All gained solutions verify the governing equation.

Keywords. The generalized Pochhammer-Chree equation, The $(1/G^{'})$-expansion method, Exact wave solutions.
2010 Mathematics Subject Classification. 35-XX, 35Qxx, 35C07.

1. Introduction

Many scientists around the world are interested in studying nonlinear partial differential equations (NPDEs). NPDEs have been used in the modeling of complex nonlinear aspects that define some of our real-life problems in distinct nonlinear sciences, including modeling of relationships between atmospheric and oceanic impacts, optical fiber, hydrodynamics, fluid mechanics, medical imaging, and plasma physics. Because of their importance in our everyday lives, it is important to investigate the behavior of these models. Different numerical and computational methods have been used to study the NPDEs like the simplified Hirota's method [15, 16, 20], homotopy method [28], $1/G'$-expansion method [1],sinh-Gordon function method [31] the sine-Gordon method [2, 13], sub-equation analytical method [7], the Bernoulli approximation method [23], the ansatz-based methods in a novel way [24], and the symbolic computation method [5, 8–10, 29].

The Pochhammer-Chree equation represents the propagation of longitudinal deformation waves in the elastic rod was introduced in 1986 by Clarkson et al. as follows [6]

$$\phi_{tt} - \phi_{ttxx} - \frac{1}{n}(\phi^n)_{xx} = 0, \tag{1.1}$$

where $\phi(x,t)$ stands for the longitudinal displacement at time t, of a material point originally lying at the point x. Clarkson et al. [6] considered $n = 3$ or 5 which reflected two probable constitutive options for the material. Bogolubsky [4] acquired the solution of the soliton-type of Eq. (1.1) by taking $n = 2, 3, 5$. Zhang et al. [33] investigated Eq. (1.1) by using $n = 5$. Triki et al. [26] have been used $n = 6$ to investigate the exact solutions of Eq. (1.1). Mohebbi [21] used the discrete Fourier transform to study the Pochhammer–Chree equation.

This paper aims to use the $(1/G')$-expansion method to obtain different exact solutions to the generalized Pochhammer-Chree equation of order n, which is given by [14, 22].

$$\phi_{tt} - \phi_{ttxx} - \left(\alpha\phi + \beta\phi^{n+1} + \gamma\phi^{2n+1}\right)_{xx} = 0, \ n \geq 1, \tag{1.2}$$

Received: 25 March 2021 ; Accepted: 30 July 2021.
* Corresponding author. Email: asfyokus@yahoo.com, asfyokus@firat.edu.tr.

where α, β, and γ are constants, Eq. (1.2) is a nonlinear model of longitudinal wave propagation of elastic rods.

Many authors have been investigated Eq. (1.2) by using different methods by giving some specific values of n, α, β and γ, such as in [22] Parand and Rad used the Exp-function method, some hyperbolic and periodic singular solutions were obtained. In [14], El-Ganaini used the first integral method and constructed some complex exponential function solutions, complex traveling solitary wave solutions, complex periodic wave solutions, and complex rational function solutions. In [27], Wazwaz used the tanh-coth and sine-cosine to obtain some exact solutions including bell-shaped solitons, kink-shaped solitons, and periodic solutions. The extended-tanh method utilized to investigate Eq. (1.2) and some traveling wave solutions, including kink-shaped solitons, bell-shaped solitons, periodic solutions, rational solutions, and singular solitons were obtained [17]. Some traveling wave solutions of distinct structures, namely bell-shaped solitons, kink-shaped solitons, and periodic solutions offered in [34] by using the extended $\left(G^{'}/G\right)$-expansion method. The bifurcation theory of planar dynamical systems was used in [18], as a result, numbers of solitary waves and kink waves solutions were given. The $\left(G^{'}/G\right)$-expansion method was applied on the mentioned equation when $n = 1$, and some hyperbolic traveling wave solutions, and kink wave solutions were derived [19]. The Adomian decomposition method to study the suggested equation was used in [25].

