*Computational Methods for Differential Equations*
http://cmde.tabrizu.ac.ir
Vol. 9, No. 4, 2021, pp. 1214-1222
DOI:10.22034/cmde.2020.24117.1294

# Second order boundary value problems of nonsingular type on time scales

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**Abstract** In this study, existence of positive solutions are considered for second order boundary value problems on any time scales even in the case when $y \equiv 0$ may also be a solution.

**Keywords.** Time scale, Boundary value problem, Fixed point theorem.

**2010 Mathematics Subject Classification.** 65L05, 34K06, 34K28.

## 1. Introduction

The theory of measure chains was introduced and developed by Hilger [7]. It has been created in order to unify continuous and discrete analysis, and it allows a simultaneous treatment of differential and difference equations, extending those theories to so-called dynamic equations. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of real numbers with the topology and ordering inherited from $\mathbb{R}$, and the cases when this time scale is equal to the reals or to the integers represent the classical theories of differential and of difference equations. Of course, many other interesting time scales exist, and they give rise to plenty of applications, for example, in the study of insect population models, neural networks, heat transfer and epidemic models. We refer the reader to the excellent introductory text by Bohner and Peterson [9] as well as the recent research monograph [3]. In recent years, there has been much research activity concerning the existence of solutions of various dynamic equations on time scales, e.g., see [8, 12] and the references cited therein. Throughout this paper, by an interval $(a, b)$, we mean the intersection of the real interval $(a, b)$ with the given time scale $\mathbb{T}$.

In this work, we'll be concerned with the following boundary value problems of non-singular type

Received: 16 November 2017 ; Accepted: 07 September 2020.
$*$ corresponding.

$$\begin{aligned} y^{\Delta\Delta}(t)+\Phi(t)f(t,y,y^{\Delta})=0, \quad t\in(a,b) \\ y(a)=y^{\Delta}(\sigma(b))=0, \end{aligned} \tag{1.1}$$

where $f:[a,b]\times[0,\infty)^2\to[0,\infty)$ and $\Phi:(a,\sigma(b))\to(0,\infty)$ are continuous. We assume that $\sigma(b)$ is right dense, so that $(\sigma(b))^2=\sigma(b)$.

This paper provides a new technique for showing that (1.1) has a solution $y>0$ on $(a,b]$. Non-singular problems have been discussed in detail in [1, 2, 4, 5, 6, 11]. Our theory complements and generalizes these results for the boundary value problem (1.1) on time scales.

In this section, we present some necessary theorems and preliminary results that will be used to prove our main result. In Section 2, we put forward and prove our main result and give some examples to illustrate the main result.

Our existence principles will be proved using the following fixed point result [10].

**Theorem 1.1.** *Assume $U$ is a relatively open subset of a convex set $C$ in a normed space $E$. Let $N:\bar{U}\to C$ be a compact map with $0\in U$. Then either*

*(A1) $N$ has a fixed point in $\bar{U}$; or*

*(A2) there is a $u\in\partial U$ and a $\lambda\in(0,1)$ such that $u=\lambda Nu$.*

**Theorem 1.2.** *Suppose the following conditions are satisfied, Assume*

$$\Phi\in C(a,\sigma(b)) \text{ with } \Phi>0 \text{ on } (a,\sigma(b)) \text{ and } \Phi\ \in L^1(a,\sigma(b)). \tag{1.2}$$

*and*

$$F:[0,1]\times\mathbb{R}^2\to\mathbb{R} \text{ is continuous.} \tag{1.3}$$

*are satisfied. In addition assume there is a constant $M$, independent of $\lambda$ with $|y|_1=max\{|y|_0,|y^{\Delta}|_0\}\neq M$ for any solutions $y\in C^1[a,\sigma(b)]\cap C^2(a,\sigma(b))$ to*

$$\begin{aligned} y^{\Delta\Delta}+\lambda\Phi(t)F(t,y,y^{\Delta})=0, \quad t\in(a,b) \\ y(a)=y^{\Delta}(\sigma(b))=0, \end{aligned} \tag{1.4}$$

