*Computational Methods for Differential Equations*
http://cmde.tabrizu.ac.ir
**Vol. 2, No. 4, 2014, pp. 227-242**

# Topological soliton solutions of the some nonlinear partial differential equations

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**Abstract** In this paper, we obtained the 1-soliton solutions of the symmetric regularized long wave (SRLW) equation and the (3+1)-dimensional shallow water wave equations. Solitary wave ansatz method is used to carry out the integration of the equations and obtain topological soliton solutions The physical parameters in the soliton solutions are obtained as functions of the dependent coefficients. Note that, this method is always useful and desirable to construct exact solutions especially soliton-type envelope for the understanding of most nonlinear physical phenomena.

**Keywords.** Exact solution; topological soliton solution; the (3+1)-dimensional shallow water wave equation; the symmetric regularized long wave (SRLW) equation.

**2010 Mathematics Subject Classification.** 26A33, 34A08, 35R11, 83C15.

## 1. Introduction

Nonlinear problems are of interest to engineers and mathematicians because most physical systems are naturally nonlinear in nature [2]. Nonlinear partial differential equations (NPDEs) are difficult to solve and give rise to interesting phenomena such as fluid mechanics, mathematical biology, diffusion process, chemical kinematics, chemical physics, plasma physics, optical fibers, neural physics, solid state physics and many other fields. It is well known that wave phenomena of optical fibers are modeled by dark shaped $\tanh^p$ solutions or by bright shaped $\mathrm{sech}^p$ solutions. There is plainly a tendency in the modern nonlinear science community to obtain exact solutions for nonlinear equations.

In recent years, new exact solutions may help to find new phenomena. A variety of powerfull methods, such as the tanh-sech method [25, 26, 40], extended tanh method [12, 13, 41], sine-cosine method [5, 42], homogeneous balance method [14, 38], first integral method [16, 34], Jacobi elliptic function method [15, 23], $\left(\frac{G'}{G}\right)$-expansion

Received: 9 October 2014 ; Accepted: 10 February 2015.

method [6,39], exp-function method [20,27] and F-expansion method [1,47] were used to develop nonlinear dispersive and dissipative problems.

Solitons are caused by a cancellation of nonlinear and dispersive effects in the medium. Solitons arise as the solutions of a widespread class of weakly nonlinear dispersive partial differential equations describing physical systems. The soliton phenomenon was first described by John Scott Russell. Solitons in a fiber optic system are described by the Manakov equations. Solitons are ubiquitous in nature, appearing in diverse systems such as shallow water waves, DNA excitations, matter waves in Bose–Einstein condensates, and ultrashort pulses in nonlinear optics [22, 24].Two different types of envelope solitons, bright and dark, can propagate in nonlinear dispersive media. Compared with the bright soliton which is a pulse on a zero-intensity background, the dark soliton appears as an intensity dip in an infinitely extended constant background [32]. Topological solitons are also known as dark solitons in the context of nonlinear optics media. Much experimentation has been done using solitons in fiber optics applications. It is known that topological solitons are more stable in presence of noise and spreads more slowly in presence of loss, in the optical communication systems, as compared to bright solitons [25-34].

This study is purposed topological soliton solutions of constant-coefficient nonlinear wave equations, so we will consider the following three constant-coefficient nonlinear wave equations.

## 2. Symmetric Regularized Long Wave (SRLW) Equation

We consider nonlinear symmetric regularized long wave (SRLW) equation is given by [3]

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0, \tag{2.1}$$

which arises in several physical applications including ion sound waves in plasma [33]. Eq.(2.1) is explicitly symmetric in the $x$ and $t$ derivatives and is very similar to the regularized long wave equation that describes shallow water waves and plasma drift waves [21]. Also, this equation was shown to describe weekly nonlinear ion acoustic and space-charge waves, and the real-valued $u(x,t)$ corresponds to the dimensionless fluid velocity with a decay condition. In 1987, 1992 Guo [18,19] and in 1989 Zheng et al [48] found and analysed the global and numerical solutions of the SRLW equation. Recently, Bekir [7] obtained some the solitary and periodic wave solutions by using $\left(\frac{G'}{G}\right)$-expansion method. Xu [46] used the exp-function method with the aid of Maple to obtain some periodic solutions and soliton solutions. Qawasmeh [30] has applied sine-cosine function method to obtain exact solutions of SRLW equation. In order to start off with the solution hypothesis, the following ansatz is assumed,

$$u(x,t) = \lambda \tanh^p \tau, \tag{2.2}$$

and

$$\tau = B(x - vt), \tag{2.3}$$

where $\lambda$ and $B$ are the free parameters and $v$ is the velocity of the soliton, respectively. These will be determined. The exponent $p$ is also unknown.