2. Fundamental concepts of the proposed method

In this portion, the main concepts of the proposed method are presented [3, 11, 30, 32].
Consider a NPDE

$$P(\phi, \phi_t, \phi_x, \phi_{xx}, \phi_{tt}, \phi_{xt}, ...) = 0. \tag{2.1}$$

Step 1. Let us consider the traveling wave transformation as

$$\phi\left(x,t\right) = \Phi\left(\xi\right), \ \xi = k(x - ct), \tag{2.2}$$

where k and c are real constants. After some procedure, Eq. (2.1) reduces to

$$Q(\Phi', \Phi'', \Phi''', ...) = 0. \tag{2.3}$$

Step 2. Let the solution of Eq. (2.3) has the following form

$$\Phi\left(\xi\right) = \sum_{i=0}^{m} a_i \left(\frac{1}{G'}\right)^i, \tag{2.4}$$

where $a_0, \ a_1, \ a_2, ..., a_m$ are constants to be determine later, and m is a balance term, as well as $G = G(\xi)$ satisfy the following second order linear ordinary differential equation

$$G^{''} + \lambda G^{'} + \mu = 0, \tag{2.5}$$

where λ and μ are constants to be determined later. Substituting Eq. (2.4) into Eq. (2.3), we get a set of algebraic equations by setting the coefficients of $\left(\frac{1}{G'}\right)$ that have likely order equating to zero. We reduce these algebraic equations to find $a_i, i \geq 0$ and k, c scalars and after that substituting a_i and the general solutions of Eq. (2.4) into Eq. (2.3), one can get the solutions of Eq. (2.1).

Furthermore, we can take the solution of Eq. (2.5) as follows

$$G(\xi) = -\frac{\xi\,\mu}{\lambda} - \frac{\mathrm{e}^{-\xi\,\lambda} c_1}{\lambda} + c_2, \tag{2.6}$$

where, c_1 and c_2 are constants. If the derivative of the solution function that given by Eq. (2.6) according to the ξ variable is taken for once and necessary arrangements are made, we get

$$\frac{1}{G'} = \frac{1}{-\frac{\mu}{\lambda} + c_1 e^{-\lambda\xi}}. \tag{2.7}$$

When we transform the algebraic expression given by Eq. (2.7) to trigonometric function, where $c_1 = A$, it can be rewritten as follows

$$\frac{1}{G'} = \frac{\lambda}{-\mu + \lambda A(Cosh(\lambda\xi) - Sinh(\lambda\xi))}. \tag{2.8}$$

3. Mathematical Analysis

In this section, we applied the $(1/G^{'})$-expansion method to fine some distinct exact solutions to the generalized Pochhammer-Chree equation.

Consider

$$\phi(x,t) = \Phi(\xi), \ \xi = k(x - ct), \tag{3.1}$$

Plugging Eq. (3.1) into Eq. (1.2), and integrating twice, then Eq. (1.2) reduces to

$$k^2 \left(c^2 - \alpha\right) \Phi - k^4 c^2 \Phi'' - \beta k^2 \Phi^{n+1} - \gamma k^2 \Phi^{2n+1} = 0, \tag{3.2}$$

where $' = \frac{d}{d\xi}$, also the constants of integration are set to zero.
Now we are using the transformation

$$\Phi^n = H, \tag{3.3}$$

then

$$\Phi'' = \frac{1-n}{n^2} H^{\frac{1}{n}-2} (H')^2 + \frac{1}{n} H^{\frac{1}{n}-1} H''. \tag{3.4}$$

Substituting the transformations Eq. (3.3) and Eq. (3.4) into Eq. (3.2), which gives

$$n^2 k^2 \left(c^2 - \alpha\right) H^2 - k^4 c^2 n H H'' - k^4 c^2 (1-n) H'^2 - \beta k^2 n^2 H^3 - \gamma k^2 n^2 H^4 = 0. \tag{3.5}$$

Balancing HH'' with H^4 in Eq. (3.5), yields $m = 1$. With $m = 1$, Eq. (2.4) becomes

$$\Phi(\xi) = a_0 + a_1 \left(\frac{1}{G'}\right), \tag{3.6}$$

inserting Eq. (3.6) and its derivatives into Eq. (3.5) yields a polynomial in powers of $(\frac{1}{G'})$. Collecting the coefficients of $(\frac{1}{G'})$ with the likely order and setting each summation to zero, we get a set of algebraic equations. Solving these equations, the following solutions can be constructed.