*for each $\lambda\in(0,1)$; here $|y|_0=sup_{[a,\sigma(b)]}|y(t)|$ and $|y^{\Delta}|_0=sup_{[a,\sigma(b)]}|y^{\Delta}(t)|$. Then (1.4) has a solution $y\in C^1[a,\sigma(b)]\cap C^2(a,\sigma(b)))$ with $|y|_1\le M$.*

**Proof.** *Solving (1.4) is equivalent to the fixed point problem $y=\lambda Ny$, where*

$$Ny(t)=\int_a^t\int_a^s\Phi(u)F(u,y(u),y^{\Delta}(u)\Delta u\Delta s.$$

*Let $K[a,\sigma(b)]=\{y\in C[a_0,\sigma(b)]:y^{\Delta}\in C(a,\sigma(b))$ with norm $|y|_1\}$ which is a Banach space. It is a easy to see that $N:K[a,\sigma(b)]\to K[a,\sigma(b)]$ is continuous and completely continuous. Let $U=\{y\in K[a,\sigma(b)]:|y|_1<M\}$ and $C=E=K[a,\sigma(b)]$. Now, apply Theorem 1.1 implies that $N$ has a fixed point in $\bar{U}$ since the condition (A2) cannot occur.* □

**Lemma 1.1.** *Let $y^{\Delta\Delta}(t)<0$ on $(a,b)$ and $y(a)\ge 0$, then $y(t)\ge\dfrac{t-a}{\sigma(b)-a}y(\sigma(b))$ for $t\in(a,\sigma(b))$.*

**Proof.** *If $y^{\Delta\Delta}(t) < 0$ on $(a,b)$, then $y^{\Delta}(t)$ is non-increasing on $(a,\sigma(b))$.*

*Thus, we get $y(t) - y(a) = \int_a^t y^{\Delta}(s)\Delta s \geq (t-a)y^{\Delta}(t)$ and so $y(t) \geq y(a) + (t-a)y^{\Delta}(t)$.*

*Similarly, we get $y(\sigma(b)) - y(t) = \int_t^{\sigma(b)} y^{\Delta}(s)\Delta s \leq (\sigma(b)-t)y^{\Delta}(t)$ and so $y(t) \geq y(\sigma(b)) - (\sigma(b)-t)y^{\Delta}(t)$.*

*Together these two inequalities, we have $y(t) \geq \frac{\sigma(b)-t}{\sigma(b)-a}y(a) + \frac{t-a}{\sigma(b)-a}y(\sigma(b)) \geq \frac{t-a}{\sigma(b)-a}y(\sigma(b))$.* □

## 2. Second Order Problems

In this section, we discuss the second order non-singular problem (1.1). Throughout this section we'll assume the following conditions hold:

$$\Phi \in C(a,\sigma(b)) \text{ with } \Phi > 0 \text{ on } (a,\sigma(b)) \text{ and } \Phi \in L^1(a,\sigma(b)), \tag{2.5}$$

$$\begin{aligned} f : [a,b] \times [0,\infty)^2 \to [0,\infty) \text{ is continuous with } f(t,u,p) > 0 \\ \text{for } (t,u,p) \in [a,b] \times (0,\infty)^2, \end{aligned} \tag{2.6}$$

$$\begin{aligned} f(t,u,p) \leq w(max\{u,p\}) \text{ with } w > 0 \\ \text{continuous and nondecreasing on } [0,\infty), \end{aligned} \tag{2.7}$$

$$sup_{c\in(0,\infty)} \frac{c}{w(c)\int_a^{\sigma(b)} \Phi(s)\Delta s} > 1, \tag{2.8}$$

$$\begin{aligned} &\text{for a constant } H > 0 \text{ there exist a function } \Psi_H \\ &\text{continuous and positive on } (a,\sigma(b)), \\ &\text{and constants } \alpha,\beta \geq 0 \text{ with } \alpha+\beta < 1 \\ &\text{and with } f(t,u,p) \geq \Psi_H(t)u^{\alpha}p^{\beta} \text{ on } [a,b]\times[0,H]^2. \end{aligned} \tag{2.9}$$

