Computational Methods for Differential Equations
http://cmde.tabrizu.ac.ir
Vol. 4, No. 1, 2016, pp. 43-53

The comparison of optimal homotopy asymptotic method and homotopy perturbation method to solve Fisher equation

Zainab Ayati*
Department of Engineering sciences,
Faculty of Technology and Engineering East of Guilan,
University of Guilan P.C.44891-Rudsar-Vajargah,Iran.
E-mail: ayati.zainab@gmail.com

Sima Ahmady
Department of Mathematics, Payame Noor University,
P.O.Box 19395-3697, Tehran, Iran.
E-mail: sima.ahmadikia@gmail.com

Abstract In recent years, numerous approaches have been applied for finding the solutions of functional equations. One of them is the optimal homotopy asymptotic method. In current paper, this method has been applied for obtaining the approximate solution of Fisher equation. The reliability of the method will be shown by solving some examples of various kinds and comparing the obtained outcomes with the results of homotopy Perturbation method.

Keywords. Optimal Homotopy Asymptotic method, Homotopy Perturbation method, Fisher equation.
2010 Mathematics Subject Classification. 34K28, 35A25, 41Axx.

1. Introduction

In the past two decades, partial differential equations have been the subject of many studies, owing to their importance in the modeling of many phenomena in the areas sciences [13-17]. Fisher's equation occurs in chemical kinetics and population dynamics which include problems such as neutron population in a nuclear reaction, nonlinear evolution of a population in a one-dimensional habitat, logistic population growth models, flame propagation, neurophysiology, autocatalytic chemical reactions, and branching Brownian motion processes [1-2].

Wazwaz et al. used Adomian Decomposition method (ADM) for the exact solutions of Fisher's equation and a nonlinear diffusion equation of the Fisher's type [3]. Matinfar et al. used Homotopy perturbation method (HPM), variational iterative method (VIM) and modified VIM for Fisher's equation, Generalized Fisher's equation and nonlinear diffusion equation of the Fisher's type [4-7].

Received: 17 July 2016 ; Accepted: 9 November 2016.
∗ Corresponding author.

The objective of this paper is to show the effectiveness of optimal homotopy asymptotic method (OHAM) for the solution of Fisher's equation. In present work, we are dealing with the approximate solution of the Fisher equation as follows

$$u_t = u_{xx} + \alpha\, u(1 - \alpha\, u), \tag{1.1}$$

where α is known constant.

2. Basic idea of OHAM

To illustrate the basic concept of optimal homotopy asymptotic method [8,9], consider the following nonlinear differential equation

$$A(u(x)) + g(x) = 0, \;\; x \in \Omega\,, \tag{2.1}$$

with boundary conditions

$$B(u, \partial u/\partial n) = 0, \;\; x \in \Gamma,$$

where A is a general differential operator, B is a boundary operator, $g(x)$ is a known analytic function, and Γ is the boundary of the domain Ω. Generally speaking the operator A can be divided into two parts L and N, where L is a linear, while N is a nonlinear operator. Therefore, Eq.(2.1) can be rewritten as follows

$$L(u) + N(u) + g(x) = 0. \tag{2.2}$$

For applying optimal homotopy asymptotic method we construct a homotopy $h(v(x,p),p) : R \times [0,1] \to R$ which satisfies

$$(1-p)\left[L(v(x,p)) + g(x)\right] = H(p)\left[L(v(x,p)) + g(x) + N(v(x,p))\right], \tag{2.3}$$

where $x \in R$and $p \in [0,1]$ is an embedding parameter, $H(p)$ is a nonzero auxiliary function for $p \neq 0$, $H(0) = 0$ and $v(x,p)$ is an unknown function. Obviously, when $p = 0$ and $p = 1$ it holds that $v(x,0) = u_0(x)$ and $v(x,1) = u(x)$ respectively.
Thus, as p varies from 0 to1, the solution $v(x,p)$ approaches from $u_0(x)$ to $u(x)$ where $u_0(x)$ is obtained from Eq.4.
For $p = 0$, we have

$$L(u_0(x)) + g(x) = 0, \; B\left(u_0, \frac{du_0}{dx}\right) = 0. \tag{2.4}$$

Next, we choose auxiliary function $H(p)$ in the form

$$H(p) = pc_1 + p^2 c_2 + ...,$$

where $c_1,\; c_2,\; ...$ constants to be determined. $H(p)$ can be expressed in many forms as reported by V. Marinca et al. [8-11].

