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On the travelling wave solutions of Ostrovsky equation

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Abstract In this paper, extended trial equation method (ETEM) is applied to find exact solutions of (1+1) dimensional nonlinear Ostrovsky equation. We constitute some exact solutions such as soliton solutions, rational, Jacobi elliptic and hyperbolic function solutions of this equation via ETEM. Then, we submit the results obtained by using this method.

Keywords. (1+1) dimensional nonlinear Ostrovsky equation, Extended trial equation method, Soliton solutions, Rational, Jacobi elliptic and hyperbolic function solutions.

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1. Introduction

During the past years, travelling wave solutions are considerably important issue in biophysics, geophysical sciences, chemical kinematics, optical fibers, technology of space, electricity, elastic media and several topics in nonlinear sciences [1, 2, 3, 4, 5, 7, 11, 14, 15, 16, 21, 23]. Lately, some scholars have given several methods to supply travelling wave solutions of NLEEs [6, 8, 17, 20]. In this study, ETEM [13] will be performed to seek exact solutions of (1+1) dimensional nonlinear Ostrovsky equation.

We tackle (1+1) dimensional nonlinear Ostrovsky equation,

$$uu_{xxt} - u_x u_{xt} + u^2 u_t = 0. \tag{1.1}$$

Eq. (1.1) is a model for weakly nonlinear surface and internal waves in a rotating ocean. It has been submitted by Vakhnenko and Parkers [18]. They have found completely integrable of Eq. (1.1) by inverse scattering method [19]. Then, some authors have used hyperbolic tangent method, exp-function method, (G'/G)-expansion method, tanh-coth function method and Bernoulli Sub-ODE method to find travelling wave solutions of this equation [9, 10, 12, 22, 24].

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The layout of this paper is organized as follows. In Sec. 2, we implement ETEM to (1+1) dimensional nonlinear Ostrovsky equation. In Sec. 3, we report the results that we obtained by using this method.

2. Application of ETEM to (1+1) Dimensional Nonlinear Ostrovsky Equation

We get the transformation via the wave variables

$$u(x,t) = u(\eta), \eta = x - ct, \tag{2.1}$$

where c is arbitrary constant.

Substituting Eq. (2.2) into Eq. (1.1),

$$u_t = -cu', u_x = u', u_{xt} = -cu'', u_{xxt} = -cu''', \tag{2.2}$$

we get following equation

$$3uu'' - 3(u')^2 + u^3 = 0. \tag{2.3}$$

Get transformation and trial equation as following:

$$u = \sum_{i=0}^{\delta} \tau_i \Gamma^i, \tag{2.4}$$

where

$$(\Gamma')^2 = \Lambda(\Gamma) = \frac{\phi(\Gamma)}{\psi(\Gamma)} = \frac{\xi_\theta \Gamma^\theta + \cdots + \xi_1 \Gamma + \xi_0}{\zeta_\epsilon \Gamma^\epsilon + \cdots + \zeta_1 \Gamma + \zeta_0}. \tag{2.5}$$

Paying regard to Eq. (2.4) and Eq. (2.5), we can obtain

$$(u')^2 = \frac{\phi(\Gamma)}{\psi(\Gamma)} \left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1} \right)^2, \tag{2.6}$$

$$u'' = \frac{\phi'(\Gamma)\psi(\Gamma) - \phi(\Gamma)\psi'(\Gamma)}{2\psi^2(\Gamma)} \left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1} \right) + \frac{\phi(\Gamma)}{\psi(\Gamma)} \left(\sum_{i}^{\delta} i(i-1)\tau_i \Gamma^{i-2} \right), \tag{2.7}$$

where $\phi(\Gamma)$ and $\psi(\Gamma)$ are polynomials. A relation of θ, ϵ and δ can be determined by paying regard to balance principle.

Reduce Eq. (2.7) to

$$\pm(\eta - \eta_0) = \int \frac{d\Gamma}{\sqrt{\Lambda(\Gamma)}} = \int \sqrt{\frac{\psi(\Gamma)}{\phi(\Gamma)}} d\Gamma. \tag{2.8}$$

Using a complete discrimination system for polynomial to sort the roots of $\phi(\Gamma)$, we unfasten Eq. (2.8) and divide the exact solutions to Eq. (1.1) by Mathematica.