From (2.2) it is possible to obtain

$$u_{tt} = pv^2\lambda B^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}, \tag{2.4}$$

$$u_{xx} = p\lambda B^2\left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\right\}, \tag{2.5}$$

$$uu_{xt} = -\lambda^2 pvB^2\left\{(p-1)\tanh^{2p-2}\tau - 2p\tanh^{2p}\tau + (p+1)\tanh^{2p+2}\tau\right\}, \tag{2.6}$$

$$u_x u_t = -p^2\lambda^2 B^2 v\{\tanh^{2p+2}\tau - 2\tanh^{2p}\tau + \tanh^{2p-2}\tau\}, \tag{2.7}$$

$$u_{xxtt} = p\lambda B^4 v^2 \left\{\begin{array}{c} (p-1)(p-2)(p-3)\tanh^{p-4}\tau \\ -4(p-1)(p^2-2p+2)\tanh^{p-2}\tau \\ +2p(p^2+5)\tanh^p\tau \\ -4(p+1)(p^2+2p+2)\tanh^{p+2}\tau \\ +(p+1)(p+2)(p+3)\tanh^{p+4}\tau \end{array}\right\}, \tag{2.8}$$

where $\tau = B(x - vt)$. Substituting (2.4)–(2.8) into (2.1), gives

$$\begin{aligned}
&pv^2\lambda B^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\}\\
&+p\lambda B^2\left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\right\}\\
&-\lambda^2 pvB^2\left\{(p-1)\tanh^{2p-2}\tau - 2p\tanh^{2p}\tau + (p+1)\tanh^{2p+2}\tau\right\}\\
&-p^2\lambda^2B^2v\{\tanh^{2p+2}\tau - 2\tanh^{2p}\tau + \tanh^{2p-2}\tau\}\\
&+p(p-1)(p-2)(p-3)\lambda B^4v^2\tanh^{p-4}\tau\\
&-4\lambda B^4v^2p(p-1)(p^2-2p+2)\tanh^{p-2}\tau\\
&+p\lambda B^4v^2\{2p(p^2+5)\tanh^{p}\tau - 4(p+1)(p^2+2p+2)\tanh^{p+2}\tau\\
&+(p+1)(p+2)(p+3)\tanh^{p+4}\tau\} = 0
\end{aligned} \tag{2.9}$$

Now, from (2.9) equating the exponents of $\tanh^{2p+2}\tau$ and $\tanh^{p+4}\tau$ gives,

$$2p + 2 = p + 4, \tag{2.10}$$

so that

$$p = 2. \tag{2.11}$$

It needs to be noted that the same value of $p$ is yielded when the exponents pairs $2p$ and $p+2$, $2p-2$ and $p$ are equated with each other. Hence setting their respective coefficients to zero yields a set of algebraic equations:

$$pv^2\lambda B^2(p-1) + p\lambda B^2(p-1) - 4\lambda B^4v^2p(p-1)(p^2-2p+2) = 0, \tag{2.12}$$

$$-\lambda^2pvB^2(p+1) - p^2\lambda^2B^2v + p(p+1)(p+2)(p+3)\lambda B^4v^2 = 0, \tag{2.13}$$

$$\begin{aligned}
p(p+1)v^2\lambda B^2 + p(p+1)\lambda B^2 - 4p(p+1)(p^2+2p+2)\lambda B^4v^2&\\
+2p^2\lambda^2vB^2 + 2p^2\lambda^2B^2v = 0.&
\end{aligned} \tag{2.14}$$

