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Finite-time stabilization of satellite quaternion attitude

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Abstract In this paper, a finite-time control scheme is presented for stabilization of the satellite chaotic attitude around its equilibrium point when its attitude is confused by a disturbed torque. Controllers and settling-time of stabilizaton are obtained, based on the Lyapunov stability theorem and finite-time control scheme. This method is satisfied for any initial condition. Numerical simulations are presented to illustrate the ability and effectiveness of proposed method.

Keywords. Finite-time stabilization, quaternion, satellite attitude.

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1. Introduction

At first this section, a brief history of finite-time stability is presented according to [3]. The concept of finite-time stability (FTS) dates back to the 1950s, when it was introduced in the Russian literature [19, 21, 22]; later, during the 1960s, this concept appeared in the western journals [16, 25, 32].

In the last two decades, FTS and stabilization have been investigated in the context of continuous-time linear systems [5, 6, 7, 9, 11, 13, 14, 18, 28] and of discrete-time linear systems [4, 8, 10, 12]; in these papers, conditions for analysis and design are generally provided in terms of feasibility problems involving Linear Matrix Inequalities (LMIs) [15] and/or Differential Linear Matrix Inequalities (DLMIs) [27] and Differential Lyapunov Equations (DLEs). More recently, an effort has been spent to extend such results to the context of nonlinear systems [13, 23, 34]. The issue of control and stabilization in most of nonlinear systems, and particularly those who argue about the control of satellitse attitude, have been analized without calculating the settling-time [24, 26].

In this paper, along with FTS analysis of satellite attitude, the settling-time also is calculated. Moreover, quaternion parameters are used instead of Euler angles in the

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satellite's kinematic equations. These parameters are applied to overcome singularity problem in the numerical solution when performing the great maneuvers is required.

Another important feature of the proposed control scheme compared to other similar control methods, it is the possibility of rapid stabilization of the satellite attitude around its equilibrium point, with applicable choice of control parameter η.

2. FTS and satellite attitude equations

A system is said to be finite-time stable if, given a bound on the initial condition, its state does not exceed a certain threshold during a specified time interval. More precisely, given the system

$$\dot{x}(t) = f(t, x), \qquad x(t_0) = x_0, \tag{2.1}$$

where $f : \mathbb{R}^+ \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is continuous nonlinear function, $x \in \mathbb{R}^n$ is the state vector, x_0 is initial state, we can give the following formal definition [3].

Definition 2.1. (FTS) Given an initial time t_0, a positive scalar T, two sets X_0 and X_t, system (2.1) is said to be finite-time stable with respect to (t_0, T, X_0, X_t) if

$$x_0 \in X_0 \Rightarrow x(t) \in X_t, \qquad t \in [t_0, t_0 + T], \tag{2.2}$$

where, $x(.)$ denotes the solution of (2.1) starting from x_0 at time t_0, X_0 is initial set, and X_t is trajectory set. Sets X_0 and X_t are generally determined from practical considerations.

We consider another definition of FTS in this paper which will be utilized in analyzing the stability of our system.

Definition 2.2. (see [31]). Consider the dynamic system (2.1), if there exists a constant $T^* > 0$ (T^* may depend on the initial state $X(0)$), such that $\lim_{t\to T^*} \parallel x(t) \parallel = 0$, and $\parallel x(t) \parallel = 0$ if $t \geq T^*$, then the system (2.1) is finite-time stable.

In the following this section, we describe the satellite's attitude equations. These equations are expressed by kinematics and kinetic equations.

2.1. kinematics equations.

