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Some new exact traveling wave solutions one dimensional modified complex Ginzburg- Landau equation

M. Mortazavi*
Department of Applied Mathematics,
School of Mathematical Sciences,
Ferdowsi University of Mashhad,
Mashhad, Iran
E-mail: m_mortazavi95@yahoo.com

M. Mirzazadeh
Department of Mathematics,
Faculty of Mathematical Sciences,
University of Guilan,
Rasht, Iran.
E-mail: Mirzazadehs2@guilan.ac.ir

Abstract In this paper, we obtain exact solutions involving parameters of some nonlinear PDEs in mathematical physics; namely the one-dimensional modified complex Ginzburg-Landau equation by using the $(G^{'}/G)$ expansion method, homogeneous balance method, extended F-expansion method. By using homogeneous balance principle and the extended F-expansion, more periodic wave solutions expressed by jacobi elliptic functions for the 1D MCGL equation are derived. Homogeneous method is a powerful method, it can be used to construct a large families of exact solutions to different nonlinear differential equations that does not involve independent variables.

Keywords. Exact traveling wave Solutions, MCGL equation, $(G^{'}/G)$ -expanson method.

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1. Introduction

Soliton generation in actively and passively mode locked fiber lasers is presently a very active area of nonlinear polarization rotation may be described by the modified complex Ginzburg-Landau equation (MCGLE). This equation also applies when describing soliton propagation in optical fiber systems with linear and nonlinear gain and spectral filtering. Different forms of the CGLE have been used, including the cubic Ginzburg-Landau equation, cubic CGLE with saturation, and move complicated models.The schrodinger equation is a special case of the Ginzburg-Landau equation for purely dispersive waves ([1]- [21]).

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* M. Mortazavi corresponding.

In the present letter, we consider a class of nonlinear partial differential equation with constant coefficients which is called one- dimensional modified complex Ginzburg-Landau equation (1D MCGLE)

$$i\Phi_t + p\Phi_{xx} + q|\Phi|^2\Phi = a\frac{\Phi_x\Phi_x^*}{\Phi^*} + b\nabla^2(\sqrt{\Phi\Phi^*})\sqrt{\frac{\Phi}{\Phi^*}} + i\gamma\Phi,$$

where the operator $\nabla^2 = \frac{\partial^2}{\partial x^2}$ and the system parameters p, q, a, b and γ may be real, complex or a combination of the two different combinations of the above system parameters describe different types of wave propagation in different physical systems. During the past decades, with the development of soliton theory, some methods for obtaining analytic solution to NPDEs have been proposed, such as the sine-cosine method [17] , first integral method ([10], [24]) , $(G^{'}/G)$- expansion method ([14], [32]) , the exp- function method ([34], [33]) , Jacobi elliptic expansion method [25] , and so on. F-expansion method and extended F-expansion method which ([18], [22], [29], [11], [12])" was proposed recently as an overall generalization of Jacobi elliptic expansion function method. Most of exact solutions were obtained by these methods, including the solitary wave solutions, shock wave solution, periodic wave solutions and so on. Very recently, the extended F-expansion method has been proposed to obtain not only the single non degenerative Jacobi elliptic function solutions, but also the combined non degenerative Jacobi elliptic solutions and their corresponding degenerative solutions. Homogeneous balance method (HBM) ([9], [28], [15], [16])" is proved to be efficient method for finding explicit solutions of NEEs. The main objective of paper are used the $(G^{'}/G)$-expansion method, extended F-expansion method and homogeneous balance method to constant the exact solutions for nonlinear evolution equation in the mathematical physics via the modified complex Ginzburg-Landau.

2. Description of the $(G^{'}/G)$-expansion method

Suppose we have the following nonlinear PDE:

$$p(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \ldots) = 0, \tag{2.1}$$

where $u = u(x, t)$ is an unknown function, p is a polynomial in $u = u(x, t)$ and its various partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of a deformation method

Step 1. The traveling wave variable

$$u(x,t) = u(\xi), \quad \xi = k(x - ct), \tag{2.2}$$

where k and c, are the wave number and the wave speed, respectively. Under the transformation (2.2) , (2.1)" becomes an ordinary differential equation (ODE) as

$$p(u, u^{'}, u^{''}, u^{'''}, \dots) = 0. \tag{2.3}$$

Step 2. Suppose that the solution Eq. (2.3)" has the following form

$$u(\xi) = \sum_{i=0}^{m} a_i (G^{'}/G)^i, \tag{2.4}$$

while $G = G(\xi)$ satisfies the second order linear differential equation

$$G^{''}(\xi) + \lambda G^{'}(\xi) + \mu G(\xi) = 0, \tag{2.5}$$

where $a_i (i = 0, 1, \dots, m)$, λ and μ are constants to be determined later. The positive integer m can be determined by considering the homogeneous balance between the highest derivative terms and the nonlinear terms appearing in (2.3)".
Step 3. The solution of the differential (2.5)" is

