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# Sensitivity analytic and synchronization of a new fractional-order financial system

**Malek Karimian***
Department of Mathematics,
Payame Noor University,
P.O. Box 19395-3697, Tehran, Iran.
E-mail: malekkarimian@gmail.com

**Bashir Naderi**
Department of Mathematics,
Payame Noor University,
P.O. Box 19395-3697, Tehran, Iran.
E-mail: b_naderi@pnu.ac.ir

**Yousef Edrisi Tabriz**
Department of Mathematics,
Payame Noor University,
P.O. Box 19395-3697, Tehran, Iran.
E-mail: yedrisy@gmail.com

**Abstract** In this paper, we present a new fractional-order financial system (FOFS) with the new parameters. We study the synchronization for commensurate order of the fractional-order financial system with disturbance observer (FOFSDO) on the new parameters. Also, the sensitivity analysis of the synchronization error was investigated by using the feedback control technique for the FOFSDO. The stability of the used method demonstrates by Lyapunov stability theorem. Numerical simulations are presented to ensure the validity and influence of the target feedback control design in the presence of extrinsic bounded unknown disturbance.

**Keywords.** Synchronization, Fractional order financial system, Disturbance observer, Control, Lyapunov stability.

**2010 Mathematics Subject Classification.** 34H10, 34Dxx, 34D06.

## 1. Introduction

In the last few years, enormously of researches showing, in the many nonlinear fractional order systems in the fields of engineering and economics chaotic behavior occurred [4, 6, 7, 15, 16, 17]. Chaotic nonlinear systems, are dynamic systems that exhibit very high sensitivity to the initial conditions. Chaos phenomenon emerges in the economy by the middle $80's$. To study an economic model, economists decided that as a first step, only the endogenous variables considered the behavior of the simple model.

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* corresponding.

The fractional-order dynamic systems have emerged as a new investigated by the researchers many of the results are analyzed. For example, in 2001, Chen et al [12, 13] a third-order financial dynamic system was presented in terms of time and three state variable; Wei-Ching Chen [5] presented non-linear dynamics and chaos in a fractional-order financial system; Sara Dadras et al [8] applied control of a fractional-order economical system via sliding mode; Zhen Wang et al [20] proposed synchronization of a chaotic fractional-order economical system with active control; Danca et al [9] presented sustaining stable dynamics of a fractional-order chaotic financial system by parameter switching; Baogui Xin et al [21] presented stabilizing a fractional-order chaotic financial system with market confidence.

In this paper, we try to investigate a new FOFS presented by [10, 18]. Our proposed is to investigate on synchronization error for commensurate FOFSDO based on the feedback control method and observe disturbance. The commensurate order means the all order of equations in fractional order system are equal. Also, we are to research the system sensitivity analysis by using new parameters and the different initial conditions. The Lyapunov stability theorem in fractional is used for stability of system.

The rest of this paper is organized as follows: Section 2 contains the fundamental definitions, lemma, theorem, and properties of fractional calculations. Section 3 analyzes the fractional-order financial system with new parameters. In section 4, we discuss the synchronization error for commensurate order the FOFSDO based on the feedback control method and using the approximation of disturbance in controller. Finally, concluding remarks are presented in Section 6.

## 2. Preliminaries

In this section, we review some fundamental definitions of fractional calculus. Also, we present some useful stability theorems and properties of fractional-order dynamical systems.

**Definition 2.1.** [2] The $qth$ order Riemann-Liouville derivative of fractional for the function $G(t)$ can be described as

$$\begin{aligned} {}^{RL}D_t^q G(t) &= D^{(m)} D^{-(m-q)} G(t) \\ &= \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-q)}\int_0^t (t-\zeta)^{m-q-1} G(\zeta)d\zeta\right], \end{aligned} \tag{2.1}$$

where $m-1<q<m, m \in N, q \in R^+, \Gamma(q)=\int_0^\infty t^{q-1}e^{-t}dt$.

**Definition 2.2.** [2] The $qth$ order Caputo derivative of fractional for the function $G(t)$ is defined as follows:

$$\begin{aligned} {}^{c}D_t^q G(t) &= D^{-(m-q)} D^{(m)} G(t) \\ &= \frac{1}{\Gamma(m-q)}\int_0^t (t-\zeta)^{m-q-1} G^{(m)}(\zeta)d\zeta, \end{aligned} \tag{2.2}$$

where $m-1<q\le m, m \in N, q \in R^+, \Gamma(q)=\int_0^\infty t^{q-1}e^{-t}dt$.