Set 1. When

$$\begin{aligned} &a_0 = \tfrac{(2+n)\left(c^2-\alpha\right)}{\beta}, \ a_1 = \tfrac{(2+n)\left(c^2-\alpha\right)\mu}{\beta\lambda}, \\ &k = \tfrac{n\sqrt{c^2-\alpha}}{c\lambda}, \ \gamma = -\tfrac{(1+n)\beta^2}{(2+n)^2(c^2-\alpha)}, \end{aligned} \tag{3.7}$$

substituting Eq. (3.7) into Eqs. (3.5) and (3.2) consequently, we get a singular solution

$$\phi_1(x,t) = \left(\frac{C_1 e^{nt\sqrt{c^2-\alpha}} (2+n)\left(c^2-\alpha\right)\lambda}{\beta\left(C_1 e^{nt\sqrt{c^2-\alpha}}\lambda - e^{\frac{nx\sqrt{c^2-\alpha}}{c}}\mu\right)}\right)^{\frac{1}{n}}. \tag{3.8}$$

Set 2. When

$$\begin{aligned} &a_0 = -\tfrac{i\sqrt{1+n}\sqrt{c^2-\alpha}}{\sqrt{\gamma}}, \ a_1 = -\tfrac{i\sqrt{1+n}\sqrt{c^2-\alpha}\mu}{\sqrt{\gamma}\lambda}, \\ &k = -\tfrac{n\sqrt{c^2-\alpha}}{c\lambda}, \ \beta = \tfrac{i(2+n)\sqrt{c^2-\alpha}\sqrt{\gamma}}{\sqrt{1+n}}, \end{aligned} \tag{3.9}$$

where $i = \sqrt{-1}$. Plugging Eq. (3.9) into with Eqs. (3.5) and (3.2) subsequently, a singular solution can be obtained

$$\phi_2(x,t) = \left(-\frac{i\sqrt{1+n}\sqrt{c^2-\alpha}}{\sqrt{\gamma}} - \frac{i\sqrt{1+n}\sqrt{c^2-\alpha}\mu}{\sqrt{\gamma}\lambda\left(-\frac{\mu}{\lambda} + C_1\cosh(B) - C_1\sinh(B)\right)}\right)^{\frac{1}{n}}, \tag{3.10}$$

where $B = \left(\frac{nt\sqrt{c^2-\alpha}}{\lambda} - \frac{nx\sqrt{c^2-\alpha}}{c\lambda}\right)\lambda$.

Set 3. When

$$a_0 = -\frac{(1+n)\,\beta}{(2+n)\,\gamma},\ a_1 = -\frac{(1+n)\,\beta\mu}{(2+n)\,\gamma\lambda},\ c = \frac{\mathrm{i}n\sqrt{1+n}\beta}{\sqrt{k^2(2+n)^2\gamma\lambda^2}},\ \alpha = \frac{(1+n)\,\beta^2\left(-n^2+k^2\lambda^2\right)}{k^2(2+n)^2\gamma\lambda^2}, \tag{3.11}$$

putting Eq. (3.11) into Eqs. (3.5) and (3.2), we get

$$\phi_3\,(x,t) = \left(-\frac{(1+n)\,\beta}{(2+n)\,\gamma} - \frac{(1+n)\,\beta\mu}{(2+n)\,\gamma\lambda\left(-\dfrac{\mu}{\lambda} + C_1\cosh\left(M\right) - C_1\sinh\left(M\right)\right)}\right)^{\frac{1}{n}}, \tag{3.12}$$

where $M = \lambda\left(kx - \frac{\mathrm{i}kn\sqrt{1+n}t\beta}{\sqrt{k^2(2+n)^2\gamma\lambda^2}}\right)$.

