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Exact travelling wave solutions for some complex nonlinear partial differential equations

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Abstract This paper reflects the implementation of a reliable technique which is called $\left(\frac{G'}{G}\right)$-expansion method for constructing exact travelling wave solutions of nonlinear partial differential equations. The proposed algorithm has been successfully tested on two two selected equations, the balance numbers of which are not positive integers namely Kundu-Eckhaus equation and Derivative nonlinear Schrödingers equation. This method is a powerful tool for searching exact travelling solutions in closed form

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1. Introduction

The study of nonlinear partial differential equations (NPDEs) is extremely important in various branches of applied sciences [1-23]. These NPDEs form the fabric of various physical phenomena in nonlinear optics, plasma physics, nuclear physics, mathematical biology, fluid dynamics, and many other areas in physical and biological sciences.

With the development of soliton theory, many useful methods for obtaining the exact solutions of nonlinear partial differential equations have been presented, some of them are: the $\left(\frac{G'}{G}\right)$-expansion method [1-8], the simplest equation method [9-11], the solitary wave ansatz method [12-14], the first integral method [15-18], the functional variable method [19-21] and so on.

Recently, a new method has been proposed by Wang et al., [4] called the $\left(\frac{G'}{G}\right)$-expansion method to study traveling wave solutions of nonlinear evolution equations. This useful method is developed successfully by many authors [1-3, 5-8] and the reference therein. The $\left(\frac{G'}{G}\right)$-expansion method [4-8] is based on the assumptions that the traveling wave solutions can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ such that $G = G(\xi)$ satisfies a second order linear ordinary differential equation (ODE). In this paper, we describe the $\left(\frac{G'}{G}\right)$-expansion method [4-8] for finding traveling wave solutions of nonlinear partial differentiae equations and then subsequently it will be applied to solve Kundu-Eckhaus equation and derivative nonlinear Schrödingers equation. The paper is arranged as follows. In section 2, we describe briefly the $\frac{G'}{G}$-expansion method. In sections 3 and 4, we apply this method to Kundu-Eckhaus equation and derivative nonlinear Schrödingers equation.

2. The $\frac{G'}{G}$-expansion method

Consider a nonlinear evolution equation:

$$F\left(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, ...\right) = 0, \tag{2.1}$$

where F is a polynomial in u and its partial derivatives. In order to solve Eq. (2.1) using the $\frac{G'}{G}$-expansion method, we give the following main steps [5-8]:

Step 1. Using the wave transformation

$$u(x,t) = u(\xi),\ \xi = x - ct, \tag{2.2}$$

from Eq. (2.1) and Eq. (2.2) we have the following ODE:

$$P\left(u, u', u'', ...\right) = 0, \tag{2.3}$$

where P is a polynomial in u and its total derivatives and $' = \frac{d}{d\xi}$.

Step 2. We suppose that Eq. (2.3) has the formal solution:

$$u(\xi) = \sum_{l=0}^{N} A_l \left(\frac{G'(\xi)}{G(\xi)}\right)^l, \tag{2.4}$$

where A_l are arbitrary constants to be determined such that $A_N \neq 0$, while $G = G(\xi)$ satisfies the second order linear ordinary equation (LODE) in the following form

$$G'' + \lambda G' + \mu G = 0, \tag{2.5}$$

where λ and μ are constants to be determined later and the prime denotes the derivative with respect to ξ.

Step 3. We determine the positive integer N in Eq. (2.4) by balancing the highest order derivatives and the nonlinear terms in Eq. (2.3).

Step 4. Substituting Eq. (2.4) into Eq. (2.3) with Eq. (2.5), then the left hand side of Eq. (2.5) is converted into a polynomial in $\frac{G'(\xi)}{G(\xi)}$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for $A_l,\ \mu,\ c,\ \lambda$.

Step 5. Solving the algebraic equations obtained in step 4, and substituting the results into Eq. (2.4), then we obtain the exact traveling wave solutions for Eq. (2.3).

Remark . The second order LODE (2.5) has the following solutions:

When $\lambda^2 - 4\mu > 0$,

$$\frac{G'}{G} = -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) + C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right), \tag{2.6}$$

When $\lambda^2 - 4\mu < 0$,

$$\frac{G'}{G} = -\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right) + C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)}\right), \tag{2.7}$$

When $\lambda^2 - 4\mu = 0$,

$$\frac{G'}{G} = -\frac{\lambda}{2} + \frac{C_1}{C_1\xi + C_2}, \tag{2.8}$$

When $\mu = 0,\ \lambda \neq 0$,

$$\frac{G'}{G} = -\lambda\frac{C_2 e^{-\lambda\xi}}{C_1 + C_2 e^{-\lambda\xi}}, \tag{2.9}$$

where C_1 and C_2 are arbitrary constants.