$$\frac{G^{'}}{G} = \begin{cases} \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\left(\dfrac{c_1 \cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi) + c_2 \sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi)}{c_1 \sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi) + c_2 \cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi)}\right) - \dfrac{\lambda}{2} & if \lambda^2 - 4\mu > 0, \\ \frac{1}{2}\sqrt{4\mu - \lambda^2}\left(\dfrac{-c_1 \sin(\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi) + c_2 \cos(\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi)}{c_1 \cos(\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi) + c_2 \sin(\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi)}\right) - \dfrac{\lambda}{2} & if \lambda^2 - 4\mu < 0, \\ \dfrac{c_2}{c_2\xi + c_1} - \dfrac{\lambda}{2} & if \lambda^2 - 4\mu = 0, \end{cases} \tag{2.6}$$

where c_1 and c_2 are arbitrary constants.

Step 4. By substituting (2.4)" into (2.3)" and using (2.5) , collecting all terms with the same power of $(G^{'}/G)$ together and then equating each coefficient of the resulted polynomial to zero, yield a set of algebraic equations for a_i, μ, λ, c and k.

3. Description of the homogeneous balance method

Step 1. We suppose that Eq. (2.3)" has the formal solution

$$u(\xi) = \sum_{i=0}^{m} a_i (G(\xi)^i, \tag{3.1}$$

where $a_i (i = 0, 1, \cdots, m)$ are constant to be determined, such that $a_m \neq 0$, and $G(\xi)$ is the solution of the equation

$$G^{'}(\xi) = G^2(\xi) - G(\xi) \tag{3.2}$$

Eq. (3.2)" has the solution

$$G(\xi) = \frac{1}{1 \pm e^{\xi}}. \tag{3.3}$$

Step 2. We determine the positive integer m in Eq. (3.1)" by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (2.3)".
Step 3. Substitute (3.1)" into Eq. (2.3) , we calculate all the necessary derivatives $u^{'}, u^{''}, \dots$ of the function $u(\xi)$. As a result of this substitution, we get a polynomial of $G^i, (i = 0, 1, \dots, m)$. In this polynomial we gather all terms of same powers and equating them to zero, we obtain a system algebraic equations which can be solved by the Maple or Mathematica to get the unknown parameters $a_i (i = 0, 1, \dots, m), k$ and w.

4. Description of an extended F- expansion method

Step 1. Supposing that $u(\xi)$ can be expressed as

$$u(\xi) = a_0 + \sum_{i=1}^{m} a_i F(\xi)^i + \sum_{i=1}^{m} b_i F(\xi)^i, \tag{4.1}$$

where a_i, b_i are constants to be determined, $F(\xi)$ satisfy the following relation

$$(F^{'})^2 = PF^4 + QF^2 + R, \tag{4.2}$$

where P, Q, R are parameters.

Step 2. Inserting F-expansion (4.1)" into Eq. (2.3)" and using (4.2) , we obtain a series in $(F^p, p = 0, 1, \dots, k)$. Equating each coefficients of (F^p) to zero yields a system of algebraic equations for $(a_i, b_i, i = 1, 2, \dots, m; c, k)$.

Step 3. Solving these equations, a_i, b_i, c and k can be expressed in terms of P, Q, R and the parameters of NODE (2.3) . Substituting these results into (2.3)" gives the general form of travelling wave solutions (See**Appendix A**).

Step 3. With the aid of Appendices A and B and the relation Eq. (4.2)" the appropriate kinds of the Jacobi elliptic function solutions of (2.3)" including the single functions and the combined function solutions can be chosen. As we know, when $m \to 1$, Jacobi elliptic function degenerate as hyperbolic functions in the manner of **Appendix C**. When $m \to 0$, Jacobi elliptic function degenerate as trigonometric functions in the manner of **Appendix C**. So we can get the corresponding hyperbolic function solutions and trigonometric function solutions.

5. One dimensional modified complex Ginzburg- Landau equation

5.1. **The $(G^{'}/G)$ method.** We will exert $(G^{'}/G)$ -expansion method to solve the MCGL equation [25]".