Setting Eq. (2.4) and Eq. (2.7) into Eq. (2.5), and paying regard to balance principle, we find

$$\theta = \delta + \epsilon + 2. \tag{2.9}$$

In attempt to get exact solutions of Eq. (1.1), if we pick $\epsilon = 0, \delta = 1$ and $\theta = 3$ in Eq. (2.9), then

$$(u')^2 = \frac{\tau_1^2(\xi_0 + \Gamma\xi_1 + \Gamma^2\xi_2 + \Gamma^3\xi_3)}{\zeta_0}, u'' = \frac{\tau_1(\xi_1 + 2\Gamma\xi_2 + 3\Gamma^2\xi_3)}{2\zeta_0}, \tag{2.10}$$

where $\xi_4 \neq 0, \zeta_0 \neq 0$. Solving the algebraic equation system (2.5) supplies

$$\begin{gathered} \xi_1 = \xi_1, \xi_2 = \frac{\tau_1(2\xi_1\tau_0 - 3\xi_0\tau_1)}{\tau_0^2}, \xi_3 = \frac{\tau_1^2(\xi_1\tau_0 - 2\xi_0\tau_1)}{\tau_0^3} \\ \tau_0 = \tau_0, \tau_1 = \tau_1, \zeta_0 = -\frac{3\tau_1(\xi_1\tau_0 - 2\xi_0\tau_1)}{2\tau_0^3}. \end{gathered} \tag{2.11}$$

Replacing these results into Eq. (2.5) and Eq. (2.8), we get

$$\pm(\eta - \eta_0) = A\int \frac{d\Gamma}{\sqrt{\frac{\xi_0}{\xi_3} + \frac{\xi_1}{\xi_3}\Gamma + \frac{\xi_2}{\xi_3}\Gamma^2 + \Gamma^3}}, \tag{2.12}$$

where $A = \sqrt{\frac{-3}{2\tau_1}}$.

Taking the integral of Eq. (2.12), we attain the solutions of Eq. (1.1) as following:

$$\pm(\eta - \eta_0) = -2\sqrt{A}\frac{1}{\sqrt{\Gamma - \alpha_1}}, \tag{2.13}$$

$$\pm(\eta - \eta_0) = 2\sqrt{\frac{A}{\alpha_2 - \alpha_1}} \arctan\sqrt{\frac{\Gamma - \alpha_2}{\alpha_2 - \alpha_1}}, \alpha_2 > \alpha_1, \tag{2.14}$$

$$\pm(\eta - \eta_0) = \sqrt{\frac{A}{\alpha_1 - \alpha_2}} \ln\left|\frac{\sqrt{\Gamma - \alpha_2} - \sqrt{\alpha_1 - \alpha_2}}{\sqrt{\Gamma - \alpha_2} + \sqrt{\alpha_1 - \alpha_2}}\right|, \alpha_1 > \alpha_2, \tag{2.15}$$

$$\pm(\eta - \eta_0) = 2\sqrt{\frac{A}{\alpha_1 - \alpha_3}}F(\varphi, l), \alpha_1 > \alpha_2 > \alpha_3, \tag{2.16}$$

where

$$F(\varphi, l) = \int_0^{\varphi} \frac{d\psi}{\sqrt{1 - l^2\sin^2\psi}}, \varphi = \arcsin\sqrt{\frac{\Gamma - \alpha_3}{\alpha_2 - \alpha_3}}, l^2 = \frac{\alpha_2 - \alpha_3}{\alpha_1 - \alpha_3}. \tag{2.17}$$

Also, α_1, α_2 and α_3 are the roots of the following equation

$$\Gamma^3 + \frac{\xi_2}{\xi_3}\Gamma^2 + \frac{\xi_1}{\xi_3}\Gamma + \frac{\xi_0}{\xi_3} = 0. \tag{2.18}$$

Inserting the solutions (2.13), (2.14), (2.15) and (2.16) into (2.4) by the help of Eq.(2.1), we can find rational function solution of Eq. (1.1)

$$u(x,t) = \tau_0 + \tau_1\alpha_1 + \frac{4\tau_1 A}{(x-ct-\eta_0)^2}, \tag{2.19}$$

hyperbolic function solutions,

$$u(x,t) = \tau_0 + \tau_1\alpha_1 + \tau_1(\alpha_2-\alpha_1)\tanh^2\left(\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A}}(x-ct-\eta_0)\right), \tag{2.20}$$

$$u(x,t) = \tau_0 + \tau_1\alpha_1 + \tau_1(\alpha_1-\alpha_2)csch^2\left(\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A}}(x-ct-\eta_0)\right), \tag{2.21}$$

and Jacobi elliptic function solutions

$$u(x,t) = \tau_0 + \tau_1\alpha_3 + \tau_1(\alpha_2-\alpha_3)sn^2\left(\mp\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_3}{A}}(x-ct-\eta_0), \frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}\right). \tag{2.22}$$