The kinematics equations explain relationship between attitude and angular velocity in inertial frame. The quaternion is a four-dimensional vector which is defined as [33]

$$\vec{q} = \begin{bmatrix} \vec{Q}^T & q_4 \end{bmatrix}^T, \tag{2.3}$$

where $\vec{Q} = [q_1 \;\; q_2 \;\; q_3]^T = \hat{e}sin(\upsilon)$, $q_4 = cos(\upsilon/2)$, $\hat{e}$ is the unit vector in the direction of the Euler axes and υ is the rotation angle between the body and orbital frames. The quaternion vector satisfy the constraint $\vec{q}^T\vec{q} = 1$. By regarding the satellite as an ideal rigid body, the kinematics equations based on quaternion parameters are expressed as [29]

$$\begin{cases} \dot{q_1} = \frac{1}{2}\left(w_z q_2 - w_y q_3 + w_x q_4\right), & \dot{q_2} = \frac{1}{2}\left(-w_z q_1 + w_x q_3 + w_y q_4\right), \\ \dot{q_3} = \frac{1}{2}\left(w_y q_1 - w_x q_2 + w_z q_4\right), & \dot{q_4} = \frac{1}{2}\left(-w_x q_1 - w_y q_2 - w_z q_3\right), \end{cases} \tag{2.4}$$

where w_x, w_y, w_z are angular velocities around axes fixed in the rigid body.

2.2. **kinetic equations.** Relation between angular velocity and torque in body frame is given by kinetic equations. These equations can be derived from a Newton-Euler formulation [17]

$$\begin{cases} \dot{w}_x = \frac{1}{I_x}\left[(I_y - I_z)w_y w_z + c_x\right], \\ \dot{w}_y = \frac{1}{I_y}\left[(I_z - I_x)w_x w_z + c_y\right], \\ \dot{w}_z = \frac{1}{I_z}\left[(I_x - I_y)w_x w_y + c_z\right], \end{cases} \tag{2.5}$$

where I_x, I_y, I_z are the inertial moments of the satellite about its principal axes, and c_x, c_y, c_z are torques applied around these axes at time t.

3. Chaos analysis based on the Lyapunov exponents (LEs)

By combining (2.4) and (2.5), the satellite attitude system is presented as

$$\begin{cases} \dot{q}_1 = \frac{1}{2}\left(w_z q_2 - w_y q_3 + w_x q_4\right), \\ \dot{q}_2 = \frac{1}{2}\left(-w_z q_1 + w_x q_3 + w_y q_4\right), \\ \dot{q}_3 = \frac{1}{2}\left(w_y q_1 - w_x q_2 + w_z q_4\right), \\ \dot{q}_4 = \frac{1}{2}\left(-w_x q_1 - w_y q_2 - w_z q_3\right), \\ \dot{w}_x = \frac{1}{I_x}\left[(I_y - I_z)w_y w_z + c_x\right], \\ \dot{w}_y = \frac{1}{I_y}\left[(I_z - I_x)w_x w_z + c_y\right], \\ \dot{w}_z = \frac{1}{I_z}\left[(I_x - I_y)w_x w_y + c_z\right]. \end{cases} \tag{3.1}$$

In the following, we use SA symbol to refer to the equation (3.1).

Now by choosing constant values and initial conditions given in Table 1, the LEs of SA system are obtained under the perturbing torques

$$\begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} = \begin{bmatrix} -1200 & 0 & (1000)\frac{\sqrt{6}}{2} \\ 0 & 350 & 0 \\ -(1000)\sqrt{6} & 0 & -400 \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}, \tag{3.2}$$

for more detail the reader can see [20, 30]. Figure 1 illustrates dynamics of LEs of the SA system. In this figure value of each of LEs is given in time $t = 50(s)$, and existing positive LE, indicate that the system is chaotic.