Set 4. When

$$\begin{aligned} &a_0 = -\tfrac{(1+n)\beta}{(2+n)\gamma},\ a_1 = -\tfrac{(1+n)\beta\mu}{(2+n)\gamma\lambda},\\ &k = \tfrac{\mathrm{i}n\sqrt{1+n}\beta}{\sqrt{c^2(2+n)^2\gamma\lambda^2}},\ \alpha = c^2 + \tfrac{(1+n)\beta^2}{(2+n)^2\gamma}, \end{aligned} \tag{3.13}$$

substituting Eq. (3.13) into Eqs. (3.5) and (3.2), we get the periodic wave solution

$$\phi_4\,(x,t) = \left(-\frac{C_1\,(1+n)\,\beta\lambda}{(2+n)\,\gamma\left(C_1\lambda - \mathrm{e}^{-\frac{\mathrm{i}n\sqrt{1+n}(ct-x)\beta\lambda}{\sqrt{c^2(2+n)^2\gamma\lambda^2}}}\mu\right)}\right)^{\frac{1}{n}}. \tag{3.14}$$

Set 5. When

$$\begin{aligned} &a_0 = \tfrac{\mathrm{i}n\sqrt{-c^2+\alpha}a_1}{ck\mu},\ \beta = \tfrac{\mathrm{i}ck(2+n)\sqrt{-c^2+\alpha}\mu}{na_1},\\ &\gamma = -\tfrac{c^2k^2(1+n)\mu^2}{n^2a_1^2},\ \lambda = \tfrac{\mathrm{i}n\sqrt{-c^2+\alpha}}{ck}, \end{aligned} \tag{3.15}$$

putting Eq. (3.15) into Eqs. (3.5) and (3.2) consequently, a periodic wave solution can produced

$$\phi_5\,(x,t) = \left(\frac{\mathrm{i}n\sqrt{-c^2+\alpha}a_1}{ck\mu} + \frac{a_1}{\dfrac{\mathrm{i}ck\mu}{n\sqrt{-c^2+\alpha}} + C_1\cos\left(N\right) - \mathrm{i}C_1\sin\left(N\right)}\right)^{\frac{1}{n}}, \tag{3.16}$$

where $N = \frac{n(-ckt+kx)\sqrt{-c^2+\alpha}}{ck}$.

4. Conclusion

In this study, the generalized Pochhammer–Chree equation with a term of order n dispersion is investigated by using the $(1/G^{'})$-expansion method. It is worth noting that the strong $(1/G')$-expansion method can be applied to a wide range of high order NPDEs. As a result, we have obtained some exact solutions to the studied equation, including singular and periodic wave solutions. We conclude that our results are new and different in comparison with reported solutions in Refs. [14, 17–19, 22, 25, 27, 34], which are presented in the introduction section. We believe that our solutions can be useful in many of research areas, including fluid dynamics and physics. Besides, 2D, 3D, and contour plots are graphed for all obtaining solutions by choosing the suitable values of the involved parameters and setting $n = 1$, $n = 2$, $n = 3$, $n = 5$ and $n = 7$. This method is a strong method of obtaining traveling wave solutions for full NPDEs.

5. Figures

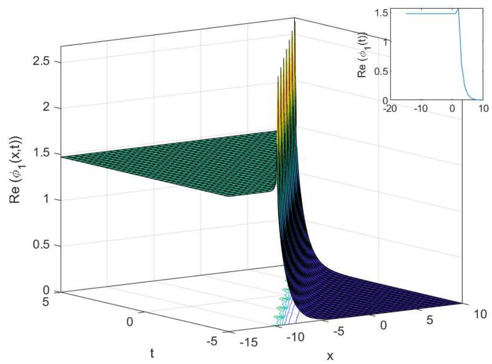

Figure 1. 2D, 3D and contour plots of Eq. (3.8), for $\mu = 0.3,\ c = 2,\ \alpha = 0.5,\ \lambda = 3,\ \beta = 2.1,\ C_1 = 0.3,$ and $n = 7$.