If we put $p = 2$ in (2.12)-(2.14) the system reduces to;

$$2\lambda B^2 + 2v^2\lambda B^2 - 16\lambda B^4v^2 = 0, \tag{2.15}$$

$$120\lambda B^4v^2 - 10\lambda^2B^2v = 0, \tag{2.16}$$

$$6v^2\lambda B^2 + 16\lambda^2vB^2 + 6\lambda B^2 - 240\lambda B^4v^2 = 0. \tag{2.17}$$

Solving the above equation (2.15), it yields

$$B = \pm\frac{\sqrt{2+2v^2}}{4v}, \tag{2.18}$$

Solving Eq. (2.16) by using (2.18) we get

$$\lambda = \frac{3+3v^2}{2v}. \tag{2.19}$$

Hence, finally, the topological soliton solution to (2.1) is given by

$$u(x,t) = \frac{3+3v^2}{2v}\tanh^2\left(\pm\frac{\sqrt{2+2v^2}}{4v}(x-vt)\right), \tag{2.20}$$

which exist provided that $v \neq 0$.

**Remark 1:** Comparing our results with Qawasmeh's and Xu's [30, 46] results, it can be seen that the results are new.

## 3. The Shallow Water Wave Equations

In [10, 43] the (2+1)- dimensional shallow water wave equations

$$u_{yt} + u_{xxxy} - 3u_{xx}u_y - 3u_xu_{xy} = 0, \tag{3.1}$$

and

$$u_{xt} + u_{xxxy} - 2u_{xx}u_y - 4u_xu_{xy} = 0, \tag{3.2}$$

were studied. Both equations reduce to the potential KdV equation for $y = x$. The difference between the two models (3.1) and (3.2) is that $x$ replaces $y$ in the term $u_{yt}$ and in the coefficients of the other terms.

In [10, 43, 44] the (3+1)- dimensional shallow water wave equations

$$u_{yzt} + u_{xxxyz} - 6u_xu_{xyz} - 6u_{xz}u_{xy} = 0, \tag{3.3}$$

and

$$u_{xzt} + u_{xxxyz} - 2(u_{xx}u_{yz} + u_yu_{xxz}) - 4(u_xu_{xyz} + u_{xz}u_{xy}) = 0, \tag{3.4}$$

were also studied. Both equations reduce to the potential KdV equation for $z = y = x$. The difference between the first terms of the two models is that $x$ replaces $y$ in the term $u_{yzt}$.

The studies about Eqs. (3.1)-(3.4) in [10, 43, 44], and some of the references therein were to show that each model is completely integrable and each one gives rice to multiple soliton solutions.

In this work, we will introduce four extended shallow water wave equations in (2+1) and (3+1) dimensions that were extended by Wazwaz [45]

$$u_{yt} + u_{xxxy} - 3u_{xx}u_y - 3u_x u_{xy} + \alpha u_{xy} = 0, \tag{3.5}$$

$$u_{xt} + u_{xxxy} - 2u_{xx}u_y - 4u_x u_{xy} + \alpha u_{xy} = 0, \tag{3.6}$$

$$u_{yzt} + u_{xxxyz} - 6u_x u_{xyz} - 6u_{xz}u_{xy} + \alpha u_{xyz} = 0, \tag{3.7}$$

and

$$u_{xzt} + u_{xxxyz} - 2(u_{xx}u_{yz} + u_y u_{xxz}) - 4(u_x u_{xyz} + u_{xz}u_{xy}) + \alpha u_{xyz} = 0. \tag{3.8}$$

The extended equations are established by adding the derivative of $u(x,t)$ with respect to the space variables $x$ and $y$ for the first two equations (3.1) and (3.2), and with respect to the space variables $x$, $y$ and $z$ for the last two equations (3.3) and (3.4). In [4], Bekir and Aksoy studied the exact solutions which include hyperbolic functions, trigonometric and rational functions for Eqs. (3.5) and (3.6) by the $\left(\frac{G'}{G}\right)$-expansion method.

Now, we will study the extended (3+1)-dimensional shallow water wave Eq. (3.7) and topological soliton solution of this equation will be obtained. The following ansatz is assumed,

$$u(x,y,z,t) = \lambda \tanh^p \tau, \tag{3.9}$$

and choosing now a suitable solitary wave ansatz with (3+1) dependent variables of the form

$$\tau = ax + by + cz - vt, \tag{3.10}$$

where $\lambda$,$a$, $b$ and $c$ are unknown free parameters and $v$ is the velocity of the soliton, that will be determined. The exponent $p$ is also unknown.