We consider the MCGL equation in the following form

$$\begin{aligned} &i\Phi_t + p\Phi_{xx} + q|\Phi|^2\Phi = \\ &a\frac{\Phi_x\Phi_x^*}{\Phi^*} + b\left(\frac{1}{2}(\Phi\Phi^*)_{xx}\Phi\Phi^* - \frac{1}{4}((\Phi\Phi^*)_x)^2\right)\frac{1}{\Phi\Phi^{*2}} + i\gamma\Phi. \end{aligned} \tag{5.1}$$

To solve (5.1) , consider the wave transformation

$$\Phi(x,t) = u(\xi)e^{i(kx-wt)}, \qquad \xi = x - ct, \tag{5.2}$$

where $u(\xi)$ is a real function, c represents the wave speed, k, w are constants that to be determined later. We substitute (5.2)" into (5.1)" to get following complex equation for u ,

$$-icu^{'} + wu + p(u^{''} + 2iku^{'} - k^2u) + qu^3 = a\frac{(u^{'})^2 + k^2u^2}{u} + bu^{''} + i\gamma u. \tag{5.3}$$

The real and imaginary parts of equation (5.2)" are separated which yield the following equations.

$$wu + p(u^{''} - k^2u)u + qu^4 = a((u^{'})^2 + k^2u^2) + bu^{''}, \tag{5.4}$$

and

$$(2pk-c)u^{'}=\gamma u \tag{5.5}$$

where $u^{'}=\dfrac{du}{d\xi}$ and $u^{''}=\dfrac{d^2u}{d\xi^2}$ with $\xi=x-ct$.

Now (5.5)" gives the velocity of the soliton as

$$c=2pk-\frac{Lnu}{t}. \tag{5.6}$$

Eq. (5.4) , by balancing $u^{''}u$ and u^4 we get

$$m+2+m=4m\Rightarrow m=1. \tag{5.7}$$

Therefore $m=1$ reduces Eq. (2.4)" to

$$u(\xi)=a_0+a_1\frac{G^{'}(\xi)}{G(\xi)}. \tag{5.8}$$

Substituting (5.8)" into Eq. (5.4) , collecting the coefficients of each power of $(\dfrac{G^{'}}{G})$, and solve the system of algebraic equations using Maple, we obtain the set of solution:

$$\begin{aligned} a_0&=-\frac{1}{3}\frac{b(a-b\lambda)}{ap},a_1=\frac{2}{3}\frac{b^2}{ap},\mu=-\frac{1}{4}\frac{a^2-\lambda^2b^2}{b^2},\\ k&=k,w=\frac{2pa^2+ak^2b^2+k^2b^2p}{b^2},q=-\frac{9}{2}\frac{p^3a^2}{b^4}.\end{aligned} \tag{5.9}$$

By substituting (5.9)" into (5.8)" and using (2.6) , we obtain

Case 1. If $\sqrt{\lambda^2-4\mu}>0$, then we have the hyperbolic solution

$$\begin{aligned}\Phi_1(x,t)=e^{i(kx-wt)}\Bigg\{&-\frac{1}{3}\frac{b(a-b\lambda)}{ap}+\frac{2}{3}\frac{b^2}{ap}\Bigg[\frac{1}{2}\sqrt{\lambda^2-4\mu}\\ &\left(\frac{c_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}(x-ct))+c_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}(x-ct))}{c_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}(x-ct))+c_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}(x-ct))}\right)-\frac{\lambda}{2}\Bigg]\Bigg\},\end{aligned} \tag{5.10}$$

where $\mu = -\frac{1}{4}\frac{a^2-\lambda^2b^2}{b^2}, w = \frac{2pa^2+ak^2b^2+k^2b^2p}{b^2}$ and $c = 2pk - \frac{Lnu}{t}$.

Case 2. If $\sqrt{\lambda^2-4\mu} < 0$, then we have the trigonometric solution

$$\begin{aligned}\Phi_2(x,t) = e^{i(kx-wt)}\Bigg\{ & -\frac{1}{3}\frac{b(a-b\lambda)}{ap} + \frac{2}{3}\frac{b^2}{ap}\Bigg[\frac{1}{2}\sqrt{4\mu-\lambda^2} \\ & \left(\frac{-c_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}(x-ct)) + c_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}(x-ct))}{c_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}(x-ct)) + c_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}(x-ct))}\right) - \frac{\lambda}{2}\Bigg]\Bigg\},\end{aligned} \tag{5.11}$$

where $\mu = -\frac{1}{4}\frac{a^2-\lambda^2b^2}{b^2}, w = \frac{2pa^2+ak^2b^2+k^2b^2p}{b^2}$ and $c = 2pk - \frac{Lnu}{t}$.