To get an approximate solution, we expand $v(x,p,c_i)$ in Taylor's series about p in the following manner,

$$v(x,p,c_i) = u_0(x) + \sum_{k=1}^{\infty} u_k(x, c_1, c_2, ..., c_k)p^k. \tag{2.5}$$

Substituting Eq. (2.5) into Eq. (2.3) and equating the coefficient of like powers of p, the following linear equations will be obtained.

Zeroth order problem is given by Eq. (2.4) and the first order problem is given by following equation

$$L(u_1(x)) + g(x) = c_1 N_0(u_0(x)),\ B\left(u_1, \frac{du_1}{dx}\right) = 0.$$

The general governing equations for $u_k(x)$ are given by:

$$L\left(u_k(x)\right) - L\left(u_{k-1}(x)\right) = c_k N_0(u_0(x))$$
$$+ \sum_{i=1}^{k-1} c_i \left[L\left(u_{k-i}(x)\right) + N_{k-i}(u_0(x), u_1(x), ..., u_{k-1}(x))\right],$$
$$k = 2, 3, ...\ \ B\left(u_k \frac{du_k}{dx}\right) = 0.$$

Where $N_m(u_0(x), u_1(x), ..., u_m(x))$ is the coefficient of p^m in the expansion of $N(v(x,p))$ about the embedding parameter p.

$$N(v(x, p, c_i)) = N_0(u_0(x)) + \sum_{m=1}^{\infty} N_m(u_0, u_1, u_2, ..., u_m)p^m.$$

It has been observed that the convergence of the series (2.5) depends upon the auxiliary constants $c_1, c_2, ...$, if it is convergent at $p = 1$, one has

$$v(x, c_i) = u_0(x) + \sum_{k=1}^{\infty} u_m(x, c_1, c_2, ..., c_k).$$

The result of the m'th order approximations are given

$$\widetilde{u}(x, c_1, c_2, ..., c_m) = u_0(x) + \sum_{i=1}^{m} u_i(x, c_1, c_2, ..., c_i). \tag{2.6}$$

Substituting Eq. (2.6) into Eq. (2.2), leads to the following equation

$$R(x, c_1, c_2, ..., c_m) = L(\widetilde{u}(x, c_1, c_2, ..., c_m)) + g(x) + N(\widetilde{u}(x, c_1, c_2, ..., c_m)). \tag{2.7}$$

If $R = 0$ then $\widetilde{u}$ will be the exact solution. Generally, it does not happen, especially in nonlinear problems. There are several methods to find the optimal c_i such as Galerkin method, Ritz method, collocation method and least square method. In present paper, least square method has been applied to achieve the goal. Therefore, the following functional equation will be constructed

$$J(c_1, c_2, ..., c_m) = \int_a^b R^2(x, c_1, c_2, ..., c_m)dx. \tag{2.8}$$

And for minimizing it, we have

$$\frac{\partial J}{\partial c_1} = \frac{\partial J}{\partial c_2} = ... = \frac{\partial J}{\partial c_m} = 0. \tag{2.9}$$

After determining these constants, the approximate solution (of order m) will be obtained.

3. Solution of the Fisher equation by HPM and OHPM

3.1. Solving by HPM

Homotopy perturbation method has been well-addressed in [12]. So we skip to introduce the method and apply it directly.

Consider Eq. (1.1) with the following initial condition

$$u(x,0) = f(x).$$

According to the homotopy perturbation method, we construct the following homotopy

$$H(v,p) = (1-p)(v_t - (u_0)_t) + p(v_t - v_{xx} - \alpha v + \alpha^2 v^2) = 0,$$

or

$$v_t - (u_0)_t + p((u_0)_t - v_{xx} - \alpha v + \alpha^2 v^2) = 0. \tag{3.1}$$

To solve Eq. (3.1) by homotopy perturbation method, let's consider the solution v as the summation of a series,

$$v = \sum_{i=0}^{\infty} v_i p^i. \tag{3.2}$$

Substituting (3.2) into (3.1) leads to

$$\sum_{i=0}^{\infty} \frac{\partial v_i}{\partial t} p^i - \frac{\partial u_0}{\partial t} = p\left(-\frac{\partial u_0}{\partial t} + \sum_{i=0}^{\infty} \frac{\partial^2 v_i}{\partial x^2} p^i + \alpha \sum_{i=0}^{\infty} v_i p^i - \alpha^2 \left(\sum_{i=0}^{\infty} v_i p^i \right)^2 \right).$$