4. Prelimination

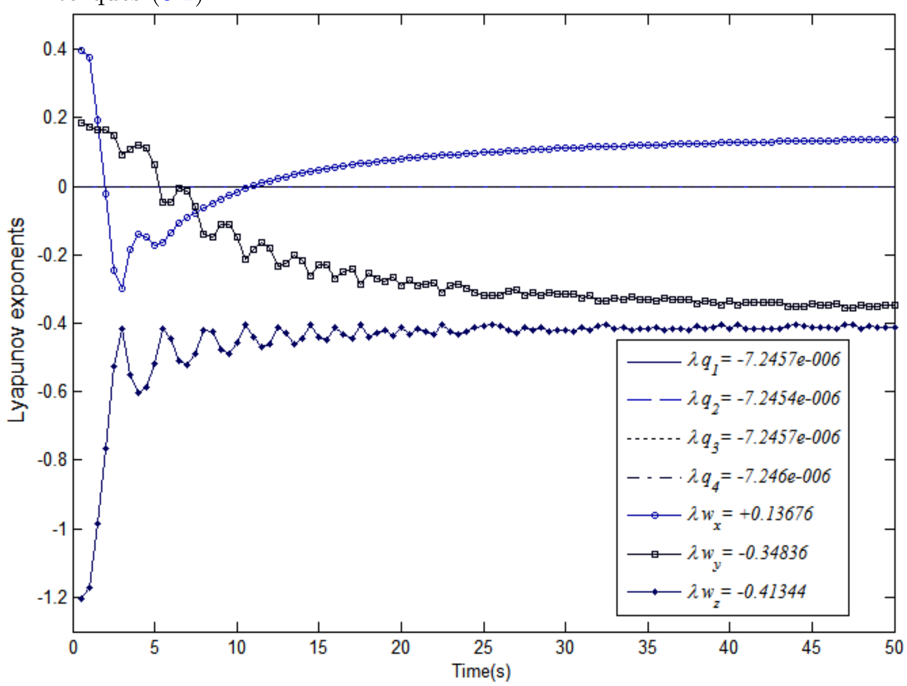
Consider dynamical system

$$\dot{x}_i(t) = f_i(x_1(t), x_2(t), ..., x_n(t)), \qquad i = 1, ..., n. \tag{4.1}$$

TABLE 1. Initial conditions and constant values of the SA system.

Attitudes	Values	Constants	Values
q_{10}	0.2425	$I_x(kgm^2)$	3000
q_{20}	0.04915	$I_y(kgm^2)$	2000
q_{30}	0.4645	$I_z(kgm^2)$	1000
q_{40}	0.8503		
$w_{x_0}(r/s)$	0.2		
$w_{y_0}(r/s)$	0.6		
$w_{z_0}(r/s)$	0.8		

FIGURE 1. Dynamics of LEs of the SA system under the perturbing torques (3.2).



where, $x_1, x_2, ..., x_n$ are components of attitude vector x, $f_i : \mathbb{R}^n \longrightarrow \mathbb{R}^n, (i = 1, ..., n)$ are continuous nonlinear functions. The finite-time controlled system with initial conditions is

$$\begin{array}{l} \dot{x}_i(t) = f_i(x_1(t), x_2(t), ..., x_n(t)) + u_i(t), \qquad i = 1, ..., n. \\ x_i(0) = x_{i,0}, \end{array} \tag{4.2}$$

where, $x_{i,0}, (i = 1, ..., n)$ are components of initial attitude x_0, and $u_i(t), (i = 1, ..., n)$ are control functions.

Let

$$e(t) = x(t) - \bar{x}, \tag{4.3}$$

is attitude error vector where, $\bar{x}$ is the equilibrium point of system (4.2). The goal is to design the appropriate controllers $u_i(t)$ such that for any initial condition x_0, we have

$$\lim_{t \to T^*} \| e(t) \| = 0,$$

where $\| \, . \, \|$ is the Euclidean norm and T^* is the settling time. Settling time dependent to initial conditions.

We need the following theorem to analyze the finite-time stability of the SA system and calculate the value of T^*.

Theorem 4.1. [35] *Suppose that function $V(t) : [0, \infty) \longrightarrow [0, \infty)$ is differentiable (the derivative of $V(t)$ at 0 is, in fact, its right derivative) and*

$$\dot{V}(t) \leq -KV(t)^{\alpha}, \tag{4.4}$$

where $K > 0$ and $0 < \alpha < 1$. Then $V(t)$ will reach to zero in finite time $T^* \leq \frac{V(0)^{(1-\alpha)}}{K(1-\alpha)}$, and $V(t) = 0$ for all $t > T^*$.