Some properties of fractional order differential equations are:

- The linear characteristic of the Caputo fractional-order derivatives satisfies the following
$$^{c}D_t^q[c_1G_1(t) + c_2G_2(t)] = c_1^c D_t^q G_1(t) + c_2^c D_t^q G_2(t), \tag{2.3}$$
where $c_1, c_2$ are constants and $G_1, G_2$ are functions of $t$ [3].
- The Caputo fractional-order derivative satisfies the following
$$^{c}D_t^q G(t) = 0, \tag{2.4}$$
where $G(t)$ is a constant function and $0 < q \leq 1$ [3].

**Lemma 2.3.** [1] *Assume that $G(t) \in R$ be a continuously differentiable function, then we have*
$$\frac{1}{2}(^{c}D_t^q G^2(t)) \leq G(t)^c D_t^q G(t), \tag{2.5}$$
*where $0 < q < 1$ .*

**Theorem 2.4.** [14] *Autonomous system $D^q x = Ax, x(0) = x_0$ is asymptotically stable if the following condition is satisfied*
$$|arg(\lambda(A))| > \frac{q\pi}{2},$$
*where $0 < q < 1$ and $\lambda(A)$ represents the eigenvalues of matrix $A$. Also, this system is stable if and only if $|arg(\lambda(A))| \geq \frac{q\pi}{2}$, and those critical eigenvalues that satisfy $|arg(\lambda(A))| = \frac{q\pi}{2}$, have geometric multiplicity of one.*

## 3. Analysis of the fractional-order financial system with new parameters

A dynamical model of financial system introduced in [18] as follows:
$$\begin{cases} \dot{x} &= z + (y - a)x, \\ \dot{y} &= 1 - by - |x|, \\ \dot{z} &= -x - cz, \end{cases} \tag{3.1}$$
recently extended form of financial system (3.1) introduced as a fractional-order financial system as follows [10]. We let it as a master system:
$$\begin{cases} D^{q_1}x &= z + (y - a)x, \\ D^{q_2}y &= 1 - by - |x|, \\ D^{q_3}z &= -x - cz, \end{cases} \tag{3.2}$$
where the state variables $x, y$ and $z$ represent interest rate, the investment demand, and the price index, respectively; The parameters $a > 0, b > 0$, and $c > 0$, are all constants, represent saving amount, the cost per investment and the elasticity of demand, respectively; $0 < q_i \leq 1$ is the fractional derivative order finance system.

Consider the new parameters as $a = 0.7, \quad b = 0.1, \quad c = 0.9$ and the different initial condition $(x(0), y(0), z(0)) = (2, 1, -1)$ [19], the phase portrait shows the chaotic behavior of the system (3.2) for the commensurate and incommensurate orders (see Figures 1 and 2 respectively). The system is chaotic when $q > 0.79$.