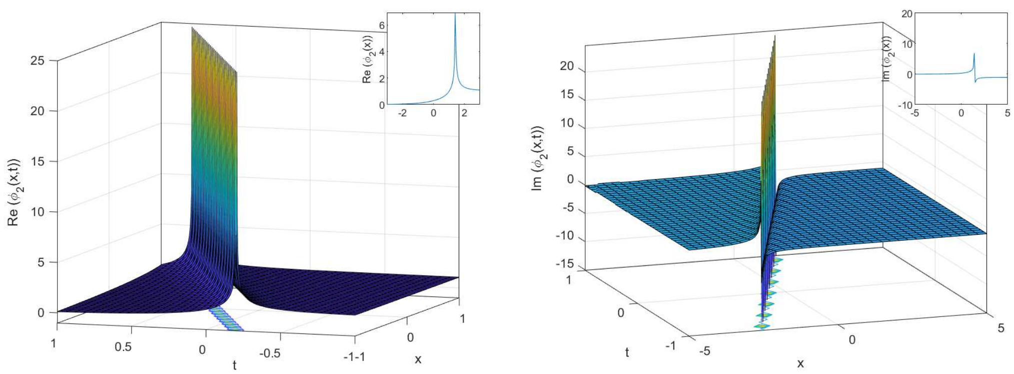

Figure 2. 3D, 2D and contour plots of Eq. (3.10), using $\mu = 0.3,\ c = 2,\ \alpha = 0.5,\ \lambda = 3,\ n = 2,\ C_1 = 0.3,$ and $\gamma = 2$.

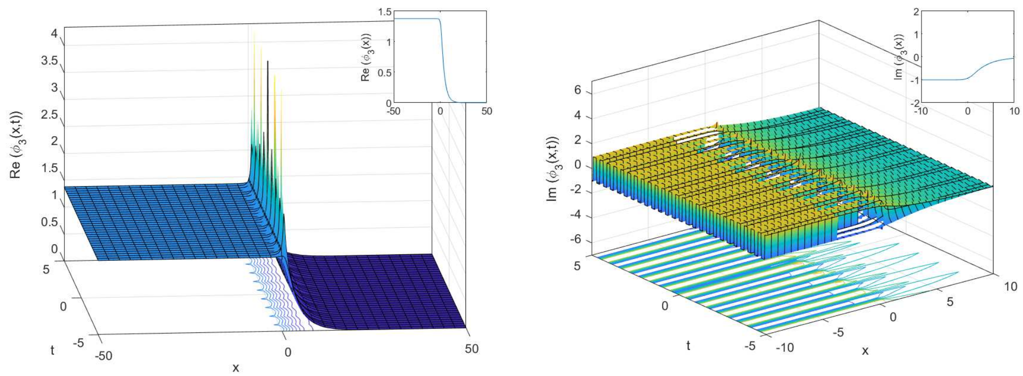

FIGURE 3. 3D, 2D and contour plots of Eq.(3.12) where $\mu = 3,\ \ \lambda = 3,\ \beta = 5,\ n = 5,\ C_1 = 3,\ \ k = 0.5, \text{and}\ \ \gamma = 0.3$.