By equating the terms with the identical powers in p, we derive

$$\begin{aligned} &p^0 : (v_0)_t - (u_0)_t = 0,\\ &p^1 : (v_1)_t + (u_0)_t - (v_0)_{xx} - \alpha v_0 + \alpha^2 (v_0)^2 = 0,\\ &p^2 : (v_2)_t - (v_1)_{xx} - \alpha v_1 + 2\ \alpha^2 v_0 v_1 = 0,\\ &p^3 : (v_3)_t - (v_2)_{xx} - \alpha v_2 + \alpha^2 \left(2 v_0 v_2 + (v_1)^2\right) = 0,\\ &\vdots \end{aligned} \tag{3.3}$$

For the sake of simplicity, let's take $v_0 = u_0 = f(x)$, so we have

$$\begin{aligned} &v_1 = \textstyle\int_0^t (-(u_0)_t + (v_0)_{xx} + \alpha v_0 - \alpha^2 (v_0)^2) dt,\\ &v_2 = \textstyle\int_0^t ((v_1)_{xx} + \alpha v_1 - 2\ \alpha^2 v_0 v_1) dt,\\ &\ \vdots \end{aligned} \tag{3.4}$$

Setting p=1, results in an approximation to the solution of Eq.(1.1). So

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots$$

3.2. Solving by OHAM:

According to Eq. (2.3), we construct the following homotopy

$$(1-p)(v_t) = (c_1 p + c_2 p^2 + c_3 p^3 ...)(v_t - v_{xx} - \alpha v + \alpha^2 v^2),$$

or

$$v_t = p v_t + (c_1 p + c_2 p^2 + c_3 p^3 ...)(v_t - v_{xx} - \alpha v + \alpha^2 v^2). \tag{3.5}$$

Let's consider the solution v as the summation of a series (2.5), and by equating the terms with the identical powers in p, the following equations will be obtained

$$\begin{aligned}
&p^0 : (v_0)_t = 0,\\
&p^1 : (v_1)_t = (v_0)_t + c_1(v_0)_t - c_1(v_0)_{xx} - c_1\alpha\, v_0 + c_1\alpha^2(v_0)^2,\\
&p^2 : (v_2)_t = (v_1)_t + c_1(v_1)_t - c_1(v_1)_{xx} - c_1\alpha v_1 + 2c_1\alpha^2 v_0 v_1 + c_2(v_0)_t\\
&\qquad -c_2(v_0)_{xx} - c_2\alpha v_0 + c_2\alpha^2(v_0)^2,\\
&p^3 : (v_3)_t = (v_2)_t + c_1(v_2)_t - c_1(v_2)_{xx} - c_1\alpha v_2 + c_1\alpha^2\left((v_1)^2 + 2v_0v_2\right) + c_2(v_1)_t\\
&\qquad -c_2(v_1)_{xx} - c_2\alpha v_1 + 2c_2\alpha^2 v_0 v_1 + c_3(v_0)_t - c_3(v_0)_{xx} - c_3\alpha v_0 + c_3\alpha^2(v_0)^2,\\
&\ \vdots
\end{aligned}$$

let's take $v_0 = u_0 = f(x)$, so we have

$$v_1 = \int_0^t ((v_0)_t + c_1(v_0)_t - c_1(v_0)_{xx} - c_1\alpha v_0 + c_1\alpha^2(v_0)^2)dt,$$

$$\begin{aligned}
v_2 = &\int_0^t ((v_1)_t + c_1(v_1)_t - c_1(v_1)_{xx} - c_1\alpha v_1 + 2c_1\alpha^2 v_0 v_1 + c_2(v_0)_t - c_2(v_0)_{xx}\\
&-c_2\alpha v_0 + c_2\alpha^2(v_0)^2)dt,\\
&\vdots
\end{aligned}$$

So, by considering an approximation with $(m+1)$ terms as
$\widetilde{u}(x, c_1, c_2, ..., c_m) = u_0(x) + \sum_{i=1}^{m} u_i(x, c_1, c_2, ..., c_i)$,
and by using eq's (2.7)-(2.9) we compute c_i's.

4. Example

To illustrate the method some examples are provided. In each example, we defined values of α in prior to applying the method.