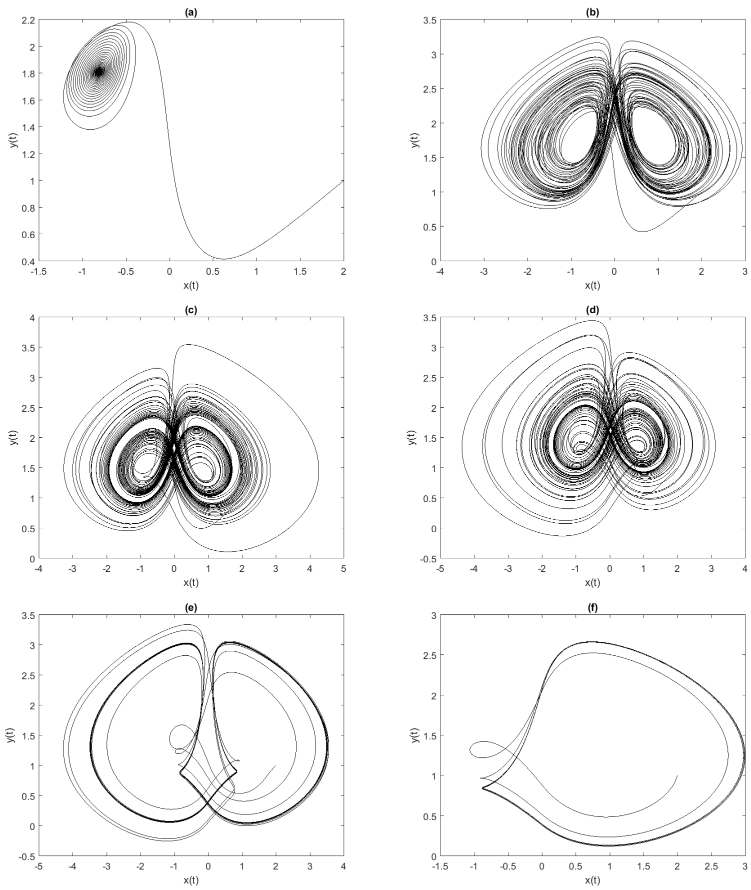


FIGURE 1. The phase portrait of fractional order finance system (3.2) for commensurate orders at $q_1 = q_2 = q_3 = q$. (a)q=0.78, (b)q=0.79, (c)q=0.87, (d)q=0.92, (e)q=0.96, (f)q=1.

## 4. Feedback control synchronization method for FOFSDO of commensurate orders

In this section, the nonlinear feedback control method is presented to synchronization of FOFS. Also, it is established that the synchronization error is stable under the adaptive feedback control method. We also showed that the timing of synchronization in return from the fractional orders is not so much different. We consider the slave system as follows:

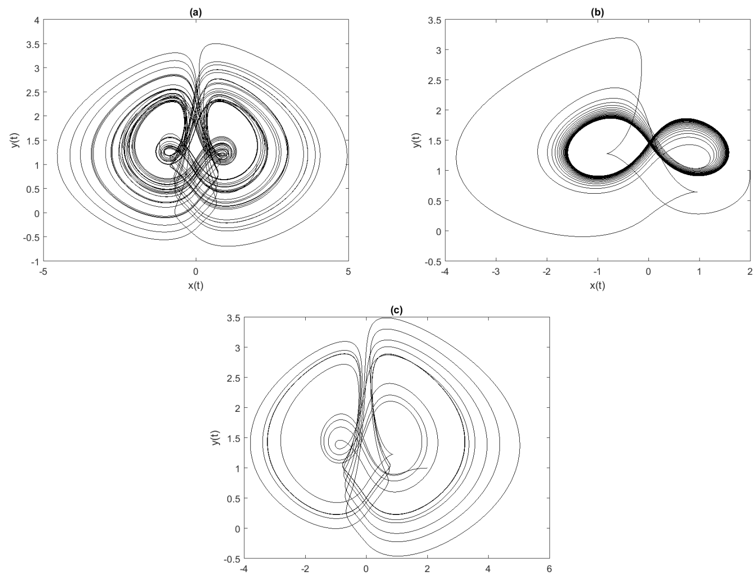


FIGURE 2. The phase portrait of fractional order finance system (3.2) for incommensurate orders.(a) $q_1 = q_2 = 1$, $q_3 = 0.9$, (b) $q_1 = q_3 = 1$, $q_2 = 0.93$, (c) $q_2 = q_3 = 1$, $q_1 = 0.9$

$$\begin{cases} D^{q_1} x_1 = z_1 + (y_1 - a)x_1 + u_1(t) + d_1(t), \\ D^{q_2} y_1 = 1 - by_1 - |x_1| + u_2(t) + d_2(t), \\ D^{q_3} z_1 = -x_1 - cz_1 + u_3(t) + d_3(t), \end{cases} \tag{4.1}$$

where $x_1(t)$, $y_1(t)$ and $z_1(t)$ are interest rate, the investment demand and the price index respectively. $u_1(t), u_2(t)$ and $u_3(t)$ are controllers, and $d_1(t), d_2(t), d_3(t)$, $(d_i \geq 0, i = 1, 2, 3)$ are unknown bounded external disturbances, such that the Caputo fractional-order derivative of the $d_i$ is bounded, i.e. $D^q d_i < \xi_i$ where $\xi$ are unknown positive constants. A disturbance is added to increase system performance resistance.