3. The Kundu-Eckhaus equation

Let us consider the Kundu-Eckhaus equation [22]

$$iQ_t + Q_{xx} - 2\sigma|Q|^2Q + \delta^2|Q|^4Q + 2i\delta\left(|Q|^2\right)_x Q = 0. \tag{3.1}$$

We may choose the following traveling wave transformation:

$$Q(x,t) = e^{i(\alpha x+\beta t)}u(\xi),\ \ \xi = ik\left(x - 2\alpha t\right), \tag{3.2}$$

where $k,\ \alpha$ and β are constants to be determined later.
On substituting these into Eq. (3.1) yields

$$Q_t = i\left(\beta u - 2k\alpha u'\right)e^{i(\alpha x+\beta t)}, \tag{3.3}$$

$$Q_{xx} = -\left(\alpha^2 u + 2k\alpha u' + k^2 u''\right)e^{i(\alpha x+\beta t)}, \tag{3.4}$$

$$\left(|Q|^2\right)_x Q = 2iku^2u'e^{i(\alpha x+\beta t)}, \tag{3.5}$$

Substituting Eqs. (3.3)-(3.5) into Eq. (3.1), we have

$$-(\beta+\alpha^2)u - k^2u'' - 2\sigma u^3 + \delta^2 u^5 - 4k\delta u^2 u' = 0. \tag{3.6}$$

When balancing u'' with u^5 then gives

$$N + 2 = 5N \Rightarrow N = \frac{1}{2}. \tag{3.7}$$

To obtain an analytic solution, N should be an integer. This requires the use of the transformation

$$u(\xi) = (v(\xi))^{\frac{1}{2}}, \tag{3.8}$$

that transforms (3.6) to

$$-4(\beta+\alpha^2)v^2 + k^2(v')^2 - 2k^2vv'' - 8\sigma v^3 + 4\delta^2 v^4 - 8k\delta v^2 v' = 0. \tag{3.9}$$

Balancing vv'' with v^4 in (3.9) gives

$$2N + 2 = 4N, \tag{3.10}$$

so that $N = 1$. Hence, we look for solutions to Eq. (3.9) in the form

$$v(\xi) = A_0 + A_1\left(\frac{G'}{G}\right), \; A_1 \neq 0. \tag{3.11}$$

By using Eq. (2.5), from Eq. (3.11) we have

$$v'(\xi) = -A_1\left(\frac{G'}{G}\right)^2 - A_1\lambda\left(\frac{G'}{G}\right) - A_1\mu, \tag{3.12}$$

$$v''(\xi) = 2A_1\left(\frac{G'}{G}\right)^3 + 3A_1\lambda\left(\frac{G'}{G}\right)^2 + \left(2A_1\mu + A_1\lambda^2\right)\left(\frac{G'}{G}\right) + A_1\lambda\mu. \tag{3.13}$$

Substituting Eqs. (3.11)-(3.13) into Eq. (3.9), collecting the coefficients of $(\frac{G'}{G})^l$, $(l = 0, 1, ..., 4)$ and set it to zero we obtain the system

$$-8\sigma A_0^3 + 4\delta^2 A_0^4 - 4(\beta+\alpha^2)A_0^2 + k^2\mu^2A_1^2 - 2k^2\lambda\mu A_0A_1 + 8k\delta\mu A_0^2A_1 = 0,$$

$$16k\delta\mu A_0A_1^2 + 8k\delta\lambda A_0^2A_1 + 16\delta^2A_0^3A_1 - 2k^2(\lambda^2-\mu)A_0A_1 - 6k^2\mu A_0A_1 - 24\sigma A_0^2A_1 - 8(\beta+\alpha^2)A_0A_1 = 0,$$

$$8k\delta\mu A_1^3 + 16k\lambda\delta A_0A_1^2 + 8k\delta A_0^2A_1 + 24\delta^2A_0^2A_1^2 - 2k^2(\lambda^2-\mu)A_1^2 - 6k^2\mu A_1^2 - 6k^2\lambda A_0A_1 - 24\sigma A_0A_1^2 + 2k^2\mu A_1^2 + k^2\lambda^2A_1^2 - 4(\beta+\alpha^2)A_1^2 = 0,$$