Example 1
If we take $\alpha = 1$ and $u(x,0) = f(x) = \lambda$, Eq. (1.1) turns into

$$u_t = u_{xx} + u - u^2,$$

where $\lambda$is constant. The exact solution is given by

$$u(x,t) = \frac{\lambda e^t}{1 - \lambda + \lambda e^t}.$$

Solution via HPM
According to HPM the following homotopy can be constructed

$$H(v,p) = (1-p)(v_t - (u_0)_t) + p(v_t - v_{xx} - v + v^2) = 0,$$

or

$$v_t - (u_0)_t + p((u_0)_t - v_{xx} - v + v^2) = 0. \; v_t - (u_0)_t + p((u_0)_t - v_{xx} - v + v^2) = 0.$$

Substitute $v = \sum_{i=0}^{\infty} v_i p^i$ in above Eqation and equating the coefficients of the terms with the identical powers of p, leads to

$$\begin{array}{l} p^0 : (v_0)_t - (u_0)_t = 0, \\ p^1 : (v_1)_t + (u_0)_t - (v_0)_{xx} - v_0 + (v_0)^2 = 0, \\ p^2 : (v_2)_t - (v_1)_{xx} - v_1 + 2v_0 v_1 = 0, \\ p^3 : (v_3)_t - (v_2)_{xx} - v_2 + 2v_0 v_2 + (v_1)^2 = 0, \\ \quad \vdots \end{array}$$

Assume $v_0 = \lambda$, then

$$v_1 = -\lambda(\lambda - 1)t,$$

$$v_2 = -\tfrac{3}{2}\lambda^2 t^2 + \tfrac{1}{2}\lambda t^2 + \lambda^3 t^2,$$

$$v_3 = -\tfrac{7}{6}\lambda^2 t^3 + \tfrac{1}{6}\lambda t^3 + 2\lambda^3 t^3 - \lambda^4 t^3,$$

$$\vdots$$

Consider approximation for four terms $v \approx v_0 + v_1 + v_2 + v_3$ solution of equation will be obtained as the following form

$$v(x,t) = \lambda - \lambda^2 t + \lambda t - \frac{3}{2}\lambda^2 t^2 + \frac{1}{2}\lambda t^2 + \lambda^3 t^2 - \frac{7}{6}\lambda^2 t^3 + \frac{1}{6}\lambda t^3 + 2\lambda^3 t^3 - \lambda^4 t^3.$$

Solution via OHAM
According to the OHAM, by applying Eq. (2.3), we derive

$$(1-p)(v_t) = (c_1 p + c_2 p^2 + c_3 p^3 \ldots)(v_t - v_{xx} - v + v^2).$$

By substituting Eq. (2.5) into above equation, and equating the coefficients of the terms with the identical powers of p, leads to

$$
\begin{aligned}
&(v_0)_t = 0,\\
&(v_1)_t = (v_0)_t + c_1(v_0)_t - c_1(v_0)_{xx} - c_1 v_0 + c_1(v_0)^2,\\
&(v_2)_t = (v_1)_t + c_1(v_1)_t - c_1(v_1)_{xx} - c_1 v_1 + 2c_1 v_0 v_1 + c_2(v_0)_t - c_2(v_0)_{xx}\\
&-c_2 v_0 + c_2(v_0)^2,\\
&(v_3)_t = (v_2)_t + c_1(v_2)_t - c_1(v_2)_{xx} - c_1 v_2 + c_1\left((v_1)^2 + 2v_0 v_2\right) + c_2(v_1)_t\\
&\quad -c_2(v_1)_{xx} - c_2 v_1 + 2c_2 v_0 v_1 + c_3(v_0)_t - c_3(v_0)_{xx} - c_3 v_0 + c_3(v_0)^2,\\
&\quad \vdots
\end{aligned}
$$

Assume $v_0 = \lambda$, then

$$
\begin{aligned}
&v_1 = c_1\lambda(\lambda-1)t,\\
&v_2 = c_1\lambda^2 t - c_1\lambda t + c_1^2\lambda^2 t - c_1^2\lambda t - \tfrac{3}{2}c_1^2\lambda^2 t^2 + \tfrac{1}{2}c_1^2\lambda t^2 + c_1^2\lambda^3 t^2 - c_2\lambda t + c_2\lambda^2 t,\\
&v_3 = -c_1\lambda t - 2c_1^2\lambda t + c_1^2\lambda t^2 - 2c_1c_2\lambda t + 2c_1c_2\lambda^2 t + c_1c_2\lambda t^2 - 3c_1c_2\lambda^2 t^2\\
&\ +2c_1c_2\lambda^3 t^2 - c_2\lambda t + c_2\lambda^2 t + c_1\lambda^2 t + 2c_1^2\lambda^2 t - 3c_1^2\lambda^2 t^2 + 2c_1^2\lambda^3 t^2 + c_1^3\lambda^4 t^3\\
&\ -c_3\lambda t + c_3\lambda^2 t + c_1^3\lambda^2 t - c_1^3\lambda t - 3c_1^3\lambda^2 t^2 + c_1^3\lambda t^2 + 2c_1^3\lambda^3 t^2 + \tfrac{7}{6}c_1^3\lambda^2 t^3\\
&-\tfrac{1}{6}c_1^3\lambda t^3 - 2c_1^3\lambda^3 t^3,\\
&\quad \vdots
\end{aligned}
$$