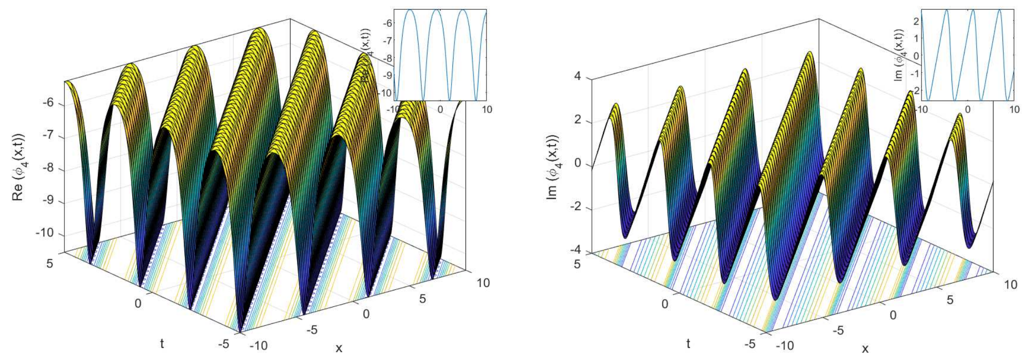

FIGURE 4. 3D, 2D and contour surfaces of Eq. (3.14) for $\mu = 0.3,\ c = 2,\ \ \lambda = 3,\ \beta = 2.1,\ n = 1,\ C_1 = 0.3, \text{and}\, \gamma = 0.2$.

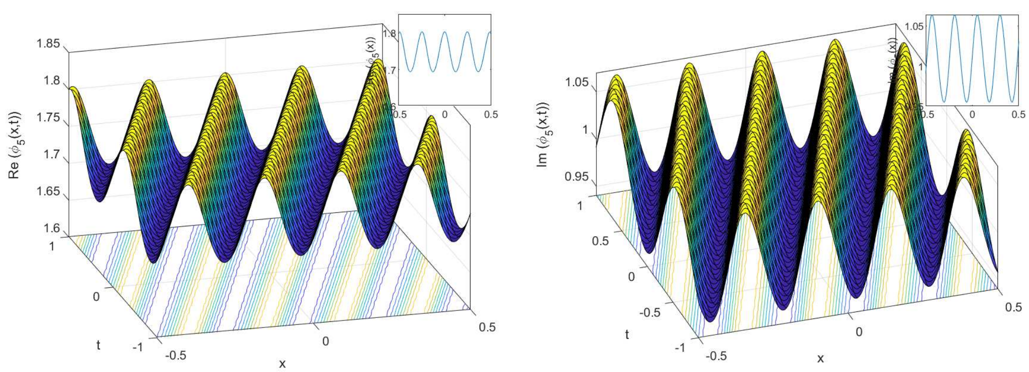

FIGURE 5. 3D, 2D and contour plots of Eq. (3.16) using $\mu = 3,\ c = 0.2,\ \alpha = 3,\ n = 3,\ C_1 = 3,\ k = 2.1, \text{and}\, a_1 = 2$

Acknowledgment

The authors are highly thankful and grateful to Firat University, Elazig, Turkey for generous support and facilities of this research work.

References

[1] K. K. Ali, A. R. Seadawy, A. Yokus, R. Yilmazer, and H. Bulut, *Propagation of dispersive wave solutions for (3+ 1)-dimensional nonlinear modified Zakharov–Kuznetsov equation in plasma physics*, International Journal of Modern Physics B, *34*(25) (2020), 2050227.
DOI: 10.1142/S0217979220502276.

[2] K. K. Ali, R. Yilmazer, H. M. Baskonus, and H. Bulut, *New wave behaviors and stability analysis of the Gilson–Pickering equation in plasma physics*, Indian Journal of Physics, *95*(5) (2021), 1003-1008.
DOI: 10.1007/s12648-020-01773-9.

[3] K. K. Ali, R. Yilmazer, A. Yokus, and H. Bulut, *Analytical solutions for the (3+ 1)-dimensional nonlinear extended quantum Zakharov–Kuznetsov equation in plasma physics* Physica A: Statistical Mechanics and its Applications, *548* (2020), 124327.
DOI: 10.1016/j.physa.2020.124327.

[4] I. L. Bogolubsky, *Some examples of inelastic soliton interaction*, Computer Physics Communications, *13*(3) (1977), 149-155.
DOI: 10.1016/0010-4655(77)90009-1.