5. Finite-time stability and control

Considering the error vector corresponding to the SA system, and choosing the equilibrium point $\bar{x} = (0, 0, 0, 1, 0, 0, 0)$, dynamic of error system is identical with SA system. Then controlled system of the SA system is presented as

$$\begin{cases} \dot{q_1} = \frac{1}{2}\left(w_z q_2 - w_y q_3 + w_x q_4\right) + u_1, \\ \dot{q_2} = \frac{1}{2}\left(-w_z q_1 + w_x q_3 + w_y q_4\right) + u_2, \\ \dot{q_3} = \frac{1}{2}\left(w_y q_1 - w_x q_2 + w_z q_4\right) + u_3, \\ \dot{q_4} = \frac{1}{2}\left(-w_x q_1 - w_y q_2 - w_z q_3\right) + u_4, \\ I_x \dot{w_x} = [(I_y - I_z) w_y w_z + c_x] + u_5, \\ I_y \dot{w_y} = [(I_z - I_x) w_x w_z + c_y] + u_6, \\ I_z \dot{w_z} = [(I_x - I_y) w_x w_y + c_z] + u_7, \\ (q_1(0), ..., q_4(0), w_x(0), w_y(0), w_z(0)) = (q_{1_0}, ..., q_{4_0}, w_{x_0}, w_{y_0}, w_{z_0}). \end{cases} \tag{5.1}$$

The control functions will be designed such that for any initial condition, we have

$$\lim_{t \to T^*} \| (q_1, q_2, q_3, q_4, w_x, w_y, w_z) - (0, 0, 0, 1, 0, 0, 0) \| = 0.$$

Lemma 5.1. [2] *Suppose* $a_1, a_2, ..., a_n$ *and* $0 < q < 2$ *are all real numbers, then the following inequality holds*

$$|a_1|^q + |a_2|^q + ... + |a_n|^q \geq ({a_1}^2 + {a_2}^2 + ... + {a_n}^2)^{\frac{q}{2}}. \tag{5.2}$$

Theorem 5.2. *System (5.1) can be finite-time stable for any different initial condition with following control functions*

$$\begin{cases} u_1 = -\eta\, |q_1|^\alpha\, sign(q_1), \\ u_1 = -\eta\, |q_2|^\alpha\, sign(q_2), \\ u_2 = -\eta\, |q_3|^\alpha\, sign(q_3), \\ u_4 = -\eta\, |1 - q_4|^\alpha\, sign(q_4 - 1), \\ u_5 = -\eta {I_x}^{\frac{\alpha+1}{2}}\, |w_x|^\alpha\, sign(w_x) - [(I_y - I_z)w_y w_z + c_x] - \frac{1}{2}q_1, \\ u_6 = -\eta {I_y}^{\frac{\alpha+1}{2}}\, |w_y|^\alpha\, sign(w_y) - [(I_z - I_x)w_x w_z + c_y] - \frac{1}{2}q_2, \\ u_7 = -\eta {I_z}^{\frac{\alpha+1}{2}}\, |w_z|^\alpha\, sign(w_z) - [(I_x - I_y)w_x w_y + c_z] - \frac{1}{2}q_3, \end{cases} \tag{5.3}$$

where $0 < \alpha < 1$, $\eta > 0$ *is control parameter, and the settling time is*

$$T^* \leq \frac{V(0)^{\frac{1-\alpha}{2}}}{\eta 2^{\frac{\alpha+1}{2}}(\frac{1-\alpha}{2})}. \tag{5.4}$$

Proof. Consider the lyapunov function as

$$V(t) = \frac{1}{2}[q_1^2 + q_2^2 + q_3^2 + (1 - q_4)^2 + I_x w_x^2 + I_y w_y^2 + I_z w_z^2], \tag{5.5}$$

therefor

$$\begin{aligned} \dot{V} &= q_1\dot{q_1} + q_2\dot{q_2} + q_3\dot{q_3} + \dot{q_4}(q_4 - 1) + I_x w_x \dot{w_x} + I_y w_y \dot{w_y} + I_z w_z \dot{w_z} \\ &= \tfrac{1}{2}(q_1 w_x + q_2 w_y + q_3 w_z) + w_x\, [(I_y - I_z)w_y w_z + c_x] \\ &+ w_y\, [(I_z - I_x)w_x w_z + c_y] + w_z\, [(I_x - I_y)w_x w_y + c_z] \\ &+ q_1 u_1 + q_2 u_2 + q_3 u_3 + (q_4 - 1)u_4 + w_x u_1 + w_y u_2 + w_z u_3. \end{aligned} \tag{5.6}$$