**Theorem 2.1.** *Assume (2.5)-(2.9) hold. Then (1.1) has a solution $y \in C^1[a,\sigma(b)] \cap C^2(a,\sigma(b))$ with $y > 0$ on $(a,\sigma(b)]$.*

**Proof.** *Choose $M > 0$ with*

$$\frac{M}{w(M)\int_a^{\sigma(b)} \Phi(s)\Delta s} > 1. \tag{2.10}$$

*Next choose* $\varepsilon > 0$ *and* $\varepsilon < \dfrac{M}{2}$ *with*

$$\frac{M}{(\sigma(b)-a+1)\left(w(M)\int\limits_{a}^{\sigma(b)}\Phi(s)\Delta s+\varepsilon\right)} > 1. \tag{2.11}$$

*Let* $n_0 \in \{1,2,...\}$ *be chosen so that* $\dfrac{1}{n_0} < \varepsilon$ *and let* $N_0 = \{n_0, n_0+1, ...\}$. *We first show that*

$$\begin{aligned} y^{\Delta\Delta}(t)+\Phi(t)f^{*}(t,y,y^{\Delta}) = 0, \qquad a<t<b \\ y(a) = y^{\Delta}(\sigma(b)) = \frac{1}{m}, \end{aligned} \tag{2.12}$$

*has a solution for each* $m \in N_0$; *here*

$$f^{*}(t,u,p) = \begin{cases} f(t,u,p), & u \geq \frac{1}{m}, \quad p \geq \frac{1}{m} \\ f(t,u,\frac{1}{m}), & u \geq \frac{1}{m}, \quad p < \frac{1}{m} \\ f(t,\frac{1}{m},p), & u < \frac{1}{m}, \quad p \geq \frac{1}{m} \\ f(t,\frac{1}{m},\frac{1}{m}), & u < \frac{1}{m}, \quad p < \frac{1}{m}. \end{cases}$$

*To show (2.12) has a solution, we consider the family of problems*

$$\begin{aligned} y^{\Delta\Delta}(t)+\lambda\Phi(t)f^{*}(t,y,y^{\Delta}) = 0, \qquad a<t<b \\ y(a) = y^{\Delta}(\sigma(b)) = \frac{1}{m}, \qquad m \in N_0 \end{aligned} \tag{2.13}$$

*for* $0 < \lambda < 1$. *Let* $y \in C^1[a,\sigma(b)] \cap C^2(a,\sigma(b))$ *be any solutions of (2.13). Then* $y^{\Delta}(t) \geq \dfrac{1}{m}$ *and* $y(t) \geq \dfrac{1}{m}$ *on* $[a,b]$. *Also from (2.7) we have* $-y^{\Delta\Delta}(t) \leq \Phi(t)w(|y|_1)$ *here* $|y|_1 = max\{|y|_0, |y^{\Delta}|_0\}$ *and* $|y|_0 = sup_{[a,\sigma(b)]}|y(t)|$.