From Eqs. (3.9) and (3.10), we have:

$$\begin{aligned} u_{yzt} =& -p(p-1)(p-2)\lambda vbc \tanh^{p-3}\tau \\ &+ \left\{p(p-1)(p-2) + 2p^3\right\} \lambda vbc \tanh^{p-1}\tau \\ &- \left\{p(p+1)(p+2) + 2p^3\right\} \lambda vbc \tanh^{p+1}\tau \\ &+ p(p+1)(p+2)\lambda vbc \tanh^{p+3}\tau, \end{aligned} \tag{3.11}$$

$$
\begin{aligned}
u_{xxxyz} = \; & p(p-1)(p-2)(p-3)(p-4)\lambda a^3 bc\tanh^{p-5}\tau \\
& -p(p+1)(p+2)(p+3)(p+4)\lambda a^3 bc\tanh^{p+5}\tau \\
& -p(p-1)(p-2)\left[2p^2+2(p-2)^2+(p-3)(p-4)\right]\lambda a^3 bc\tanh^{p-3}\tau \\
& +p(p+1)(p+2)\left[2p^2+2(p+2)^2+(p+3)(p+4)\right]\lambda a^3 bc\tanh^{p+3}\tau \\
& +\left\{\begin{array}{c} 2p(p-1)(p-2)\left[p^2+(p-2)^2\right] \\ +4p^5+p^2(p-1)^2(p-2)+p^2(p+1)^2(p+2) \end{array}\right\}\lambda a^3 bc\tanh^{p-1}\tau \\
& -\left\{\begin{array}{c} 2p(p+1)(p+2)\left[p^2+(p+2)^2\right] \\ +4p^5+p^2(p-1)^2(p-2)+p^2(p+1)^2(p+2) \end{array}\right\}\lambda a^3 bc\tanh^{p+1}\tau
\end{aligned} \tag{3.12}
$$

$$
\begin{aligned}
u_x u_{xyz} = & p^2(p-1)(p-2)\lambda^2 a^2 bc\tanh^{2p-4}\tau \\
& -\left\{2p^2(p-1)(p-2)+2p^4\right\}\lambda^2 a^2 bc\tanh^{2p-2}\tau \\
& +\left\{2p^2(p^2+2)+4p^4\right\}\lambda^2 a^2 bc\tanh^{2p}\tau \\
& -\left\{2p^2(p+1)(p+2)+2p^4\right\}\lambda^2 a^2 bc\tanh^{2p+2}\tau \\
& +p^2(p+1)(p+2)\lambda^2 a^2 bc\tanh^{2p+4}\tau,
\end{aligned} \tag{3.13}
$$

$$
\begin{aligned}
u_{xz}u_{xy} = & p^2(p-1)^2\lambda^2 a^2 bc\tanh^{2p-4}\tau - p^2(4p^2-4p)\lambda^2 a^2 bc\tanh^{2p-2}\tau \\
& +p^2(6p^2-2)\lambda^2 a^2 bc\tanh^{2p}\tau - p^2(4p^2+4p)\lambda^2 a^2 bc\tanh^{2p+2}\tau \\
& +p^2(p+1)^2\lambda^2 a^2 bc\tanh^{2p+4}\tau,
\end{aligned} \tag{3.14}
$$

$$
\begin{aligned}
u_{xyz} = & p(p-1)(p-2)\lambda abc\tanh^{p-3}\tau \\
& -\left\{p(p-1)(p-2)+2p^3\right\}\lambda abc\tanh^{p-1}\tau \\
& +\left\{p(p+1)(p+2)+2p^3\right\}\lambda abc\tanh^{p+1}\tau \\
& -p(p+1)(p+2)\lambda abc\tanh^{p+3}\tau,
\end{aligned} \tag{3.15}
$$

where $\tau = ax+by+cz-vt$. Substituting Eqs. (3.9)-(3.15) into Eq.(3.7), we obtain