For synchronization, we define the synchronization error between the master system (3.2) and slave system (4.1) as follows:

$$\begin{cases} e_1(t) = x_1(t) - x(t), \\ e_2(t) = y_1(t) - y(t), \\ e_3(t) = z_1(t) - z(t). \end{cases} \tag{4.2}$$

The dynamic error between master system (3.2) and slave system (4.1) is

$$\begin{cases} D_t^{q_1} e_1(t) = e_3 + y_1 e_1 + x e_2 - a e_1 + u_1 + d_1, \\ D_t^{q_2} e_2(t) = -b e_2 - |x_1| + |x| + u_2 + d_2, \\ D_t^{q_3} e_3(t) = -e_1 - c e_3 + u_3 + d_3, \end{cases} \tag{4.3}$$

and we consider adaptive feedback controller as follow

$$\begin{cases} u_1(t) = -y_1 e_1 - x e_2 - k_1 e_1 - \hat{d}_1, \\ u_2(t) = |x_1| - |x| - k_2 e_2 - \hat{d}_2, \\ u_3(t) = -k_3 e_3 - \hat{d}_3, \end{cases} \tag{4.4}$$

where $k_i \geq 0, i = 1, 2, 3$ are gain constants and $\hat{d}_i$ is approximation of $d_i$ for $i = 1, 2, 3$ .

By substituting (4.4) into (4.3), we have

$$\begin{cases} D_t^{q_1} e_1(t) = e_3 - a e_1 - k_1 e_1 + \tilde{d}_1, \\ D_t^{q_2} e_2(t) = -b e_2 - k_2 e_2 + \tilde{d}_2, \\ D_t^{q_3} e_3(t) = -e_1 - c e_3 - k_3 e_3 + \tilde{d}_3, \end{cases} \tag{4.5}$$

where $\tilde{d}_i = d_i - \hat{d}_i, (i = 1, 2, 3)$. so we have the following result.

- If $\tilde{d}_i = d_i - \hat{d}_i = 0$, the Jacobian matrix of (4.5) is

$$J = \begin{bmatrix} -a - k_1 & 0 & 1 \\ 0 & -b - k_2 & 0 \\ -1 & 0 & -c - k_3 \end{bmatrix}, \tag{4.6}$$

When $a = 0.7, b = 0.1, c = 0.9$, and $k_1 = k_2 = k_3 = 10$, the eigenvalues are, $\lambda_1 \approx -10.8 + 0.995i$, $\lambda_2 \approx -10.8 - 0.995i$, $\lambda_3 = -10.1$, which satisfies $|arg(\lambda_i)| > \frac{q\pi}{2} (i = 1, 2, 3)$ for $0 \leq q \leq 1$. Thus the synchronization error system converges to zero as $t \longrightarrow \infty$ and therefore, synchronization between the master system (3.2) and the slave system (4.1) is achieved.

- If $\tilde{d}_i = d_i - \hat{d}_i \neq 0$. We define a observe disturbance role, as follow [11]:

$$\begin{cases} \phi_1(t) = d_1(t) - \sigma_1 x_1, \\ \phi_2(t) = d_2(t) - \sigma_2 y_1, \\ \phi_3(t) = d_3(t) - \sigma_3 z_1, \end{cases} \Longrightarrow \begin{cases} D^q \phi_1(t) = D^q d_1(t) - \sigma_1 D^q x_1, \\ D^q \phi_2(t) = D^q d_2(t) - \sigma_2 D^q y_1, \\ D^q \phi_3(t) = D^q d_3(t) - \sigma_3 D^q z_1. \end{cases} \tag{4.7}$$

By substituting (4.1) in (4.7), we have:

$$\begin{cases} D^q \phi_1(t) = D^q d_1(t) - \sigma_1 [z_1 + (y_1 - a) x_1 + u_1 + d_1(t)], \\ D^q \phi_2(t) = D^q d_2(t) - \sigma_2 [1 - b y_1 - |x_1| + u_2 + d_2(t)], \\ D^q \phi_3(t) = D^q d_3(t) - \sigma_3 [-x_1 - c z_1 + u_3 + d_3(t)]. \end{cases} \tag{4.8}$$