Therefore, the four terms approximation using OHAM for solution will be obtained as follows

$$
\begin{aligned}
&v = -3c_1\lambda t - 3c_1^2\lambda t + \tfrac{3}{2}c_1^2\lambda t^2 - 2c_1c_2\lambda t + 2c_1c_2\lambda^2 t + c_1c_2\lambda t^2 - 3c_1c_2\lambda^2 t^2\\
&\quad +2c_1c_2\lambda^3 t^2 - 2c_2\lambda t + 2c_2\lambda^2 t + 3c_1\lambda^2 t + 3c_1^2\lambda^2 t - \tfrac{9}{2}c_1^2\lambda^2 t^2 + 3c_1^2\lambda^3 t^2 + \lambda\\
&\quad +c_1^3\lambda^4 t^3 - c_3\lambda t + c_3\lambda^2 t + c_1^3\lambda^2 t - c_1^3\lambda t - 3c_1^3\lambda^2 t^2 + c_1^3\lambda t^2 + 2c_1^3\lambda^3 t^2 + \tfrac{7}{6}c_1^3\lambda^2 t^3\\
&-\tfrac{1}{6}c_1^3\lambda t^3 - 2c_1^3\lambda^3 t^3.
\end{aligned}
$$

The values of $c-i$'s are obtained by least square method

$$c_1 = -0/5654150115,\ c_2 = 0/005621158248,\ c_3 = 0/007399517569.$$

Table 1. Comparison of OHAM and HPM for $\lambda = 1/5$.

t	Exact	HPM	OHAM	Error(HPM)	Error(OHAM)
0/0	2	2	2	0	0
0/1	1/82621286	1/82566666	1/83582544	0/00054620	0/00961257
0/2	1/69309410	1/68533333	1/70242349	0/00776077	0/00932939
0/3	1/58833302	0/55300000	1/59594406	0/03533302	0/00676138
0/4	1/50412134	0/40266666	1/50913840	0/10145467	0/00501706
0/5	1/43526659	0/20833333	1/43985575	0/22693326	0/00458915

It should be noted that by increasing the amount of$t$, the errors of OHAM have been less than the error of HPM.

Example 2
In Eq. (1.1), set $\alpha = 6$ and $u(x,0) = f(x) = \frac{1}{(1+e^x)^2}$. The exact solution is given by

$$u(x,t) = \frac{1}{\left(1+e^{x-5t}\right)^2},$$

$$u_t = u_{xx} + 6u - 36u^2.$$

Solution via HPM
According to the HPM the following homotopy can be constructed

$$v_t - (u_0)_t + p\left((u_0)_t - v_{xx} - 6v + 36v^2\right) = 0.$$

Substitute $v = v_0 + pv_1 + p^2 v_2 + ...$ in above equation and equating the coefficients of the terms with the identical powers of p, leads to

$$\begin{array}{l}(v_0)_t - (u_0)_t = 0,\\ (v_1)_t + (u_0)_t - (v_0)_{xx} - 6v_0 + 36v_0^2 = 0,\\ (v_2)_t - (v_1)_{xx} - 6v_1 + 72v_0v_1 = 0,\\ (v_3)_t - (v_2)_{xx} - 6v_2 + 72v_0v_2 + 36v_1^2 = 0,\\ \vdots\end{array}$$

Assume $v_0 = \frac{1}{(1+e^x)^2}$, then

$$v_1 = \frac{10\left(e^{2x}+e^x-3\right)t}{\left(1+e^x\right)^4},$$

$$v_2 = \frac{5t^2\left(10e^{4x}+15e^{3x}-126e^{2x}+89e^x+198\right)}{(1+e^x)^6},$$

$$v_3 = \frac{5}{3}\frac{\left(100e^{6x}+125e^{5x}-4172e^{4x}-3794e^{3x}+20084e^{2x}+11293e^x-19548\right)}{(1+e^x)^8},$$

$$\vdots$$

Consider approximation for four terms $v \approx v_0 + v_1 + v_2 + v_3$ solution of equation will be obtained as the following form