By replacing control functions (5.3) in (5.6), we have

$$\begin{aligned} \dot{V} &= -\eta(|q_1|^{\alpha+1} + |q_2|^{\alpha+1} + |q_3|^{\alpha+1} + |1 - q_4|^{\alpha+1} \\ &+ \left|\sqrt{I_x} w_x\right|^{\alpha+1} + \left|\sqrt{I_y} w_y\right|^{\alpha+1} + \left|\sqrt{I_z} w_z\right|^{\alpha+1}). \end{aligned} \tag{5.7}$$

From Lemma 5.1, equality (5.7) be changed as

$$\begin{aligned} \dot{V} &\leq -\eta(|I_x w_x|^2 + |I_y w_y|^2 + |I_z w_z|^2 + |q_1|^2 + |q_2|^2 + |q_3|^2 \\ &+ |1 - q_4|^2)^{\frac{\alpha+1}{2}}. \end{aligned} \tag{5.8}$$

Therefore

$$\dot{V} \leq -\eta(2V(t))^{\frac{\alpha+1}{2}}, \tag{5.9}$$

or

$$\dot{V} \leq -(\eta 2^{\frac{\alpha+1}{2}})V(t)^{\frac{\alpha+1}{2}}. \tag{5.10}$$

Due to $0 < \alpha < 1$, it yields $0 < \frac{\alpha+1}{2} < 1$. Therefore, from theorem 4.1, and (5.10), it is obtained

$$V(t) \equiv 0, \qquad t > T^*, \tag{5.11}$$

where settling time T^* is given as

$$T^* \leq \frac{V(0)^{\frac{1-\alpha}{2}}}{\eta 2^{\frac{\alpha+1}{2}}(\frac{1-\alpha}{2})}. \tag{5.12}$$

□

Remark 5.3. [1] The $sign(.)$ function, as a rigid switcher, in the control functions (5.3), may cause undesirable oscillations. In order to avoid this problem, they are approximated by the $tanh(\rho(.))$ function, where $\rho > 0$ is a constant. Hence, we rewrite the control functions (5.3) as

$$\begin{cases} u_1 = -\eta \left|q_1\right|^{\alpha} tanh(\rho q_1), \\ u_1 = -\eta \left|q_2\right|^{\alpha} tanh(\rho q_2), \\ u_2 = -\eta \left|q_3\right|^{\alpha} tanh(\rho q_3), \\ u_4 = -\eta \left|1-q_4\right|^{\alpha} tanh(\rho(q_4-1)), \\ u_5 = -\eta I_x^{\frac{\alpha+1}{2}} \left|w_x\right|^{\alpha} tanh(\rho w_x) - [(I_y - I_z)w_y w_z + c_x] - \frac{1}{2}q_1, \\ u_6 = -\eta I_y^{\frac{\alpha+1}{2}} \left|w_y\right|^{\alpha} tanh(\rho w_y) - [(I_z - I_x)w_x w_z + c_y] - \frac{1}{2}q_2, \\ u_7 = -\eta I_z^{\frac{\alpha+1}{2}} \left|w_z\right|^{\alpha} tanh(\rho w_z) - [(I_x - I_y)w_x w_y + c_z] - \frac{1}{2}q_3. \end{cases} \tag{5.13}$$

6. Numerical simulation of finite-time control

To demonstrate and verify the validity of the proposed scheme, we present the numerical results for finite-time control of satellite chaotic attitude. Assuming $\alpha = 0.7$, $\rho = 100$, the initial point and the constant values given in Table 1, system (5.1) is solved with control functions (5.13) by Maple software. We control satellite from an arbitrary initial attitude to its equilibrium point $(0, 0, 0, 1, 0, 0, 0)$ in a finite time.