In the following, for calculating the estimation of $\phi_i(t) (i = 1, 2, 3)$, we assume the estimation law as follow

$$
\begin{cases}
D^q\hat{\phi}_1(t) = -\sigma_1[z_1 + (y_1 - a)x_1 + \sigma_1 x_1] - \sigma_1 u_1 - \sigma_1\hat{\phi}_1(t), \\
D^q\hat{\phi}_2(t) = -\sigma_2[1 - by_1 - |x_1| + \sigma_2 y_1] - \sigma_2 u_2 - \sigma_2\hat{\phi}_2(t), \\
D^q\hat{\phi}_3(t) = -\sigma_3[-x_1 - cz_1 + \sigma_3 z_1] - \sigma_3 u_3 - \sigma_3\hat{\phi}_3(t),
\end{cases} \tag{4.9}
$$

where $\hat{\phi}_i$ is the estimation of $\phi_i$, and let $\tilde{\phi}_i = \phi - \hat{\phi}$

Using the observe law (4.7), we have

$$
\begin{cases}
\hat{\phi}_1(t) = \hat{d}_1(t) - \sigma_1 x_1, \\
\hat{\phi}_2(t) = \hat{d}_2(t) - \sigma_2 y_1, \\
\hat{\phi}_3(t) = \hat{d}_3(t) - \sigma_3, z_1
\end{cases}
\Longrightarrow
\begin{cases}
\tilde{\phi}_1(t) = \phi_1 - \hat{\phi}_1 = d_1 - \hat{d}_1 = \tilde{d}_1, \\
\tilde{\phi}_2(t) = \phi_2 - \hat{\phi}_2 = d_2 - \hat{d}_2 = \tilde{d}_2, \\
\tilde{\phi}_3(t) = \phi_3 - \hat{\phi}_3 = d_3 - \hat{d}_3 = \tilde{d}_2.
\end{cases} \tag{4.10}
$$

By applying fractional order Caputo derivative on (4.10), and using (4.9) and (4.8), the error dynamic of disturbance will be as follow:

$$
\begin{cases}
D^q\tilde{\phi}_1(t) = D^q\phi_1 - D^q\hat{\phi}_1(t) = D^q d_1 - \sigma_1\tilde{\phi}_1 \\
D^q\tilde{\phi}_2(t) = D^q\phi_2 - D^q\hat{\phi}_2(t) = D^q d_2 - \sigma_2\tilde{\phi}_2 \\
D^q\tilde{\phi}_3(t) = D^q\phi_3 - D^q\hat{\phi}_3(t) = D^q d_3 - \sigma_3\tilde{\phi}_3.
\end{cases} \tag{4.11}
$$

In (4.7)-(4.11), for simplicity, we let $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$.

The stability of error systems via designed controllers and estimation law of disturbance are discussed in the following theorems.

**Theorem 4.1.** *Consider the financial fractional order system (3.2)and the system (4.1) with unknown bounded disturbance. Then the synchronization error system (4.5) is stable under the feedback controller (4.4) and disturbance approximation low (4.9)* .

*Proof.* We consider the Lyapunov function as follow

$$
V(t) = \frac{1}{2}\sum_{i=1}^{3}[e_i^2(t) + \tilde{d}_i^{\,2}] = \frac{1}{2}\sum_{i=1}^{3}[e_i^2(t) + \tilde{\phi}_i^{\,2}(t)]. \tag{4.12}
$$

By applying relationship (2.3) and lemma 2.3, we have

$$
D^q V(t) \leq \sum_{i=1}^{3}[e_i(t)D^q e_i(t) + \tilde{\phi}_i D^q\tilde{\phi}_i]. \tag{4.13}
$$

By substituting (4.5) and (4.11) in (4.13), we get

$$
\begin{aligned}
D^{q_i}V(t) \leq\ & e_1[e_3 - ae_1 - k_1 e_1 + \tilde{\phi}_1] + e_2[-be_2 - k_2 e_2 + \tilde{\phi}_2] \\
& + e_3[-e_1 - ce_3 - k_3 e_3 + \tilde{\phi}_3] + \tilde{\phi}_1(-\sigma\tilde{\phi}_1 + D^q d_1(t)) \\
& + \tilde{\phi}_2(-\sigma\tilde{\phi}_2 + D^q d_2(t)) + \tilde{\phi}_3(-\sigma\tilde{\phi}_3 + D^q d_3(t)).
\end{aligned} \tag{4.14}
$$