*If we integrate in* $-y^{\Delta\Delta}(t) \leq \Phi(t)w(|y|_1)$ *from* $t$ *to* $\sigma(b)$, *we obtain*

$$-\int\limits_{t}^{\sigma(b)} y^{\Delta\Delta}(s)\Delta s \leq \int\limits_{t}^{\sigma(b)} \Phi(s)w(|y|_1)\Delta s$$

$$-y^{\Delta}(\sigma(b))+y^{\Delta}(t) \leq w(|y|_1)\int\limits_{t}^{\sigma(b)}\Phi(s)\Delta s$$

$$y^{\Delta}(t) \leq w(|y|_1)\int\limits_{t}^{\sigma(b)}\Phi(s)\Delta s+\frac{1}{m} \quad \text{for } t \in [a,\sigma(b)]. \tag{2.14}$$

*In particular for* $t = a$, *we have*

$$y^{\Delta}(a) \leq w(|y|_1) \int\limits_a^{\sigma(b)} \Phi(s)\Delta s + \varepsilon < (\sigma(b) - a + 1)(w(|y|_1) \int\limits_a^{\sigma(b)} \Phi(s)\Delta s + \varepsilon. \tag{2.15}$$

*Also again, integrate from $t$ to $\sigma(b)$ in (2.14), we get*

$$\int\limits_t^{\sigma(b)} y^{\Delta}(t)\Delta s \leq \int\limits_t^{\sigma(b)} w(|y|_1) \left( \int\limits_s^{\sigma(b)} \Phi(u)\Delta u \right) \Delta s + \frac{1}{m} \int\limits_t^{\sigma(b)} \Delta s$$

$$y(\sigma(b)) - y(t) \leq w(|y|_1) \int\limits_t^{\sigma(b)} \int\limits_s^{\sigma(b)} \Phi(u)\Delta u \Delta s + \frac{1}{m}(\sigma(b) - t)$$

*and*

$$y(\sigma(b)) - y(a) \leq w(|y|_1) \int\limits_a^{\sigma(b)} \int\limits_s^{\sigma(b)} \Phi(u)\Delta u \Delta s + \frac{\sigma(b) - a}{m} fort = a.$$

*So we have,*

$$y(\sigma(b)) \leq w(|y|_1)(\sigma(b) - a) \int\limits_a^{\sigma(b)} \Phi(u)\Delta u + (\sigma(b) - a + 1)\varepsilon$$

$$< (\sigma(b) - a + 1) \left( w(|y|_1) \int\limits_a^{\sigma(b)} \Phi(u)\Delta u + \varepsilon \right). \tag{2.16}$$

*Combine (2.15) and (2.16) to obtain*

$$\frac{|y|_1}{(\sigma(b) - a + 1) \left( w(|y|_1) \int\limits_a^{\sigma(b)} \Phi(u)\Delta u + \varepsilon \right)} \leq 1. \tag{2.17}$$

*Now, (2.11) together with (2.17) implies $|y|_1 \neq M$.*

*Thus Theorem 1.2 implies (2.12) has a solution $y_m$ with $|y_m|_1 \leq M$. In fact*

$$\frac{1}{m} \leq y_m(t) \leq M \text{ and } \frac{1}{m} \leq y_m^{\Delta}(t) \leq M \text{ for } t \in [a, \sigma(b)] \tag{2.18}$$

*and $y_m$ satisfies*

$$y^{\Delta\Delta}(t) + \Phi(t) f(t, y, y^{\Delta}) = 0 \qquad a < t < b$$

$$y(a) = y^{\Delta}(\sigma(b)) = \frac{1}{m}.$$

*Now, (2.10) guarantees existence of a continuous and positive function* $\Psi_m(t)$ *on* $(a, \sigma(b))$*, and constants* $\alpha \geq 0$*,* $\beta \geq 0$ *with* $\alpha + \beta < 1$ *and with* $f(t, y_m(t), y_m^{\Delta}(t)) \geq \Psi_m(t)[y_m(t)]^{\alpha}[y_m^{\Delta}(t)]^{\beta}$ *for* $(t, y_m(t), y_m^{\Delta}(t)) \in [a, b] \times [0, M]^2$*.*

*Together with the differential equation and Lemma (1.1), we get*

$$-[y_m^{\Delta}(t)]^{-\beta} y_m^{\Delta\Delta}(t) \geq \Psi_m(t)\Phi(t)\left(\frac{t-a}{\sigma(b)-a}\right)^{\alpha}[y_m(\sigma(b))]^{\alpha} fort \in (a, \sigma(b)).$$