Case 3. If $\lambda^2-4\mu = 0$ then we have the rational solution

$$\Phi_3(x,t) = \left\{-\frac{1}{3}\frac{b(a-b\lambda)}{ap} + \frac{2}{3}\frac{b^2}{ap}\left[\frac{c_2}{c_2(x-ct)+c_1} - \frac{\lambda}{2}\right]\right\}e^{i(kx-wt)}, \tag{5.12}$$

where $\mu = -\frac{1}{4}\frac{a^2-\lambda^2b^2}{b^2}, w = \frac{2pa^2+ak^2b^2+k^2b^2p}{b^2}$ and $c = 2pk - \frac{Lnu}{t}$.

In particular, if we set $c_2 = 0$, $c_1 \neq 0$, $\lambda > 0$ and $\mu = 0$, in (5.10) , then we get

$$\Phi_4(x,t) = \left\{-\frac{1}{3}\frac{b(a-b\lambda)}{ap} + \frac{2}{3}\frac{b^2}{ap}\left[\frac{1}{2}\lambda csch(\lambda(x-ct)) - \frac{\lambda}{2}\right]\right\}e^{i(kx-wt)}, \tag{5.13}$$

where $w = \frac{2pa^2+ak^2b^2+k^2b^2p}{b^2}$ and $c = 2pk - \frac{Lnu}{t}$.

We make graphs of obtained solutions, so that they can depict the importance of each obtained solution and physically interpret the importance of parameters. Some of our obtained traveling wave solutions are represented in **Figure1** and **Figure2** whit the aid of Maple. We choose $a = 1$, $b = -1$, $p = 1$, $\lambda = 5$, $k = 2$, $w = 10$, $i = -1$, $c = 3$, $c_1 = -c_2$.

5.2. **The homogeneous balance method.** According to step 2, we assume that Eq. (5.4)" possesses the solutions in the form

$$u(\xi) = a_0 + a_1G(\xi). \tag{5.14}$$

Substituting (5.14)" with (3.2)" into Eq. (5.4)" and equating each of the coefficients of $G^i(\xi), i = 0,1,\ldots,4$ to zero, we obtain system of algebraic equation. To avoid tediousness, we omit the overdetermined algebraic equations. From the output of Maple, we obtain

FIGURE 1. Graphic of the periodic solutions Φ_1, Φ_2.

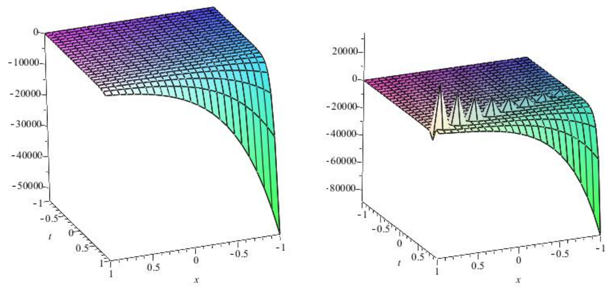
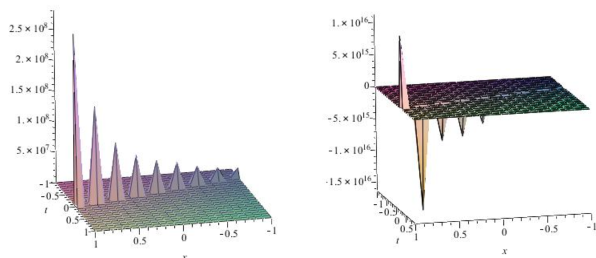


FIGURE 2. Graphic of the periodic solutions Φ_3, Φ_4.



Case 1.

$$a_0 = 0 \quad a_1 = -\frac{2w(-2+k^2)}{p(5k^2+1}, \tag{5.15}$$

and k, w are arbitrary constants.
The solution of Eq. (5.1)" corresponding to (5.15)" is

$$u_1(x,t) = -\frac{w(-2+k^2)}{p(5k^2+1} \left[1 - \tanh(\frac{x}{2} - \frac{c}{2}t)\right] e^{i(kx-wt)}. \tag{5.16}$$

$$u_2(x,t) = -\frac{w(-2+k^2)}{p(5k^2+1} \left[1 - \coth(\frac{x}{2} - \frac{c}{2}t)\right] e^{i(kx-wt)}. \tag{5.17}$$

Case 2.

$$a_0 = -\frac{2w(-2+k^2)}{p(5k^2+1}, \quad a_1 = \frac{2w(-2+k^2)}{p(5k^2+1}, \tag{5.18}$$