By sorting (4.14), the expression (4.15) can be deduced

$$
\begin{aligned}
D^qV(t) \leq & -[ae_1^2 + be_2^2 + ce_3^2] - [k_1e_1^2 + k_2e_2^2 + k_3e_3^2] \\
& - \sigma[\tilde{\phi_1}^2 + \tilde{\phi_2}^2 + \tilde{\phi_3}^2] \\
& + \tilde{\phi_1}[e_1 + D^qd_1] + \tilde{\phi_2}[e_1 + D^qd_2] + \tilde{\phi_3}[e_1 + D^qd_3].
\end{aligned} \tag{4.15}
$$

We know that the $D^qd_i \leq \xi$ and system (3.2) is chaotic, so all state variables are bounded. Therefor we can choose $\sigma$ and $k_i$'s such that

$$
\begin{aligned}
[k_1e_1^2 + k_2e_2^2 + k_3e_3^2] + \sigma[\tilde{\phi_1}^2 + \tilde{\phi_2}^2 + \tilde{\phi_3}^2] \geq & \tilde{\phi_1}[e_1 + D^qd_1] \\
& + \tilde{\phi_2}[e_1 + D^qd_2] + \tilde{\phi_3}[e_1 + D^qd_3].
\end{aligned} \tag{4.16}
$$

So

$$D^qV(t) < 0.$$

Then the synchronization error $e(t)$ is stable at zero. This complete the proof. □

## 5. Numerical simulation

In this section, numerical simulation is given to illustrate the validity of the proposed method.

We choose the commensurate order $q_1 = q_2 = q_3 = 0.79$, the new parameters are considered as $a = 0.7, \quad b = 0.1, \quad c = 0.9$, initial conditions $(x(0), y(0), z(0)) = (2, 1, -1)$, $(x_1(0), y_1(0), z_1(0)) = (1, -2, 1)$, $(\hat{\phi_1}(0), \hat{\phi_2}(0), \hat{\phi_3}(0)) = (0.5, 0, 0.3)$ $(k_1, k_2, k_3) = (100, 100, 100)$, $\sigma = 3$, and $d_1(t) = sin^2(t), \quad d_2(t) = cos^2(t), \quad d_3(t) = sin^2(t)$.

In Figure 3 Phase portraits shows synchronization with the commensurate order of the FOFSDO at $q_1 = q_2 = q_3 = 0.79$. Figure 4 and Figure 5 show the synchronization of the states for the master system (3.2) and the slave system (4.1) after applying the controller (4.4)and approximation of disturbance respectively.

For more explanation, we are investigating some examples of synchronization based on feedback control technique for different values of order as commensurate orders. In Figure 6 synchronization errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ are shown. Such that, Figure 6(a), 6(b) and 6(c) shows the error converges to zero in short time, for $q_1 = q_2 = q_3 = 0.79$, $q_1 = q_2 = q_3 = 0.82$ and $q_1 = q_2 = q_3 = 0.9$ respectively.

## 6. Conclusion

The nonlinear feedback control method is presented the synchronization error of FOFSDO. It is established that the synchronization error is stable under the feedback control method, and showed that the timing of synchronization in return from the fractional orders is not so much different. We also show that the synchronization error system converges to zero as $t \longrightarrow \infty$ and, synchronization between the master and the slave systems is achieved.