[5] H. Bulut, T. A. Sulaiman, H. M. Baskonus, and T. Akturk, *Complex acoustic gravity wave behaviors to some mathematical models arising in fluid dynamics and nonlinear dispersive media*, Optical and Quantum Electronics, *50*(1) (2018), 19.
DOI: 10.1007/s11082-017-1286-y.

[6] P. A. Clarkson, R. J. LeVeque, and R. Saxton, *Solitary-Wave Interactions in Elastic Rods*, Studies in Applied Mathematics, *75*(2) (1986), 95-121.
DOI: 10.1002/sapm198675295.

[7] S. Duran and B. Karabulut, *Nematicons in liquid crystals with Kerr Law by sub-equation method*, Alexandria Engineering Journal, 2021.
DOI: 10.1016/j.aej.2021.06.077.

[8] S. Duran, *Extractions of travelling wave solutions of (2+ 1)-dimensional Boiti–Leon–Pempinelli system via (G'/G,1/G)-expansion method*, Optical and Quantum Electronics, *53*(6) (2021), 1-12.
DOI: 10.1007/s11082-021-02940-w.

[9] S. Duran, *Breaking theory of solitary waves for the Riemann wave equation in fluid dynamics*, International Journal of Modern Physics B, (2021), 2150130.
DOI: 10.1142/S0217979221501307.

[10] S. Duran, M. Askin, and T. A. Sulaiman, *New soliton properties to the ill-posed Boussinesq equation arising in nonlinear physical science*, An International Journal of Optimization and Control: Theories & Applications (IJOCTA), *7*(3) (2017), 240-247.
DOI: 10.11121/ijocta.01.2017.00495.

[11] H. Durur and A. Yokuş, *Discussions on diffraction and the dispersion for traveling wave solutions of the (2+ 1)-dimensional paraxial wave equation*, Mathematical Sciences, 2021, 1-11.
DOI: 10.1007/s40096-021-00419-z.

[12] H. Durur, *Different types analytic solutions of the (1+1)-dimensional resonant nonlinear Schrodinger's equation using* $(G'/G)-$ *expansion method*, Modern Physics Letters B, *34*(03) (2020), 2050036.
DOI: 10.1142/S0217984920500360

[13] H. Dutta, H. Günerhan, K. K. Ali, and R. Yilmazer, *Exact Soliton Solutions to the Cubic-Quartic Non-linear Schrödinger Equation With Conformable Derivative*, Frontiers in Physics, *8* (2020), 62.

[14] S. I. A. El-Ganaini *Travelling wave solutions to the generalized Pochhammer-Chree (PC) equations using the first integral method* Mathematical Problems in Engineering, *2011* (2011).
DOI: 10.1155/2011/629760.

[15] H. F. Ismael, A. Seadawy, and H. Bulut, *Multiple soliton, fusion, breather, lump, mixed kink-lump and periodic solutions to the extended shallow water wave model in (2+ 1)-dimensions*, Modern Physics Letters B. *35*(08) (2021), 2150138.
DOI: 10.1142/S0217984921501384.

[16] H. F. Ismael, H. Bulut, C. Park, and M. S. Osman, *M-lump, N-soliton solutions, and the collision phenomena for the (2+ 1)-dimensional Date-Jimbo-Kashiwara-Miwa equation*, Results in Physics, *19*, 103329.
DOI: 10.1016/j.rinp.2020.103329.

[17] B. Li, Y. Chen, and H. Zhang, *Travelling wave solutions for generalized pochhammer-chree equations*, Zeitschrift für Naturforschung A, *57*(11) (2002), 874-882.
DOI: 10.1515/zna-2002-1106.

[18] J. Li and L. Zhang, *Bifurcations of traveling wave solutions in generalized Pochhammer–Chree equation*, Chaos, Solitons & Fractals, *14*(4) (2002), 581-593.
DOI: 10.1016/S0960-0779(01)00248-X.

[19] A. Malik, F. Chand, H. Kumar, and S. C. Mishra, *Exact solutions of some physical models using the* (G'/G)*-expansion method*, Pramana, *78*(4) (2012), 513-529.
DOI: 10.1007/s12043-011-0253-6.