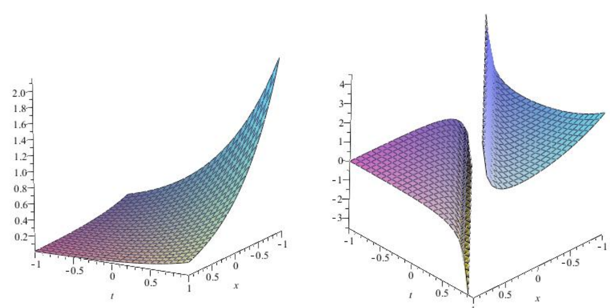
and k, w are arbitrary constants.
The solution of Eq. (5.1)" corresponding to (5.18)" is

$$u_3(x,t) = -\frac{2w(-2+k^2)}{p(5k^2+1} + \frac{w(-2+k^2)}{p(5k^2+1} \left[1 - \tanh(\frac{x}{2} - \frac{c}{2}t)\right] e^{i(kx-wt)}. \tag{5.19}$$

$$u_4(x,t) = -\frac{2w(-2+k^2)}{p(5k^2+1} + \frac{w(-2+k^2)}{p(5k^2+1} \left[1 - \coth(\frac{x}{2} - \frac{c}{2}t)\right] e^{i(kx-wt)}. \tag{5.20}$$

We choose $p = 1$, $c = 1$, $k = 1$, $w = 1$.

FIGURE 3. Graphic of the periodic solutions u_1, u_2.



5.3. **The extended F-expansion method.** We suppose that the solution to ODE (5.4)" can be expressed by

$$u(\xi) = a_0 + a_1 F(\xi) + b_1 F^{-1}(\xi), \tag{5.21}$$

where a_0, a_1, a_2 are constants to be determined. Substituting (5.21)" into Eq. (5.4) , and using (4.2) , the left-hand side of Eq. (5.4)" can be converted into a finite series in $F^p(\xi)(p = -4, \ldots, 4)$. Equating each coefficient of F^p to zero yields a system of algebraic equations. From the output of Maple, we obtain

Case 1.

$$a_1 = \frac{1}{12}\frac{(2bQ+w)b_1}{bR} \quad k = \sqrt{\frac{w}{6b}}, \tag{5.22}$$

a_0, b_1, w is an arbitrary real constant.

Case 2.

$$k=\sqrt{\frac{-7w}{2b}},\quad a_0=-\frac{1}{3}\frac{p}{q},\quad b_1=\frac{2}{p}\sqrt{\frac{bR}{2w}}, \tag{5.23}$$

a_1, w is an arbitrary real constant.

Substituting (5.22)" and (5.23)" into Eq. (5.21)" we obtain the following traveling wave solutions:

$$u_1(x,t)=\left[a_0+\frac{1}{12}\frac{(2bQ+w)b_1}{bR}F(\xi)+b_1F^{-1}(\xi)\right]e^{i(\sqrt{\frac{w}{6b}}x-wt)}, \tag{5.24}$$

$$u_2(x,t)=\left[-\frac{1}{3}\frac{p}{q}+a_1F(\xi)+\frac{2}{p}\sqrt{\frac{bR}{2w}}F^{-1}(\xi)\right]e^{i(\sqrt{\frac{-7w}{2b}}x-wt)}, \tag{5.25}$$

where $\xi=x-ct$.

With the aid of Appendices A and B, from the concentration formulae of the solutions we can obtain the exact solutions expressed by Jacobi elliptic functions for the 1D-MCGL equation.

With the aid of **Appendix A**, selecting

$$F(\xi)=sn(\xi),\quad F^{-1}(\xi)=ns(\xi),\quad P=m^2,\quad Q=-(1+m^2),\quad R=1,$$

and inserting these into (5.24)" and (5.25)" yields

$$\begin{cases} u_3(x,t)=\left[a_0+\frac{1}{12}\frac{(-2b(1+m^2)+w)b_1}{b}sn(\xi)+b_1ns(\xi)\right]e^{i(\sqrt{\frac{w}{6b}}x-wt)}, \\ u_4(x,t)=\left[-\frac{1}{3}\frac{p}{q}+a_1sn(\xi)+\frac{2}{p}\sqrt{\frac{b}{2w}}ns(\xi)\right]e^{i(\sqrt{\frac{-7w}{2b}}x-wt)} \end{cases} \tag{5.26}$$