If we take $\tau_0 = -\tau_1\alpha_1$ and $\eta_0 = 0$, then the solutions (2.19), (2.20) and (2.21) can be reduced to rational function solution

$$u(x,t) = \left(\frac{2\sqrt{\tilde{A}}}{x-ct}\right)^2, \tag{2.23}$$

1-soliton solution

$$u(x,t) = \frac{A_1}{\cosh^2\left[B(x-ct)\right]}, \tag{2.24}$$

and singular soliton solution

$$u(x,t) = \frac{A_2}{\sinh^2\left[B(x-ct)\right]}, \tag{2.25}$$

where $\tilde{A} = \tau_1 A, A_1 = \tau_1(\alpha_2-\alpha_1), A_2 = \tau_1(\alpha_1-\alpha_2), B = \frac{1}{2}\frac{\sqrt{\alpha_1-\alpha_2}}{A}$. Here, A_1 and A_2 remark the amplitudes of the solitons, while c describes the velocity and B defines the inverse width of the solitons. So, it can be said that the solitons exist for $\tau_1 > 0$. Also, when we take $\tau_0 = -\tau_1\alpha_3$ and $\eta_0 = 0$, Eq.(2.22) can be reduced to

$$u_i(x,t) = A_3 sn^2\left(B_i(x-ct), \frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}\right), \tag{2.26}$$

where $A_3 = \tau_1(\alpha_2-\alpha_3)$ and $B_i = \frac{(-1)^i}{2}\frac{\sqrt{\alpha_1-\alpha_3}}{A}, (i=1,2)$.

Remark When $l \to 1$, the solution (2.26) can be demeaned to dark soliton solutions

$$u_i(x,t) = A_3\tanh^2\left[B_i(x-ct)\right], \tag{2.27}$$

where $\alpha_1 = \alpha_2$, and c identifies the velocity of the dark soliton.

FIGURE 1. Graph of Eq. (2.23) for $\tau_1 = 3, c = 1, -55 < x < 55, -1 < t < 1$ and $t = 1$ for 2D surface.

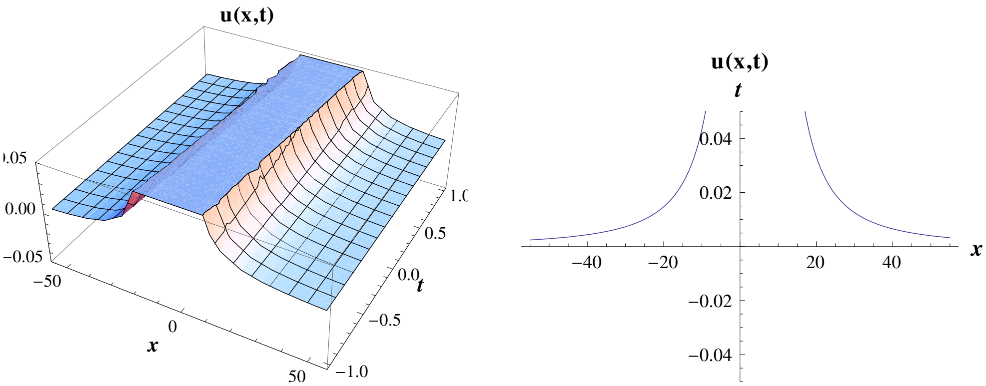
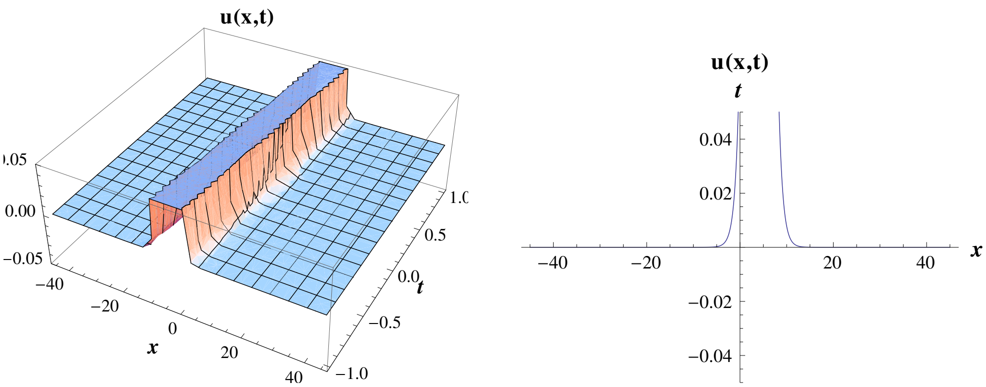