$$
\begin{aligned}
&-p(p-1)(p-2)\lambda vbc\tanh^{p-3}\tau + \left\{p(p-1)(p-2)+2p^3\right\}\lambda vbc\tanh^{p-1}\tau \\
&-\left\{p(p+1)(p+2)+2p^3\right\}\lambda vbc\tanh^{p+1}\tau + p(p+1)(p+2)\lambda vbc\tanh^{p+3}\tau \\
&+p(p-1)(p-2)(p-3)(p-4)\lambda a^3bc\tanh^{p-5}\tau \\
&-p(p+1)(p+2)(p+3)(p+4)\lambda a^3bc\tanh^{p+5}\tau \\
&-p(p-1)(p-2)\left[2p^2+2(p-2)^2+(p-3)(p-4)\right]\lambda a^3bc\tanh^{p-3}\tau \\
&+p(p+1)(p+2)\left[2p^2+2(p+2)^2+(p+3)(p+4)\right]\lambda a^3bc\tanh^{p+3}\tau \\
&+\left\{\begin{array}{c} 2p(p-1)(p-2)\left[p^2+(p-2)^2\right]+4p^5 \\ +p^2(p-1)^2(p-2)+p^2(p+1)^2(p+2) \end{array}\right\}\lambda a^3bc\tanh^{p-1}\tau \\
&-\left\{\begin{array}{c} 2p(p+1)(p+2)\left[p^2+(p+2)^2\right]+4p^5 \\ +p^2(p-1)^2(p-2)+p^2(p+1)^2(p+2) \end{array}\right\}\lambda a^3bc\tanh^{p+1}\tau \\
&-6p^2(p-1)(p-2)\lambda^2a^2bc\tanh^{2p-4}\tau \\
&+6\left\{2p^2(p-1)(p-2)+2p^4\right\}\lambda^2a^2bc\tanh^{2p-2}\tau \\
&-6\left\{2p^2(p^2+2)+4p^4\right\}\lambda^2a^2bc\tanh^{2p}\tau \\
&+6\left\{2p^2(p+1)(p+2)+2p^4\right\}\lambda^2a^2bc\tanh^{2p+2}\tau \\
&-6p^2(p+1)(p+2)\lambda^2a^2bc\tanh^{2p+4}\tau - 6p^2(p-1)^2\lambda^2a^2bc\tanh^{2p-4}\tau \\
&+6p^2(4p^2-4p)\lambda^2a^2bc\tanh^{2p-2}\tau - 6p^2(6p^2-2)\lambda^2a^2bc\tanh^{2p}\tau \\
&+6p^2(4p^2+4p)\lambda^2a^2bc\tanh^{2p+2}\tau - 6p^2(p+1)^2\lambda^2a^2bc\tanh^{2p+4}\tau \\
&+\alpha p(p-1)(p-2)\lambda abc\tanh^{p-3}\tau - \alpha\left\{p(p-1)(p-2)+2p^3\right\}\lambda abc\tanh^{p-1}\tau
\end{aligned}
$$

$$
+\alpha\left\{p(p+1)(p+2)+2p^3\right\}\lambda abc\tanh^{p+1}\tau - \alpha p(p+1)(p+2)\lambda abc\tanh^{p+3}\tau, \tag{3.16}
$$

Thus, from matching the exponents of $\tanh^{2p+4}\tau$ and $\tanh^{p+5}\tau$ terms in Eq (3.16), we obtain

$$2p+4 = p+5, \tag{3.17}$$

which yields

$$p = 1. \tag{3.18}$$

It needs to be noted that the same value of $p$ is yielded when the exponents pairs $2p+2$ and $p+3$, $2p$ and $p+1$, $2p-2$ and $p-1$, $2p-4$ and $p-3$ are equated with each other. Thus (47) has the linearly independent functions $\tanh^{p+j}\tau$ for $j=\pm1,\ \pm3$ and $\tanh^{2p+k}\tau$ for $k=0,\ \pm2,\ \pm4$.Now, $\tanh^{p-5}\tau$ is a stand alone linearly independent function. Therefore setting its coefficients to zero also yields the same value of $p$ as in (3.18) Hence setting their respective coefficients to zero which gives the following set of equations:

$$-p(p+1)(p+2)(p+3)(p+4)\lambda a^3bc-6p^2(p+1)^2\lambda^2a^2bc$$

$$-6p^2(p+1)(p+2)\lambda^2a^2bc=0, \tag{3.19}$$

$$\begin{aligned}&p(p+1)(p+2)\lambda vbc\\&+p(p+1)(p+2)\left[2p^2+2(p+2)^2+(p+3)(p+4)\right]\lambda a^3bc\\&+6\left\{2p^2(p+1)(p+2)+2p^4\right\}\lambda^2a^2bc+6p^2(4p^2+4p)\lambda^2a^2bc\\&-\alpha p(p+1)(p+2)\lambda abc=0\end{aligned} \tag{3.20}$$

$$\begin{aligned}&-\left\{p(p+1)(p+2)+2p^3\right\}\lambda vbc\\&-\left\{2p(p+1)(p+2)\left[p^2+(p+2)^2\right]+4p^5+p^2(p-1)^2(p-2)+p^2(p+1)^2(p+2)\right\}\lambda a^3bc\\&-6\left\{2p^2(p^2+2)+4p^4\right\}\lambda^2a^2bc-6p^2(6p^2-2)\lambda^2a^2bc+\alpha\left\{p(p+1)(p+2)+2p^3\right\}\lambda abc\end{aligned}$$

$$=0, \tag{3.21}$$

$$\begin{aligned}&\left\{p(p-1)(p-2)+2p^3\right\}\lambda vbc\\&+\left\{\begin{array}{c}2p(p-1)(p-2)\left[p^2+(p-2)^2\right]+4p^5\\+p^2(p-1)^2(p-2)+p^2(p+1)^2(p+2)\end{array}\right\}\lambda a^3bc\\&+6\left\{2p^2(p-1)(p-2)+2p^4\right\}\lambda^2a^2bc\\&+6p^2(4p^2-4p)\lambda^2a^2bc-\alpha\left\{p(p-1)(p-2)+2p^3\right\}\lambda abc=0\end{aligned} \tag{3.22}$$

Solving the above equation (3.19) by using (3.18), we get

$$\lambda = -2a. \tag{3.23}$$

Lastly, solving Eq. (3.20) by using (3.18) and (3.23) we obtain

$$v = a(4a^2 + \alpha). \tag{3.24}$$

Hence, we can determine the topological (dark) 1-soliton solution for the extended shallow water wave equation

$$u(x, y, z, t) = -2a \tanh(ax + by + cz - a(4a^2 + \alpha)t). \tag{3.25}$$

Now, we present the extended (3+1)-dimensional shallow water wave Eq. (3.8) and topological soliton solution of this equation will be obtained. The following ansatz is assumed,

$$u(x, y, z, t) = \lambda \tanh^p \tau, \tag{3.26}$$

and choosing now a suitable solitary wave ansatz with (3+1) dependent variables of the form

$$\tau = ax + by + cz - vt, \tag{3.27}$$

where $\lambda$,$a$, $b$ and $c$ are unknown free parameters and $v$ is the velocity of the soliton, that will be determined.

From Eqs. (3.26) and (3.27), we have:

$$\begin{aligned} u_{xzt} =& -p(p-1)(p-2)\lambda vac \tanh^{p-3}\tau \\ &+ \left\{p(p-1)(p-2) + 2p^3\right\} \lambda vac \tanh^{p-1}\tau \\ &- \left\{p(p+1)(p+2) + 2p^3\right\} \lambda vac \tanh^{p+1}\tau \\ &+ p(p+1)(p+2)\lambda vac \tanh^{p+3}\tau, \end{aligned} \tag{3.28}$$

$$\begin{aligned} u_{xx}u_{yz} =& p^2(p-1)^2\lambda^2a^2bc \tanh^{2p-4}\tau - p^2(4p^2-4p)\lambda^2a^2bc \tanh^{2p-2}\tau \\ &+ p^2(6p^2-2)\lambda^2a^2bc \tanh^{2p}\tau - p^2(4p^2+4p)\lambda^2a^2bc \tanh^{2p+2}\tau \\ &+ p^2(p+1)^2\lambda^2a^2bc \tanh^{2p+4}\tau, \end{aligned} \tag{3.29}$$