*If we integrate from* $t$ *to* $\sigma(b)$*, we obtain*

$$-\int_t^{\sigma(b)} [y_m^{\Delta}(s)]^{-\beta} y_m^{\Delta\Delta}(s)\Delta s \geq \int_t^{\sigma(b)} \Psi_m(s)\Phi(s)\left(\frac{s-a}{\sigma(b)-a}\right)^{\alpha}[y_m(\sigma(b))]^{\alpha}\Delta s.$$

*For the left side of this inequality, using Theorem* 5.45 [3]*, we get*

$$\begin{aligned}\int_t^{\sigma(b)} [y_m^{\Delta}(s)]^{-\beta} y_m^{\Delta\Delta}(s)\Delta s &= [y_m^{\Delta}(t)]^{-\beta} - [y_m^{\Delta}(\sigma(b))]^{-\beta}]\Gamma \\ &\quad + [y_m^{\Delta}(\sigma(b))]^{-\beta} \int_t^{\sigma(b)} y_m^{\Delta\Delta}(s)\Delta s,\end{aligned}$$

*where* $\Gamma$ *satisfies*

$$inf\left(\int_t^{\sigma(b)} y_m^{\Delta\Delta}(s)\Delta s\right) < \Gamma < sup\left(\int_t^{\sigma(b)} y_m^{\Delta\Delta}(s)\Delta s\right),$$

*so* $\Gamma < 0$.

*Since the function* $y_m^{\Delta}(t)$ *is non-decreasing on* $(a, \sigma(b))$ *and* $\alpha, \beta \geq 0$ *with* $\alpha + \beta < 1$*, for* $t < \sigma(b)$ *we get*

$$\begin{aligned}y_m^{\Delta}(t) > y_m^{\Delta}(\sigma(b)) &\Rightarrow [y_m^{\Delta}(t)]^{-\beta} < [y_m^{\Delta}(\sigma(b))]^{-\beta} \\ &\Rightarrow [y_m^{\Delta}(t)]^{-\beta} - [y_m^{\Delta}(\sigma(b))]^{-\beta} < 0.\end{aligned}$$

*Thus, we get*

$$\begin{aligned}\int_t^{\sigma(b)} [y_m^{\Delta}(s)]^{-\beta} y_m^{\Delta\Delta}(s)\Delta s &\geq [y_m^{\Delta}(\sigma(b))]^{-\beta} \int_t^{\sigma(b)} y_m^{\Delta\Delta}(s)\Delta s \\ &= [y_m^{\Delta}(\sigma(b))]^{-\beta}[y_m^{\Delta}(\sigma(b)) - y_m^{\Delta}(t)] \\ &= [y_m^{\Delta}(\sigma(b))]^{-\beta+1} - [y_m^{\Delta}(\sigma(b))]^{-\beta} y_m^{\Delta}(t).\end{aligned}$$

*Since* $[y_m^{\Delta}(\sigma(b))]^{-\beta+1} > 0$*, then*

$$-\int_{t}^{\sigma(b)}[y_m^{\Delta}(s)]^{-\beta}y_m^{\Delta\Delta}(s)\Delta s \le [y_m^{\Delta}(\sigma(b))]^{-\beta}y_m^{\Delta}(t).$$

*Thus, we have*

$$\begin{aligned}[y_m^{\Delta}(\sigma(b))]^{-\beta}y_m^{\Delta}(t) &\ge -\int_{t}^{\sigma(b)}[y_m^{\Delta}(s)]^{-\beta}y_m^{\Delta\Delta}(s)\Delta s\\ &\ge \int_{t}^{\sigma(b)}\Psi_m(s)\Phi(s)\left(\frac{s-a}{\sigma(b)-a}\right)^{\alpha}[y_m(\sigma(b))]^{\alpha}\Delta s,\end{aligned}$$