Inserting

$$F(\xi)=sc(\xi),\quad F^{-1}(\xi)=cs(\xi),\quad P=1-m^2,\quad Q=2-m^2,\quad R=1,$$

into (5.24)" and (5.25)" yields

$$\begin{cases} u_5(x,t)=\left[a_0+\frac{1}{12}\frac{(2b(2-m^2)+w)b_1}{b}sc(\xi)+b_1cs(\xi)\right]e^{i(\sqrt{\frac{w}{6b}}x-wt)}, \\ u_6(x,t)=\left[-\frac{1}{3}\frac{p}{q}+a_1sc(\xi)+\frac{2}{p}\sqrt{\frac{b}{2w}}cs(\xi)\right]e^{i(\sqrt{\frac{-7w}{2b}}x-wt)} \end{cases} \tag{5.27}$$

Inserting

$$F(\xi)=dn(\xi),\quad F^{-1}(\xi)=nd(\xi),\quad P=-1,\quad Q=2-m^2,\quad R=m^2-1,$$

into Eq. (5.24)" and (5.25)" yields

$$\begin{cases} u_7(x,t) = \left[a_0 + \frac{1}{12}\frac{(2b(2-m^2)+w)b_1}{b(m^2-1)}dn(\xi) + b_1 nd(\xi)\right] e^{i(\sqrt{\frac{w}{6b}}x - wt)}, \\ u_8(x,t) = \left[-\frac{1}{3}\frac{p}{q} + a_1 dn(\xi) + \frac{2}{p}\sqrt{\frac{b(m^2-1)}{2w}}nd(\xi)\right] e^{i(\sqrt{\frac{-7w}{2b}}x - wt)} \end{cases} \tag{5.28}$$

Inserting

$$F(\xi) = cn(\xi), \quad F^{-1}(\xi) = nc(\xi), \quad P = -m^2, \quad Q = 2m^2 - 1, \quad R = 1 - m^2,$$

into Eq. (5.24)" and (5.25)" yields

$$\begin{cases} u_9(x,t) = \left[a_0 + \frac{1}{12}\frac{(2b(-1+2m^2)+w)b_1}{b(-m^2+1)}cn(\xi) + b_1 nc(\xi)\right] e^{i(\sqrt{\frac{w}{6b}}x - wt)}, \\ u_{10}(x,t) = \left[-\frac{1}{3}\frac{p}{q} + a_1 cn(\xi) + \frac{2}{p}\sqrt{\frac{b(-m^2+1)}{2w}}nc(\xi)\right] e^{i(\sqrt{\frac{-7w}{2b}}x - wt)} \end{cases} \tag{5.29}$$

If $sn(\xi)$ and $ns(\xi)$ are replaced by $cd(\xi)$ and $dc(\xi)$ respectively, we have

$$\begin{cases} u_{11}(x,t) = \left[a_0 + \frac{1}{12}\frac{(-2b(1+m^2)+w)b_1}{b}cd(\xi) + b_1 dc(\xi)\right] e^{i(\sqrt{\frac{w}{6b}}x - wt)}, \\ u_{12}(x,t) = \left[-\frac{1}{3}\frac{p}{q} + a_1 cd(\xi) + \frac{2}{p}\sqrt{\frac{b}{2w}}dc(\xi)\right] e^{i(\sqrt{\frac{-7w}{2b}}x - wt)} \end{cases} \tag{5.30}$$

where $\xi = x - ct$.

In the limit case when $m \to 1$, some solitary wave solutions can be obtained, for example:

$$u_3, u_4 \to \begin{cases} u_{13}(x,t) = \left[a_0 + \frac{1}{12}\frac{(-4b+w)b_1}{b}\tanh(\xi) + b_1\coth(\xi)\right] e^{i(\sqrt{\frac{w}{6b}}x - wt)}, \\ u_{14}(x,t) = \left[-\frac{1}{3}\frac{p}{q} + a_1\tanh(\xi) + \frac{2}{p}\sqrt{\frac{b}{2w}}\coth(\xi)\right] e^{i(\sqrt{\frac{-7w}{2b}}x - wt)} \end{cases} \tag{5.31}$$

$$u_5, u_6 \to \begin{cases} u_{15}(x,t) = \left[a_0 + \frac{1}{12}\frac{(2b+w)b_1}{b}\sinh(\xi) + b_1 csch(\xi)\right] e^{i(\sqrt{\frac{w}{6b}}x - wt)}, \\ u_{16}(x,t) = \left[-\frac{1}{3}\frac{p}{q} + a_1\sinh(\xi) + \frac{2}{p}\sqrt{\frac{b}{2w}}csch(\xi)\right] e^{i(\sqrt{\frac{-7w}{2b}}x - wt)} \end{cases} \tag{5.32}$$

$$u_7, u_8 \to \begin{cases} u_{17}(x,t) = [a_0 + b_1\cosh(\xi)]\, e^{i(\sqrt{\frac{w}{6b}}x - wt)}, \\ u_{18}(x,t) = \left[-\frac{1}{3}\frac{p}{q} + a_1 sech(\xi)\right] e^{i(\sqrt{\frac{-7w}{2b}}x - wt)} \end{cases} \tag{5.33}$$

where $\xi = x - ct$.