$$
\begin{aligned}
u_y u_{xxz} =& p^2(p-1)(p-2)\lambda^2 a^2 bc \tanh^{2p-4}\tau \\
&- \left\{2p^2(p-1)(p-2)+2p^4\right\}\lambda^2 a^2 bc \tanh^{2p-2}\tau \\
&+ (6p^4+4p^2)\lambda^2 a^2 bc \tanh^{2p}\tau \\
&- \left\{2p^2(p+1)(p+2)+2p^4\right\}\lambda^2 a^2 bc \tanh^{2p+2}\tau \\
&+ p^2(p+1)(p+2)\lambda^2 a^2 bc \tanh^{2p+4}\tau,
\end{aligned} \tag{3.30}
$$

where $\tau = ax+by+cz-vt$. Substituting Eqs. (3.12)-(3.15) and (3.28)-(3.30) into Eq.(3.8), we obtain

$$
\begin{aligned}
&-p(p-1)(p-2)\lambda vac\tanh^{p-3}\tau + \left\{p(p-1)(p-2)+2p^3\right\}\lambda vac\tanh^{p-1}\tau \\
&-\left\{p(p+1)(p+2)+2p^3\right\}\lambda vac\tanh^{p+1}\tau + p(p+1)(p+2)\lambda vac\tanh^{p+3}\tau \\
&+p(p-1)(p-2)(p-3)(p-4)\lambda a^3 bc\tanh^{p-5}\tau \\
&-p(p+1)(p+2)(p+3)(p+4)\lambda a^3 bc\tanh^{p+5}\tau \\
&-p(p-1)(p-2)\left[2p^2+2(p-2)^2+(p-3)(p-4)\right]\lambda a^3 bc\tanh^{p-3}\tau \\
&+p(p+1)(p+2)\left[2p^2+2(p+2)^2+(p+3)(p+4)\right]\lambda a^3 bc\tanh^{p+3}\tau \\
&+\left\{\begin{array}{c} 2p(p-1)(p-2)\left[p^2+(p-2)^2\right]+4p^5 \\ +p^2(p-1)^2(p-2)+p^2(p+1)^2(p+2) \end{array}\right\}\lambda a^3 bc\tanh^{p-1}\tau \\
&-\left\{\begin{array}{c} 2p(p+1)(p+2)\left[p^2+(p+2)^2\right]+4p^5 \\ +p^2(p-1)^2(p-2)+p^2(p+1)^2(p+2) \end{array}\right\}\lambda a^3 bc\tanh^{p+1}\tau \\
&-2p^2(p-1)^2\lambda^2 a^2 bc\tanh^{2p-4}\tau + 2p^2(4p^2-4p)\lambda^2 a^2 bc\tanh^{2p-2}\tau \\
&-2p^2(6p^2-2)\lambda^2 a^2 bc\tanh^{2p}\tau + 2p^2(4p^2+4p)\lambda^2 a^2 bc\tanh^{2p+2}\tau \\
&-2p^2(p+1)^2\lambda^2 a^2 bc\tanh^{2p+4}\tau - 2p^2(p-1)(p-2)\lambda^2 a^2 bc\tanh^{2p-4}\tau \\
&+2\left\{2p^2(p-1)(p-2)+2p^4\right\}\lambda^2 a^2 bc\tanh^{2p-2}\tau - 2(6p^4+4p^2)\lambda^2 a^2 bc\tanh^{2p}\tau \\
&+2\left\{2p^2(p+1)(p+2)+2p^4\right\}\lambda^2 a^2 bc\tanh^{2p+2}\tau - 2p^2(p+1)(p+2)\lambda^2 a^2 bc\tanh^{2p+4}\tau \\
&-4p^2(p-1)(p-2)\lambda^2 a^2 bc\tanh^{2p-4}\tau + 4\left\{2p^2(p-1)(p-2)+2p^4\right\}\lambda^2 a^2 bc\tanh^{2p-2}\tau
\end{aligned}
$$