$$\begin{array}{l}v = \frac{1}{3}\frac{1}{(1+e^x)^8}(3 - 90t + 18e^x + 150e^{6x}t^2 + 625e^{5x}t^3 + 30e^{6x}t + 525e^{5x}t^2\\ -20860e^{4x}t^3 + 150e^{5x}t - 1290e^{4x}t^2 - 18970e^{3x}t^3 + 210e^{4x}t\\ -4890e^{3x}t^2 + 100420e^{2x}t^3 - 60e^{3x}t - 1590e^{2x}t^2 - 390e^{2x}t\\ +500e^{6x}t^3 + 56465e^x t^3 + 4605e^x t^2 - 330e^x t + 18e^{5x}\\ +3e^{6x} - 97740t^3 + 60e^{3x} + 45e^{4x} + 2970t^2 + 45e^{3x}).\end{array}$$

Solution via OHAM
According to the OHAM, we derive

$$(1-p)(v_t) = (c_1p + c_2p^2 + c_3p^3...)(v_t - v_{xx} - 6v + 36v^2).$$

By substituting Eq.(2.5) into above equation, and equating the coefficients of the terms with the identical powers of p, leads to

$$
\begin{aligned}
&(v_0)_t = 0,\\
&(v_1)_t = (v_0)_t + c_1\,(v_0)_t - c_1\,(v_0)_{xx} - 6c_1 v_0 + 36c_1(v_0)^2,\\
&(v_2)_t = (v_1)_t + c_1\,(v_1)_t - c_1\,(v_1)_{xx} - 6c_1 v_1 + 72c_1 v_0 v_1\\
&+c_2\,(v_0)_t - c_2\,(v_0)_{xx} - 6c_2 v_0 + 36c_2(v_0)^2,\\
&(v_3)_t = (v_2)_t + c_1\,(v_2)_t - c_1\,(v_2)_{xx} - 6c_1 v_2 + 72c_1 v_0 v_2\\
&+36c_1(v_1)^2 + c_2\,(v_1)_t - c_2\,(v_1)_{xx} - 6c_2 v_1\\
&+72c_2 v_0 v_1 - c_3\,(v_0)_t - c_3\,(v_0)_{xx} - 6c_3 v_0 + 36c_3(v_0)^2,\\
&\vdots
\end{aligned}
$$

By considering $v_0 = \frac{1}{(1+e^x)^2}$, the following results will be obtained

$$
\begin{aligned}
&v_1 = \frac{10c_1\left(e^{2x}+e^x-3\right)t}{(1+e^x)^4},\\
&v_2 = \frac{1}{(1+e^x)^6}(5t(10e^{4x}c_1^2 t - 2e^{4x}c_1^2 + 15e^{3x}tc_1^2 - 2e^{4x}c_1 - 2e^{4x}c_2 - 6e^{3x}c_1^2 - 126e^{2x}tc_1^2\\
&-6e^{3x}c_1 - 6e^{3x}c_2 - 89e^x tc_1^2 + 10e^x c_1^2 + 198tc_1^2 + 10e^x c_1 + 10e^x c_2 + 6c_1^2 + 6c_1 + 6c_2)),\\
&v_3 = -\frac{5}{3}\frac{1}{(1+e^x)^8}(t(-1188tc_1^2 + 30e^{5x}c_1 + 30e^{5x}c_2 + 30e^{5x}c_3 - 12e^{3x}c_1^3 + 42e^{4x}c_3\\
&-78e^{2x}c_1 - 78e^{2x}c_2 - 78e^{2x}c_3 + 6e^{6x}c_1^3 + 12e^{6x}c_1^2 + 30e^{5x}c_1^3 + 6e^{6x}c_1 + 6e^{6x}c_2\\
&+6e^{6x}c_3 + 60e^{5x}c_1^2 + 42e^{4x}c_1^3 - 24e^{3x}c_1c_2 - 156e^{2x}c_1c_2 + 60e^{5x}c_1c_2 + 1956e^{3x}tc_1^3\\
&+20084e^{2x}t^2c_1^3 + 84e^{4x}t^2c_1c_2 + 636e^{2x}tc_1^3 - 4172e^{4x}t^2c_1^3 + 12e^{6x}c_1c_2 - 210e^{5x}tc_1^2\\
&+516e^{4x}tc_1^3 - 3794e^{3x}t^2c_1^3 - 60e^{6x}tc_1^3 + 125e^{5x}t^2c_1^3 - 60e^{6x}tc_1^2 - 210e^{5x}tc_1^3\\
&+100e^{6x}t^2c_1^3 - 1188tc_1^3 - 66e^x c_1^3 - 66e^x c_3 - 36c_1c_2 + 11293e^x t^2c_1^3\\
&-1842e^x tc_1^3 - 132e^x c_1c_2 - 19548t^2c_1^3 + 516e^{4x}tc_1c_2 + 1956e^{3x}tc_1c_2\\
&+636e^{2x}tc_1c_2 - 60e^{6x}tc_1c_2 - 210e^{5x}tc_1c_2 - 1842e^x tc_1c_2 - 18c_1^3 - 1188tc_1c - 18c_3\\
&+516e^{4x}tc_1^2 + 1956e^{3x}tc_1^2 - 36c_1^2 - 18c_2 + 42e^{4x}c_1 + 42e^{4x}c_2 - 24e^{3x}c_1^2\\
&-12e^{3x}c_1 - 12e^{3x}c_2 + 84e^{4x}c_1^2 - 132e^x c_1^2 - 66e^x c_1 - 66e^x c_2 - 156e^{2x}c_1^2 + 636e^{2x}tc_1^2\\
&-1842e^x c_1^2 t - 18c_1)),\\
&\vdots
\end{aligned}
$$