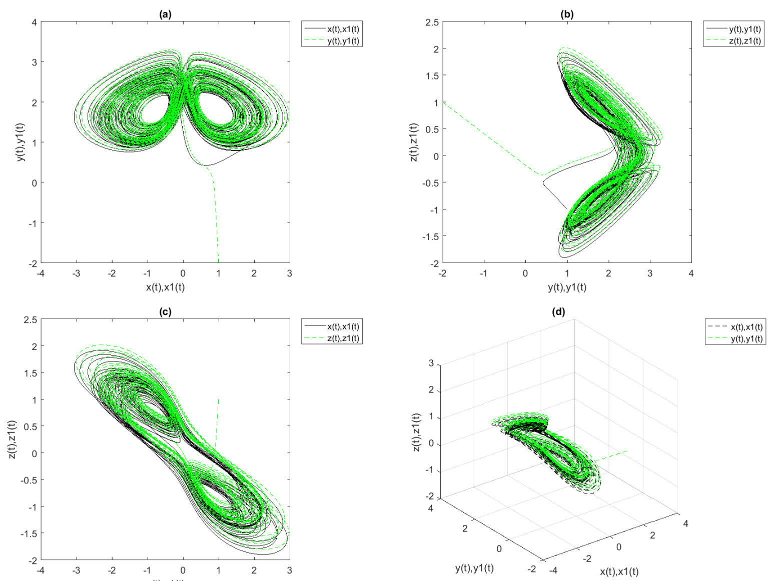


FIGURE 3. Depicts the phase portraits of synchronization of the master and slave systems.

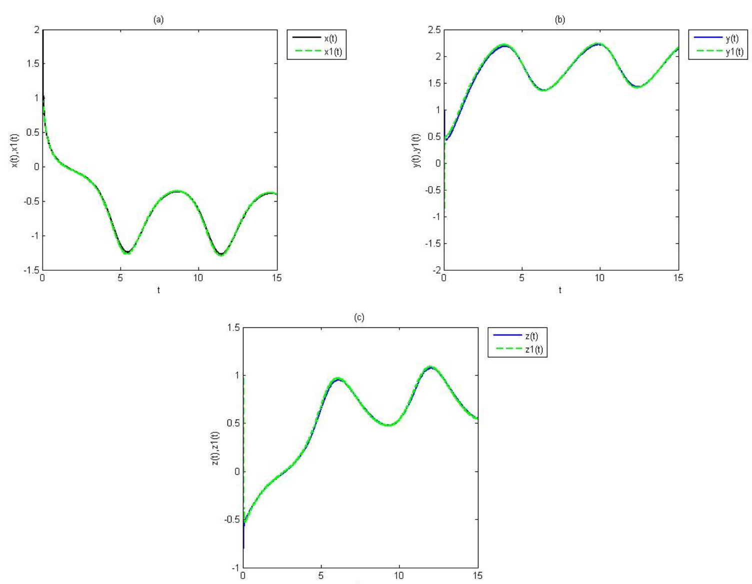


FIGURE 4. (a), (b) and (c) are the evolution curves of the states for the master system(3.2) and the slave system(4.1).

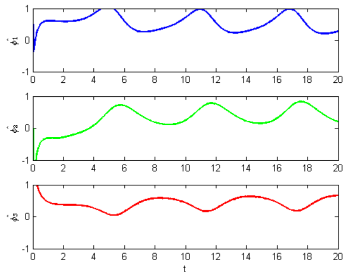


FIGURE 5. Approximation of disturbance $\hat{\phi}_i$.

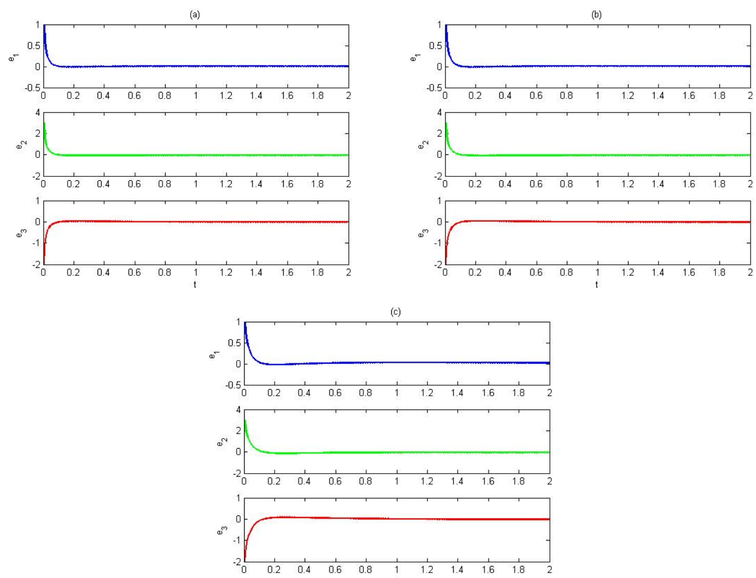


FIGURE 6. Synchronization error for the commensurate orders, (a): $q_1 = q_2 = q_3 = 0.79$, (b): $q_1 = q_2 = q_3 = 0.82$, (c): $q_1 = q_2 = q_3 = 0.9$.