[20] W. X. Ma and L. Zhang, *Lump solutions with higher-order rational dispersion relations,* Pramana, *94*(1) (2020), 1-7.
DOI: 10.1007/s12043-020-1918-9.

[21] A. Mohebbi, *Solitary wave solutions of the nonlinear generalized Pochhammer–Chree and regularized long wave equations*, Nonlinear Dynamics, *70*(4) (2012), 2463-2474.
DOI: 10.1007/s11071-012-0634-5.

[22] K. Parand and J. A. Rad, *Some solitary wave solutions of generalized Pochhammer-Chree equation via Exp-function method* International Journal of Mathematical and Computational Sciences, *4*(7) (2010), 991-996.
DOI: 10.5281/zenodo.1079408.

[23] Z. Pinar, *Analytical study on the balancing principle for the nonlinear Klein–Gordon equation with a fractional power potential* Journal of King Saud University-Science, *32*(3) (2020), 2190-2194.
DOI: 10.1016/j.jksus.2020.02.032.

[24] Z. Pinar, *An Analytical Studies of the Reaction-Diffusion Systems of Chemical Reactions*, International Journal of Applied and Computational Mathematics, *7*(3) (2021), 1-10.
DOI: 10.1007/s40819-021-01028-z.

[25] N. Shawagfeh and D. Kaya, *Series solution to the Pochhammer-Chreeequation and comparison with exact solutions*, Computers & Mathematics with Applications, *47*(12) (2004), 1915-1920.
DOI: 10.1016/j.camwa.2003.02.012.

[26] H. Triki, A. Benlalli, and A. M. Wazwaz, *Exact solutions of the generalized Pochhammer-Chree equation with sixth-order dispersion*, Rom. J. Phys, *60* (2015), 935-951.

[27] A. M. Wazwaz, *The tanh–coth and the sine–cosine methods for kinks, solitons, and periodic solutions for the Pochhammer–Chree equations*, Applied Mathematics and Computation, *195*(1) (2008), 24-33.
DOI: 10.1016/j.amc.2007.04.066.

[28] M. Yavuz, *Dynamical behaviors of separated homotopy method defined by conformable operator*, Konuralp Journal of Mathematics (KJM), *7*(1) (2019), 1-6.

[29] G. Yel and T. Aktürk, *A New Approach to (3+ 1) Dimensional Boiti–Leon–Manna–Pempinelli Equation*, Applied Mathematics and Nonlinear Sciences, *5*(1) (2020), 309-316.

[30] A. Yokus and M. Yavuz, *Novel comparison of numerical and analytical methods for fractional Burger–Fisher equation*, Discrete & Continuous Dynamical Systems-S, *14*(7) (2021), 2591.
DOI: 10.3934/dcdss.2020258.

[31] A. Yokuş, H. Durur, K. A. Abro, and D. Kaya, *Role of Gilson–Pickering equation for the different types of soliton solutions: a nonlinear analysis*, The European Physical Journal Plus, *135*(8) (2020), 1-19.
DOI: 10.1140/epjp/s13360-020-00646-8.

[32] A. Yokuş, H. Durur, and K. A. Abro, *Symbolic computation of Caudrey–Dodd–Gibbon equation subject to periodic trigonometric and hyperbolic symmetries*, The European Physical Journal Plus, *136*(4) (2021), 1-16.
DOI: 10.1140/epjp/s13360-021-01350-x.

[33] W. Zhang, Y. Zhao, G. Liu, and T. Ning, *Periodic solutions for Pochhammer–Chree equation with five order nonlinear term and their relationship with solitary wave solutions*, International Journal of Modern Physics B, *24*(19) (2010), 3769-3783.
DOI: 10.1142/S0217979210056268.

[34] J. M. Zuo, *Application of the extended* $(G'/G)-$*expansion method to solve the Pochhammer–Chree equations*, Applied mathematics and computation, *217*(1) (2010), 376-383.
DOI: 10.1016/j.amc.2010.05.072.