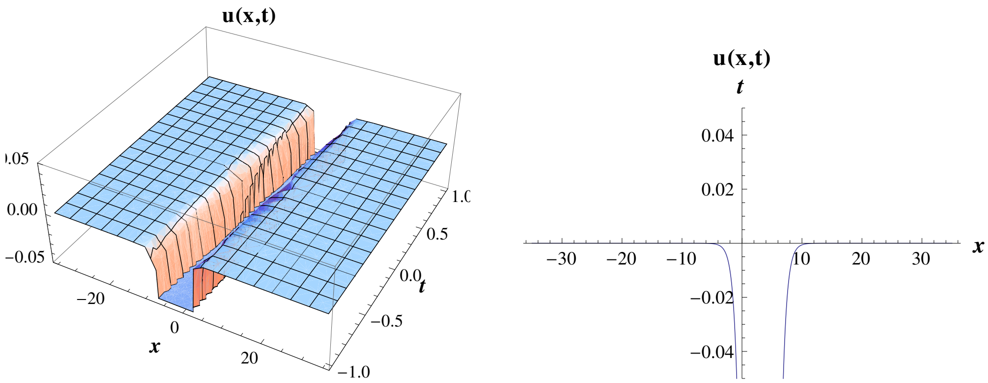
FIGURE 2. Graph of Eq. (2.24) for $\alpha_1 = 1, \alpha_2 = 2, \tau_1 = 2, c = 4, -45 < x < 45, -1 < t < 1$ and $t = 1$ for 2D surface.



3. Physical Explanations of the Solutions

In this paper, we obtained the solutions of rational function solutions, soliton solution, Jacobi elliptic function solution and hyperbolic function solution of (1+1) dimensional nonlinear Ostrovsky equation by using ETEM. Particularly, Jacobi elliptic function solutions are of substantial applications of periodic meromorphic functions. There are a lot of examples of these functions in the applied sciences such as fluid dynamics, optical fibers, electromagnetic theory, special relativity and heat transfer in several fields of physics. These solutions give to us several aspects of the solutions of NLEEs. By these solutions, we contributed some novel solutions. Also, we plot 2D and 3D surfaces of some solutions, which show the vitality of solutions with appropriate parameters. Numerical results together with the graphical demonstrations clearly present the reliability of this method.

FIGURE 3. Graph of Eq. (2.25) for $\alpha_1 = 2, \alpha_2 = 5, \tau_1 = 1$, $c = 3, -35 < x < 35, -1 < t < 1$ and $t = 1$ for 2D surface.



4. Results and Discussions

We illustrate accuracy and efficiency of ETEM by implementing the method to (1+1) dimensional nonlinear Ostrovsky equation. For this goal, we check the results via Wolfram Mathematica 9. Also, The graphical display shows validness of proposed method. When we compare with the exact solutions of Eq.(1.1) reported by the other authors, our solution (2.24) is the similar solution with the solution (4.15) in [22] and the solution (73) in [24]. Also, our solution (2.25) is the similar solution with the solution (71) in [24]. According to us, other solutions of Eq. (1.1) are novel and are not represented before.

5. Conclusion

In this work, we find travelling wave solutions of (1+1) dimensional nonlinear Ostrovsky equation by using ETEM. It is necessary to note that ETEM supplies powerful mathematical devices for finding the analytical solutions of this equation and this method is highly efficient in the way of finding new exact solutions.

References

[1] M. Dehghan and J. Manafian, *The solution of the variable coefficients fourth order parabolic partial differential equations by homotopy perturbation method*, Z. Naturforsch., *64* (2009), 420–430.

[2] M. Dehghan, J. Manafian Heris, and A. Saadatmandi, *Application of semi-analytic methods for the Fitzhugh-Nagumo equation, which models the transmission of nerve impulses*, Math. Meth. Appl. Sci., *33* (2010), 1384–1398.

[3] M. Dehghan, J. Manafian Heris, and A. Saadatmandi, *Application of the Exp-function method for solving a partial differential equation arising in biology and population genetics*, Int. J. Num. Meth. Heat Fluid Flow, *21* (2011), 736–753.

[4] M. Dehghan, J. Manafian, and A. Saadatmandi, *Analytical treatment of some partial differential equations arising in mathematical physics by using the Exp-function method*, Int. J. Modern Phys. B, *25* (2011), 2965–2981.