*and so*

$$y_m^{\Delta}(t) \ge (y_m(\sigma(b)))^{\alpha+\beta}\int_{t}^{\sigma(b)}\Psi_m(s)\Phi(s)\left(\frac{s-a}{\sigma(b)-a}\right)^{\alpha}\Delta s.$$

*If we integrate from $a$ to $\sigma(b)$, we get*

$$\begin{aligned}\int_{a}^{\sigma(b)}y_m^{\Delta}(s)\Delta s &\ge \frac{(y_m(\sigma(b)))^{\alpha+\beta}}{(\sigma(b)-a)^{\alpha}}\int_{a}^{\sigma(b)}\left(\int_{s}^{\sigma(b)}\Psi_m(u)\Phi(u)(u-a)^{\alpha}\Delta u\right)\Delta s\\ y_m(\sigma(b)) \ge y_m(\sigma(b)) - y_m(a) &\\ &\ge \frac{(y_m(\sigma(b)))^{\alpha+\beta}}{(\sigma(b)-a)^{\alpha}}\int_{a}^{\sigma(b)}\left(\int_{t}^{\sigma(b)}\Psi_m(s)\Phi(s)(s-a)^{\alpha}\Delta s\right)\Delta t\\ (y_m(\sigma(b)))^{1-\alpha-\beta} &\ge \frac{1}{(\sigma(b)-a)^{\alpha}}\int_{a}^{\sigma(b)}\left(\int_{t}^{\sigma(b)}\Psi_m(s)\Phi(s)(s-a)^{\alpha}\Delta s\right)\Delta t.\end{aligned}$$

*The other words, we get*

$$y_m(\sigma(b)) \ge \left\{\frac{1}{(\sigma(b)-a)^{\alpha}}\int_{a}^{\sigma(b)}\left(\int_{t}^{\sigma(b)}\Psi_m(s)\Phi(s)(s-a)^{\alpha}\Delta s\right)\Delta t\right\}^{\frac{1}{1-\alpha-\beta}} := d_0. \tag{2.19}$$

*From Lemma (1.1), we have*

$$y_m(t) \ge \frac{t-a}{\sigma(b)-a}y_m(\sigma(b)) \ge d_0\frac{t-a}{\sigma(b)-a} \ \ for\ t\in[a,\sigma(b)]. \tag{2.20}$$

*Of course, it is immediate that* $y_m^{\Delta^j}{}_{m\in N_0}$ *is a bounded, equicontinuous family on* $[a,\sigma(b)]$ *for each* $j=0,1$.

*The Arzela-Ascoli Theorem guarantees the existence of a subsequence* $N$ *of* $N_0$ *and a function* $y\in C^1[a,\sigma(b)]$ *with* $\{y_m^{\Delta^j}\}$ *converging uniformly on* $[a,\sigma(b))$ *to* $\{y^{\Delta^j}\}$ *as* $m$ *goes infinity through* $N$*; here* $j=0,1$. *Also* $y(a)=0=y^{\Delta}(\sigma(b))$ *and* $y(t)\geq d_0\dfrac{t-a}{\sigma(b)-a}$ *for* $t\in[a,\sigma(b)]$ *(especially* $y>0$ *on* $(a,\sigma(b)]$*). Now* $y_m, m\in N$ *satisfies*

$$y_m(t)=\frac{1}{m}+\frac{t-a}{m}+\int_a^t\frac{s-a}{\sigma(b)-a}\Phi(s)f(s,y_m(s),y_m^{\Delta}(s))\Delta s$$
$$+\frac{t-a}{\sigma(b)-a}\int_t^{\sigma(b)}\Phi(s)f(s,y_m(s),y_m^{\Delta}(s))\Delta s,$$

*for* $t\in[a,\sigma(b)]$.