## REFERENCES


[1] N. Aguila-Camacho, M. A. Duarte-Mermoud, and J. A. Javier Gallegos, *Lyapunov functions for fractional order system*, Common Nonlinear sci. Numer. sim., *19* (2014), 2951–2957.

[2] B. Bandyopadhyay and S. Kamal, *Stabilization and control of fractional-order systems: A sliding mode Approach*, springer, 2015.

[3] L. Changpin and D. Weihua, *Remarks on fractional derivatives*, App. Math. Comput., *187* (2007), 777–784.

[4] G. Chen and X. Dong, *From chaos to order methodologies, perspectives, and applications*, Word Scientific, Singapore, 1998.

[5] W. C. Chen, *Nonlinear dynamics and chaos in a fractional-order financial system*, Chaos Solitons Fractal, *36* (2006), 1305–1314.

[6] A. L. Chian, E. L. Rempel, and C. Rogers, *Complex economic dynamics: Chaotic saddle, crisis, and intermittency*, Chaos Solitons Fractals, *29* (2006), 1194–1218.

[7] A. L. Chian, F. A. Zorotta, E. L. Rempel, and C. Rogers, *Attractor emerging crisis in chaotic business cycles*, Chaos Solitons Fractal, *24*, (2005), 869–875.

[8] S. Dadras and H. R. Momeni, *Control of a fractional-order economical system via sliding mode*, Physical A, *389* (2010), 2434–2442.

[9] M. F. Danca, R. Garrappa, W. K. S Tang, and G. Chen, *Sustaining stable dynamics of a fractional-order chaotic financial system by parameter switching*, computers and mathematics with Applications, *66* (2013), 702–716.

[10] A. Hajipour and H. Tavakoli, *Analysis and circuit simulation of a novel nonlinear fractional incommensurate order financial system*, Optik *127*(22) ()2016, 10643–10652.

[11] A. Khan and A. Tyagi, *Disturbance observer-based adaptive sliding mode hybrid projective synchronisation of identical fractional-order financial systems*, Pramana J. Phys., *90*(67) (2018), DOI:10.1007/s12043-018-1555-8.

[12] J. H. Ma and Y. S. Chen, *Study for the bifurcation topological structure and the global complicated character of a kind of non-linear finance system(I)*, Appl. Math., *22*(11) (2001), 1119–1128.

[13] J. H. Ma and Y. S. Chen,*Study for the bifurcation topological structure and the global complicated character of a kind of non-linear finance system(II)*, Appl. Math., *22*(12) (2001), 1236–1242.

[14] D. Matignon, *Stability results for fractional differential equations with applications to control processing*, Computational Engineering in Systems Applications, *2* (1996), 963–968.

[15] R. A. Meyers, *Complex system in Finance and Econometrics*, Springer, New York, 2009.

[16] B Naderi, H. Kheiri, and V. Vafaei, *Modified Sliding-Mode Control Method for Synchronization a Class of Chaotic Fractional-Order Systems with Application in Encryption*, The ISC International Journal of Information Security, *12*(1) (2020), 55–66.

[17] I. Petras, *Fractional-order nonlinear systems, modeling, analysis, and simulation*, Higher Education Press and Springer, Beijing, 2011.

[18] C. K. Tacha, I. M. Volos, I. N. Kyaprianidis, S. Stouboulos, and V. T. P. Vaidyanathan, *Analysis, adaptive control and circuit simulation of a novel nonlinear finance system*m, App. Math. Comput., *276* (2016), 200–217.

[19] O. I. Tacha, J. M-Munoz-Pacheco, E. Zambrano-Serrano, I. N. Stouboulos, and T. Pham-V *Determining the chaotic behavior in a fractional-order finance system with negative parameters*, Nonlinear Dyn., (2018), DOI:10.1007/s11071-018-4425-5.

[20] Z. Wang and X. Huang, *Synchronization of a chaotic fractional-order economical system with active control*, Procida Engineering, *15* (2010), 516–520,

[21] B. Xin, and J. Zhang, *Finite-time stabilizing a fractional-order chaotic financial system with market confidence*, Nonlinear Dyn., *79*(2) (2015), 1399–1409.