In the limit case when $m \to 0$, some trigonometric function solutions (single periodic wave solutions) can be obtained, for example:

$$u_3, u_4 \to \begin{cases} u_{19}(x,t) = \left[a_0 + \dfrac{1}{12}\dfrac{(-2b+w)b_1}{b}\sin(\xi) + b_1\csc(\xi)\right] e^{i(\sqrt{\frac{w}{6b}}x - wt)}, \\ u_{20}(x,t) = \left[-\dfrac{1}{3}\dfrac{p}{q} + a_1\sin(\xi) + \dfrac{2}{p}\sqrt{\dfrac{b}{2w}}\csc(\xi)\right] e^{i(\sqrt{\frac{-7w}{2b}}x - wt)} \end{cases} \tag{5.34}$$

$$u_5, u_6 \to \begin{cases} u_{21}(x,t) = \left[a_0 + \dfrac{1}{12}\dfrac{(4b+w)b_1}{b}\tan(\xi) + b_1\cot(\xi)\right] e^{i(\sqrt{\frac{w}{6b}}x - wt)}, \\ u_{22}(x,t) = \left[-\dfrac{1}{3}\dfrac{p}{q} + a_1\tan(\xi) + \dfrac{2}{p}\sqrt{\dfrac{b}{2w}}\cot(\xi)\right] e^{i(\sqrt{\frac{-7w}{2b}}x - wt)} \end{cases} \tag{5.35}$$

$$u_9, u_{10} \to \begin{cases} u_{23}(x,t) = \left[a_0 + \dfrac{1}{12}\dfrac{(-2b+w)b_1}{b}\cos(\xi) + b_1\sec(\xi)\right] e^{i(\sqrt{\frac{w}{6b}}x - wt)}, \\ u_{24}(x,t) = \left[-\dfrac{1}{3}\dfrac{p}{q} + a_1\cos(\xi) + \dfrac{2}{p}\sqrt{\dfrac{b}{2w}}\sec(\xi)\right] e^{i(\sqrt{\frac{-7w}{2b}}x - wt)} \end{cases} \tag{5.36}$$

where $\xi = x - ct$.

We will provide the figures of u_{13}, u_{15}, u_{18}, u_{19}, u_{22}, u_{24} for direct-viewing analysis. We choose $a_0 = 0$, $b_1 = \dfrac{1}{2}$, $a_1 = w = 1$, $p = q = 1$.

FIGURE 4. Graphic of the periodic solutions u_{13}, u_{15}.

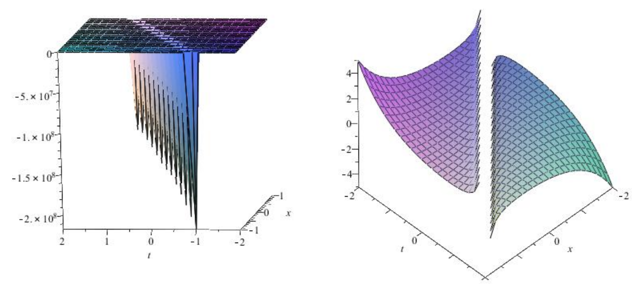


FIGURE 5. Graphic of the periodic solutions u_{18}, u_{19}.

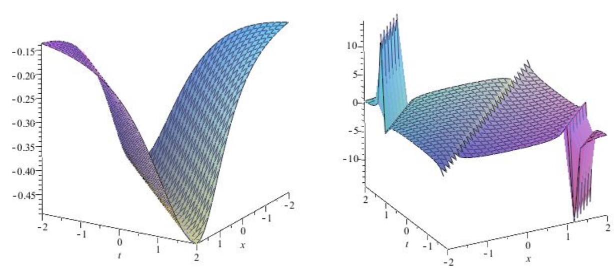


FIGURE 6. Graphic of the periodic solutions u_{22}, u_{24}.