Figure 2 and Figure 3 illustrate the simulation results of the system (5.1) based on the control functions (5.13) for $\eta = 0.25$ and $\eta = 5$, respectively. In these figures, time series responses corresponding to quaternion parameters and angular velocities demonstrate the appropriate performance of the controllers in order to return satellite attitude to its equilibrium point. Simple calculations show the settling time for $\eta = 0.25$ and $\eta = 5$, is $T^* = 39.8557s$, and $T^* = 1.9928s$, respectively. Thus, if possible, by choosing a large and allowable amount from control parameter η, we can more quickly converge to the equilibrium point of system.

Figure 2. Time series responses corresponding to angular velocities and quaternion parameters of the system (5.1) with control parameter $\eta = .25$.

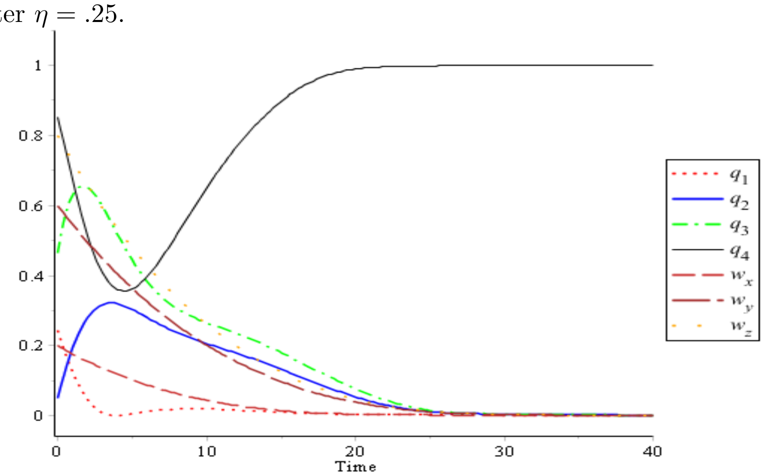
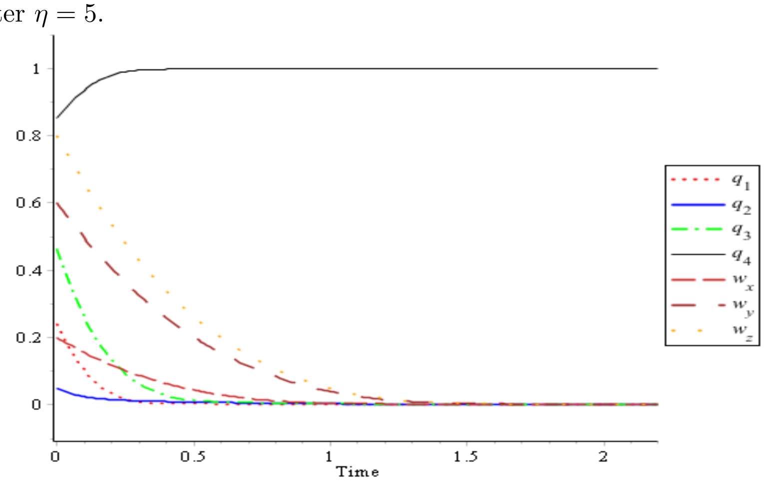


Figure 3. Time series responses corresponding to angular velocities and quaternion parameters of the system (5.1) with control parameter $\eta = 5$.



7. Conclusion

A finite-time control scheme along with quaternion parameters were applied on the SA system for stabilization the satellite chaotic attitude around its equilibrium point, when this system is confused by a disturbed torque. In this method, it is possible to calculate the settling time. Whilst this method can solve the singularity problem in the numerical solution of system, the simulation results illustrated quick stability the SA system as for achieving the its equilibrium point, by choosing a large and allowable amount from control parameter η.

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