[5] M. Foroutan,I. Zamanpour, and J. Manafian, *Applications of IBSOM and ETEM for solving the nonlinear chains of atoms with long-range interactions*, Eur. Phys. J. Plus, *132*(421) (2017), 1–18.

[6] E. Fan, *Extended tanh-function method and its applications to nonlinear equations*, Phys. Lett. A, *277*(4-5) (2000), 212–218.

[7] K. Hosseini, J. Manafian, F. Samadani, M. Foroutan, M. Mirzazadeh, and Q. Zhou, *Resonant optical solitons with perturbation terms and fractional temporal evolution using improved tan* $(\phi(\eta)/2)$*-expansion method and exp function approach*, Optik, *158* (2018), 933–939.

[8] J. H. He and X. H. Wu, *Exp-function method for nonlinear wave equations*, Chaos Soliton. Fract, *30* (2006), 700–708.

[9] C. Koroglu and T. Ozis, *A novel traveling wave solution for Ostrovsky equation using Exp-function method*, Computers and Mathematics with Applications, *58* (2009), 2142–2146.

[10] J. Lee and R. Sakthivel, *Exact traveling wave solutions for some nonlinear evolution equations*, Commun. Math. Sci., *7*(4) (2009), 1053-1062.

[11] J. Manafian, *On the complex structures of the Biswas-Milovic equation for power, parabolic and dual parabolic law nonlinearities*, Eur. Phys. J. Plus, *130* (2015), 1–20.

[12] S. Marzan, F. Farhana, Md. T. Ahmed, K. Khan, and M. A. Akbar, *Study of Nonlinear Evolution Equations in Mathematical Physics*, Global Journal of Science Frontier Research (F) , *13*(9) (2013), 45–53.

[13] Y. Pandir, Y. Gurefe, U. Kadak, and E. Misirli, *Classification of exact solutions for some nonlinear partial differential equations with generalized evolution*, Abstr. Appl. Anal, *2012* (2012), 1–12.

[14] S. H. Seyedi, B. N. Saray, and A. Ramazani, *On the multiscale simulation of squeezing nanofluid flow by a highprecision scheme*, Powder Technology, *340* (2018), 264–273.

[15] S. H. Seyedi, B. N. Saray, and M. R. H. Nobari, *Using interpolation scaling functions based on Galerkin method for solving non-Newtonian fluid flow between two vertical flat plates*, Applied Mathematics and Computation, *269* (2015), 488–496.

[16] C. T. Sendi, J. Manafian, H. Mobasseri, M. Mirzazadeh, Q. Zhou, and A. Bekir, *Application of the ITEM for solving three nonlinear evolution equations arising in fluid mechanics*, Nonlinear Dynamics, *95* (2019), 669–684.

[17] S. Tuluce Demiray and H. Bulut, *Soliton solutions of some nonlinear evolution problems by GKM*, Neural Comput. Applic, (2017),1–8 DOI 10.1007/s00521-017-2999-3.

[18] V. O. Vakhnenko and E. J. Parkers, *The two loop soliton solutions of the Vakhnenko equation*, Nonlinearity, *11* (1998), 1457–1464.

[19] V. O. Vakhnenko and E. J. Parkers, *The calculation of multi-soliton solutions of the Vakhnenko equation by the inverse scattering method*, Chaos Solutions and Fractals, *13* (2003), 1819–1826.

[20] M. Wang, X. Li and J. Zhang, *The G'/G-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics*, Phys. Lett. A, *372* (2008), 417–423.

[21] W. Yu, M. Ekici, M. Mirzazadeh, Q. Zhou, and W. Liu, *Periodic oscillations of dark solitons in nonlinear optics*, Optik, *165* (2018), 341–344.

[22] E. Yusufoglu and A. Bekir, *A traveling wave solution of the Ostrovsky equation*, Appl. Math. Comput, *186* (2007), 256–260.

[23] Y. Zhang, C. Yang, W. Yu, M. Mirzazadeh, Q. Zhou, and W. Liu, *Interactions of vector anti-dark solitons for the coupled nonlinear Schr dinger equation in inhomogeneous fibers*, Nonlinear Dynamics, *94*(2) (2018), 1351 -1360.

[24] E. M. E. Zayed and S. Al-Joudi, *The Traveling Wave Solutions for Nonlinear Partial Differential Equations Using the (G'/G)-expansion Method*, International Journal of Nonlinear Science, *8*(4) (2009), 435–447.