Therefore, the four terms, approximation using OHAM for solution will be obtained as follows

$$v = \frac{1}{3}\frac{1}{(1+e^x)^8}(-3 - 270c_1^2t - 18e^x - 390e^{2x}tc_3 + 210e^{4x}tc_3 - 60e^{3x}tc_3$$
$$-1170e^{2x}tc_1 - 780e^{2x}tc_2 + 30e^{6x}tc_3 + 100420e^{2x}t^3c_1^3 + 450e^{5x}tc_1 + 300e^{5x}tc_2$$
$$+150e^{5x}tc_3 - 20860e^{4x}t^3c_1^3 - 1575e^{5x}t^2c_1^2 - 18970e^{3x}t^3c_1^3 + 90e^{6x}tc_1 + 60e^{6x}tc_2$$
$$+500e^{6x}t^3c_1^3 + 625e^{5x}t^3c_1^3 - 450e^{6x}t^2c_1^2 - 330e^xtc_3 - 5940t^2c_1c_2 + 56465e^xt^3c_1^3$$
$$-300e^{6x}t^2c_1c_2 - 1050e^{5x}t^2c_1c_2 + 2580e^{4x}t^2c_1c_2 + 9780e^{3x}t^2c_1c_2 + 3180e^{2x}t^2c_1c_2$$
$$-9210e^xt^2c_1c_2 - 97740t^3c_1^3 - 60e^{3x}tc_1^3 + 3180e^{2x}t^2c_1^3 - 390e^{2x}tc_1^3 + 2580e^{4x}t^2c_1^3$$
$$+450e^{5x}tc_1^2 + 210e^{4x}tc_1^3 + 9780e^{3x}t^2c_1^3 + 30e^{6x}tc_1^3 - 1050e^{5x}t^2c_1^3 + 90e^{6x}tc_1^2$$
$$+150e^{5x}tc_1^3 - 300e^{6x}t^2c_1^3 - 18e^{5x} - 3e^{6x} - 90tc_1^3 - 9210e^xt^2c_1^3$$
$$-330e^xtc_1^3 - 5940t^2c_1^3 + 420e^{4x}tc_1c_2 - 120e^xtc_1c_2 - 780e^xtc_1c_2 + 60e^xtc_1c_2$$
$$+300e^xtc_1c_2 - 660e^xtc_1c_2 - 180tc_1c_2 - 90tc_3 - 8910t^2c_1^2 - 270tc_1$$
$$+3870e^{4x}t^2c_1^2 + 14670e^{3x}t^2c_1^2 + 630e^{4x}tc_1 + 420e^{4x}tc_2 + 4770e^{2x}t^2c_1^2 - 180e^{3x}tc_1$$
$$-120e^{3x}tc_1 - 13815e^xt^2c_1^2 - 990e^xtc_1 - 660e^xtc_2 - 1170e^{2x}tc_1^2 + 630e^{4x}tc_1^2$$
$$-180e^{3x}tc_1^2 - 60e^{3x} - 45e^{4x} - 180tc_2 - 990e^xtc_1^2 - 45e^{2x}).$$

We use least squares method to obtain the unknown convergent constants c_1, c_2 and c_3. So

$$c_1 = 0/1904265795,\ c_2 = 0/2972665626,\ c_3 = -2/962386224.$$

Table 2. Comparison of OHAM and HPM for $x = 1$

t	Exact	HPM	OHAM	Error(HPM)	Error(OHAM)
0/00	0/07232948	0/07232948	0.07232948	0/00000000	0/00000000
0/02	0/08355019	0/07744470	0/08396484	0/00610548	0/00041465
0/04	0/09611582	0/08217936	0/09555592	0/01393645	0/00055989
0/06	0/11009935	0/00863461	0/10710401	0/02375318	0/00299534
0/08	0/12555945	0/08975782	0/81861041	0/03580162	0/00694904
0/10	0/14253695	0/09222702	0/13007641	0/05030992	0/01246054

As it clear table like the previous example, by increasing the amount of t, the error of OHAM is less than the error of HPM.