*Fix* $t\in[a,\sigma(b)]$ *and let* $m$ *goes infinity through* $N$ *to obtain*

$$y(t)=\int_a^t\frac{s-a}{\sigma(b)-a}\Phi(s)f(s,y(s),y^{\Delta}(s))\Delta s$$
$$+\frac{t-a}{\sigma(b)-a}\int_t^{\sigma(b)}s\Phi(s)f(s,y(s),y^{\Delta}(s))\Delta s.$$

**Example 2.1.** *Consider the boundary value problem*

$$\begin{aligned} y^{\Delta\Delta}(t)+y^{\alpha}(t)(y^{\Delta}(t))^{\beta}=0 \quad for\ a<t<b\\ y(a)=y^{\Delta}(\sigma(b))=0\end{aligned}\tag{2.21}$$

*with* $\alpha\geq 0,\ \beta\geq 0$ *and* $\alpha+\beta<1$. *Then, (2.21) has a solution* $y\in C^1[a,\sigma(b)]\cap C^2(a,\sigma(b))$ *with* $y>0$ *on* $[a,\sigma(b)]$.

*To see we'll apply Theorem 2.1. Notice (2.5), (2.6), (2.7)* $(w(x)=x^{\alpha+\beta})$ *and (2.9)* $(\Psi_H=1,\ \alpha=\alpha$ *and* $\beta=\beta)$ *hold. Also from (2.8),*

$$1<sup_{c\in(0,\infty)}\frac{c}{w(c)\int_a^{\sigma(b)}\Phi(s)\Delta s}=sup_{c\in(0,\infty)}\frac{c}{c^{\alpha+\beta}(\sigma(b)-a)}=\infty$$

*hold. Again Theorem 2.1 establish the result.*

**Remark 2.1.** *Notice* $y\equiv 0$ *is also a solution of (2.21) if* $\alpha+\beta\neq 0$.

**Example 2.2.** *Consider the boundary value problem*

$$\begin{aligned} y^{\Delta\Delta}(t)+\mu(e^{-(t-a)})(y^{\alpha}(t)(y^{\Delta})^{\beta}(t)+\eta_0 y^{\gamma}(t)+\eta_1)=0,\ a<t<b\\ y(a)=y^{\Delta}(\sigma(b))=0\end{aligned}\tag{2.22}$$

*with* $\alpha, \beta, \eta_0, \eta_1 \geq 0,\ \gamma, \mu > 0$ *and* $\alpha + \beta < 1$. *If*

$$(\sigma(b) - a)\mu \leq sup_{c\in(0,\infty)} \frac{c}{c^{\alpha+\beta} + \eta_0 c^{\gamma} + \eta_1} \tag{2.23}$$

*then, (2.22) has a solution* $y \in C^1[a, \sigma(b)] \cap C^2[a, \sigma(b))$ *with* $y > 0$ *on* $(a, \sigma(b)]$. *Again, we apply Theorem 2.1.*

*It is easy to check (2.5), (2.6), (2.7)* $(w(x) = x^{\alpha+\beta} + \eta_0 x^{\gamma} + \eta_1)$, *and (2.9)* $(\Psi_H = e^{-(t-a)}, \alpha = \alpha$ *and* $\beta = \beta)$ *hold. Also,*

$$sup_{c\in(0,\infty)} \frac{c}{w(c) \int\limits_a^{\sigma(b)} \Phi(s)\Delta s} \geq sup_{c\in(0,\infty)} \frac{c}{\mu(\sigma(b) - a)[c^{\alpha+\beta} + \eta_0 c^{\gamma} + \eta_1]},$$

*since* $\int\limits_a^{\sigma(b)} e^{-(t-a)}\delta t \leq \sigma(b) - a$. *So (2.23) guarantees that (2.8) holds. Theorem 2.1 now establishes the result.*

## 3. Conclusion

In this study, we showed the existence of positive solutions of the nonsingular type second order boundary value problem on the time scale and we supported it with some examples.

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