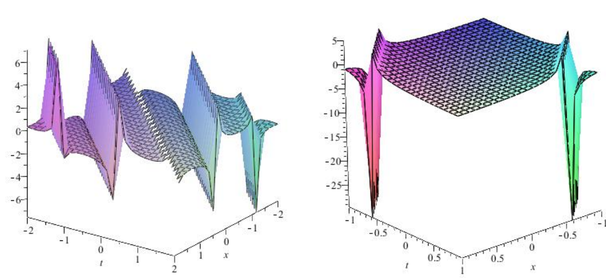


Table 1. Relations between values of (P, Q, R) and corresponding $F(\xi)$ in ODE $(F^{'})^2 = PF^4 + QF^2 + R$

P	Q	R	F
m^2	$-(1+m^2)$	1	$sn(\xi), cd(\xi)$
$-m^2$	$2m^2-1$	$1-m^2$	$cn(\xi)$
-1	$2-m^2$	$-1+m^2$	$dn(\xi)$
1	$-(1+m^2)$	m^2	$ns(\xi), (sn(\xi)^{-1}$
$1-m^2$	$2m^2-1$	$-m^2$	$nc(\xi), cn(\xi)^{-1}$
m^2-1	$2-m^2$	-1	$nd(\xi)$
$1-m^2$	$2-m^2$	1	$sc(\xi) = \frac{sn(\xi)}{cn(\xi)}$
$-m^2(1-m^2)$	$-2+2m^2$	14	$sd(\xi) = \frac{sn(\xi)}{dn(\xi)}$
1	$2-m^2$	$1-m^2$	$cs(\xi) = \frac{cn(\xi)}{sn(\xi)}$
1	$2m^2-1$	$-m^2(1-m^2)$	$ds(\xi) = \frac{dn(\xi)}{sn(\xi)}$

Appendix A

Appendix B

Jacobi elliptic functions with modulus $0 < m < 1$.

$$dn^2(\xi) = -m^2 sn^2(\xi) + 1,\ dn^2(\xi) = m^2 cn^2(\xi) + (1-m^2),$$
$$dc^2(\xi) = (1-m^2)ns^2(\xi) + m^2,\ ns^2(\xi) = cs^2(\xi) + 1$$
$$cd^2(\xi) = -\frac{m^2-1}{m^2} nd^2(\xi) + \frac{1}{m},\ nd^2(\xi) = m^2 sd^2(\xi) + 1,$$
$$c^2(\xi) = (1-m^2)nc^2(\xi) + m^2,\ dc^2(\xi) = (1-m^2)sc^2(\xi) + 1,$$
$$cn^2(\xi) = -sn^2(\xi) + 1,\ cd^2(\xi) = (m^2-1)sd^2(\xi) + 1,\ nc^2(\xi) = sc^2(\xi) + 1.$$

Appendix C

Table 2. Jacobi elliptic functions degenerate as hyperbolic functions when $m \to 1$

$sn(\xi)$	$\tanh(\xi)$	$dc(\xi)$	1
$cn(\xi)$	$sech(\xi)$	$ds(\xi)$	$csch(\xi)$
$dn(\xi)$	$sech(\xi)$	$cs(\xi)$	$csch(\xi)$
$sc(\xi)$	$\sinh(\xi)$	$nd(\xi)$	$\cosh(\xi)$
$sd(\xi)$	$\sinh(\xi)$	$nc(\xi)$	$\cosh(\xi)$
$cd(\xi)$	1	$ns(\xi)$	$\coth(\xi)$

TABLE 3. Jacobi elliptic functions degenerate as trigonometric functions when $m \to 0$

$sn(\xi)$	$\sin(\xi)$	$dc(\xi)$	$\sec(\xi)$
$cn(\xi)$	$\cos(\xi)$	$ds(\xi)$	$\csc(\xi)$
$dn(\xi)$	1	$cs(\xi)$	$\cot(\xi)$
$sc(\xi)$	$\tan(\xi)$	$nd(\xi)$	1
$sd(\xi)$	$\sin(\xi)$	$nc(\xi)$	$\sec(\xi)$
$cd(\xi)$	$\cos(\xi)$	$ns(\xi)$	$\csc(\xi)$

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6. Conclusion

In this work, we have obtain the exact traveling wave solutions in terms of hyperbolic, trigonometric, and rational functions for the 1D-MCGL equation by using the $(\frac{G'}{G})$-expansion method, extended F-expansion method and homogeneous balance method. By introducing appropriate transformations and using extended F-expansion method, we have been able to obtain in a unified way with the aid of symbolic computation system Maple, a series of solutions including single and boned nondegenerative Jacobi elliptic function solutions and their degenerative solutions to a some class of nonlinear evolution equations of special interest in mathematical physics. It seems that the extended F-expansion is more effective than the F-expansion and jacobi elliptic function expansion. On comparing this method with the other methods, we see that the homogeneous balance method is much more simpler than these methods. Also we deduce that the homogeneous balance method is direct, effective and can be applied to many other nonlinear evolution equations. In future, this methods provides a powerful mathematical tool to obtain more general exact solutions of a great many nonlinear PDEs in mathematical physics.

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