5. Conclusion

In this paper, the Fisher equation has been solved by HPM and OHAM. The results obtained by OHAM are very consistent in comparison to HPM. It is found that OHAM compared with HPM is a reliable efficient and powerful method for solving nonlinear partial differential equations, especially for the Fisher equation. Therefore, we believe that the OHAM is an expectable technique for solving linear and nonlinear Fisher equation.

References

[1] P. Brazhnik, and J. Tyson, *on traveling wave solutions of Fisher's equation in two spatial dimensions*, SIAM J. Appl. Math., *60* (1999), 371–391.

[2] W. Malfliet, *Solitary wave solutions of nonlinear wave equations*, J. Phys., *60* (1992), 650–654.

[3] A.M. Wazwaz, A. Gorguis, *An analytic study of Fisher's equation by using Adomian decomposition method*, J. Phys., *154* (2004), 609–620.

[4] M. Matinfar, M. Ghanbari, *Homotopy Perturbation Method for the Fisher's Equation and Its Generalized*, Int.J. Nonlin. Sci., *8(4)* (2009), 448–455.

[5] M. Matinfar, M. Ghanbari, *HHomotopy Perturbation Method for the Generalized Fisher's Equation*, Islamic Azad University of Lahijan, *4(27)* (2011), 39–44.

[6] M. Matinfar, M. Ghanbari, *Solving the Fisher's Equation by Means of Variation Iteration Method*, Int. J. Contemp. Math.Sciences, *4(7)* (2009), 343–348.

[7] M. Matinfar, M. Ghanbari, *SThe application of the modified variational iteration method on the generalized Fisher's equation*, J. Appl. Math. Comput., *5(19)* (2008), 34–39.

[8] V. Marinca, N. Herisanu, *An Optimal Homotopy Asymptotic Method for solving nonlinear equations arising in heat transfer*, Int. Comm. Heat Mass Transfer , *35* (2008), 710–715.

[9] V. Marinca, N. Herisanu, I. Nemes, *optimal Homotopy Asymptotic Method with application to thin film flow*, Cent. Eur. J. Phys. , *6(3)* (2008), 648–653.

[10] N. Herisanu, V. Marinca, T. Dordea, G. Madescu, *A new analytical approach to nonlinear vibration of an electric machine*, Proc. Romanian. Acad. Ser.A: Math. Phys. Tech. Sci., Inf. Sci. , *9(3)* (2008), 229–236.

[11] V. Marinca, N. Herisanu, C. Bota, B. Marinca, *An Optimal Homotopy Asymptotic Method applied to the steady flow of fourth-grade fluid past a porous plate*, Appl. Math. Lett., *22(2)* (2009), 245–251.

[12] J.H. He, *Homotopy perturbation technique*, Computer Methods in Applied Mechanics and Engineering, *178* (1999), 257–262.

[13] J. Manafian, M. Lakestani, *Dispersive dark optical soliton with Tzitzeica type nonlinear evolution equations arising in nonlinear optics*,ptical and Quantum Electronics, *48* (2016), 116.

[14] J. Manafian, *Optical soliton solutions for Schrodinger type nonlinear evolution equations by the tan*($\phi/2$)*-expansion method,*,Optik – International journal for Light and Electron Optics, *127* (2016), 4222-4245.

[15] J. Manafian, M. Lakestani , *Abundant soliton solutions for the Kundu-Eckhaus equation via tan*($\phi/2$)*-expansion method*, Optik – International journal for Light and Electron Optics, *127* (2016), 5543-5551.

[16] M. Dehghan, J. Manafian, *AThe solution of the variable coefficients fourth-order parabolic partial differential equations by homotopy perturbation method*, Z. Naturforsch64a, *127* (2009), 420–430.

[17] M. Dehghan, J. Manafian, A. Saadatmandi, *Application of semi-analytic methods for the Fitzhugh–Nagumo equation, which models the transmission of nerve impulses*, Math. Methods Appl. Sci., *33* (2010), 1384–1398.