$$
\begin{aligned}
&-4\left\{2p^2(p^2{+}2)+4p^4\right\}\lambda^2a^2bc\tanh^{2p}\tau\\
&+4\left\{2p^2(p+1)(p+2)+2p^4\right\}\lambda^2a^2bc\tanh^{2p+2}\tau\\
&-4p^2(p+1)(p+2)\lambda^2a^2bc\tanh^{2p+4}\tau\\
&-4p^2(p-1)^2\lambda^2a^2bc\tanh^{2p-4}\tau+4p^2(4p^2{-}4p)\lambda^2a^2bc\tanh^{2p-2}\tau\\
&-4p^2(6p^2{-}2)\lambda^2a^2bc\tanh^{2p}\tau+4p^2(4p^2{+}4p)\lambda^2a^2bc\tanh^{2p+2}\tau\\
&-4p^2(p+1)^2\lambda^2a^2bc\tanh^{2p+4}\tau+\alpha p(p-1)(p-2)\lambda abc\tanh^{p-3}\tau\\
&-\alpha\left\{p(p-1)(p-2)+2p^3\right\}\lambda abc\tanh^{p-1}\tau\\
&+\alpha\left\{p(p+1)(p+2)+2p^3\right\}\lambda abc\tanh^{p+1}\tau\\
&-\alpha p(p+1)(p+2)\lambda abc\tanh^{p+3}\tau=0
\end{aligned}
\tag{3.31}
$$

Thus, from matching the exponents of $\tanh^{2p+4}\tau$ and $\tanh^{p+5}\tau$ terms in Eq (3.31), we obtain

$$2p+4=p+5, \tag{3.32}$$

which yields

$$p=1. \tag{3.33}$$

It needs to be noted that the same value of $p$ is yielded when the exponents pairs $2p+2$ and $p+3$, $2p$ and $p+1$, $2p-2$ and $p-1$, $2p-4$ and $p-3$ are equated with each other. Thus (3.31) has the linearly independent functions $\tanh^{p+j}\tau$ for $j=\pm1,\ \pm3$ and $\tanh^{2p+k}\tau$ for $k=0,\ \pm2,\ \pm4$.Now, $\tanh^{p-5}\tau$ is a stand alone linearly independent function. Therefore setting its coefficients to zero also yields the same value of $p$ as in (3.33) Hence setting their respective coefficients to zero and put $p=2$ which gives the following set of equations:

$$132\lambda^2a^2bc+6\lambda vac+240\lambda a^3bc-\alpha\lambda abc=0, \tag{3.34}$$

$$60\lambda^2a^2bc+120\lambda a^3bc=0, \tag{3.35}$$

$$-136\lambda a^3bc-84\lambda^2a^2bc+8\alpha\lambda abc-8\lambda vac=0, \tag{3.36}$$

$$12\lambda^2a^2bc-2\alpha\lambda abc+16\lambda a^3bc+2\lambda vac=0 \tag{3.37}$$

Solving the above equation (3.34), we get

$$v = b\alpha - 22\lambda ab - 40a^2 b. \tag{3.38}$$

And we solving Eq. (3.35), we obtain

$$\lambda = -2a \tag{3.39}$$

It needs to be noted that by equating the two values of the velocity $v$ from (3.36) and (3.37) also implies (3.38).

Thus, finally the topological soliton solution to the extended (3+1)-dimensional shallow water wave equation (3.8) is given by

$$u(x, y, z, t) = \lambda \tanh(ax + by + cz - vt), \tag{3.40}$$

where the free parameters $\lambda$ is given by (3.39) and the velocity $v$ of the wave is given by (3.38).

**Remark 2:** Comparing our results with Wazwaz's [45] results, it can be seen that the results are new.

**Remark 3:** Similarly, for the extended (2+1)-dimensional shallow water wave equations, using the wave variable

$$\tau = ax + by - vt, \tag{3.41}$$

Eqs. (3.5) and (3.6) and the ansatz method is applied to these equations, it can be seen that the obtained solutions are the same with solutions of Equations (3.7) and (3.8), respectively.

## 4. Conclusions

We have derived the exact topological soliton solutions of the three nonlinear equations. The 1-soliton solution is obtained by solitary wave ansatz method. With the aid of Maple, it is confirmed that the solutions are correct since these solutions satisfy the original equation. To our knowledge, these new solutions have not been reported in former literature. In view of the analysis, we see that the used method is an efficient method of integrability for constructing exact soliton solutions.

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