$$8k\lambda\delta A_1^3 + 16k\delta A_0A_1^2 + 16\delta^2A_0A_1^3 - 4k^2\lambda A_1^2 - 4k^2A_0A_1 - 8\sigma A_1^3 = 0,$$

$$4\delta^2A_1^4 + 8k\delta A_1^3 - 3k^2A_1^2 = 0. \tag{3.14}$$

Solving this system by Maple gives

$$A_0 = 0, \; A_1 = \left(\frac{-2\pm\sqrt{7}}{\delta}\right)\left(\frac{k}{2}\right), \; \mu = 0, \; \lambda = \left(\frac{-2\pm\sqrt{7}}{-3\pm\sqrt{7}}\right)\left(\frac{\sigma}{k\delta}\right), \tag{3.15}$$

$$\beta = -\frac{\left(24\delta^2\alpha^2 + 4\sigma^2\right)\left(-2\pm\sqrt{7}\right) - 3\sigma^2 - 16\delta^2\alpha^2}{8\delta^2\left(-8\pm3\sqrt{7}\right)},$$

where k and α are arbitrary constants.
By using Eq. (3.15), expression (3.11) can be written as

$$v(\xi) = \frac{k\left(-2\pm\sqrt{7}\right)}{2\delta}\left(\frac{G'}{G}\right), \tag{3.16}$$

From Eq. (2.9), Eq. (3.8) and Eq. (3.16), we obtain:

$$v(\xi) = \frac{k\lambda\left(2 \mp \sqrt{7}\right)}{2\delta}\left(\frac{C_2 e^{-\lambda\xi}}{C_1 + C_2 e^{-\lambda\xi}}\right), \tag{3.17}$$

and

$$u(\xi) = \sqrt{\frac{k\lambda\left(2 \mp \sqrt{7}\right)}{2\delta}}\left(\frac{C_2 e^{-\lambda\xi}}{C_1 + C_2 e^{-\lambda\xi}}\right)^{\frac{1}{2}}. \tag{3.18}$$

Thus, in (x,t)- variables we have the exact traveling wave solution of the Kundu-Eckhaus equation as follows:

$$Q(x,t) = \sqrt{\frac{k\lambda\left(2 \mp \sqrt{7}\right)}{2\delta}}\left(\frac{C_2 e^{-i\lambda k(x-2\alpha t)}}{C_1 + C_2 e^{-i\lambda k(x-2\alpha t)}}\right)^{\frac{1}{2}} e^{i\{\alpha x - At\}} \tag{3.19}$$

where

$$\lambda = \left(\frac{-2 \pm \sqrt{7}}{-3 \pm \sqrt{7}}\right)\left(\frac{\sigma}{k\delta}\right), \quad A = \frac{\left(24\delta^2\alpha^2 + 4\sigma^2\right)\left(-2 \pm \sqrt{7}\right) - 3\sigma^2 - 16\delta^2\alpha^2}{8\delta^2\left(-8 \pm 3\sqrt{7}\right)}.$$

If we choose $C_1 = C_2$ in Eq. (3.19), we obtain the exact solution of the Kundu-Eckhaus equation as follows:

$$Q(x,t) = \sqrt{\frac{k\lambda\left(2 \mp \sqrt{7}\right)}{2\delta}}\left(\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{ik\lambda}{2}\left(x - 2\alpha t\right)\right)\right)^{\frac{1}{2}} e^{i\{\alpha x - At\}} \tag{3.20}$$

where

$$\lambda = \left(\frac{-2 \pm \sqrt{7}}{-3 \pm \sqrt{7}}\right)\left(\frac{\sigma}{k\delta}\right).$$

4. Derivative nonlinear Schrödingers equation

In this section we study the Derivative nonlinear Schrödingers equation [23] in the following form

$$q_t + iq_{xx} + \left(|q|^2 q\right)_x = 0. \tag{4.1}$$

We use the transformation

$$q(x,t) = e^{i(\alpha x + \beta t)}u(\xi), \ \xi = ik\left(x + 2\alpha t\right), \tag{4.2}$$

where k, α and β are constants to be determined later.
On substituting these into Eq. (4.1) yields

$$q_t = i\left(\beta u + 2k\alpha u'\right)e^{i(\alpha x + \beta t)}, \tag{4.3}$$

$$q_{xx} = -(\alpha^2 u + 2k\alpha u' + k^2 u'')e^{i(\alpha x + \beta t)}, \tag{4.4}$$

$$\left(|q|^2 q\right)_x = i(\alpha u^3 + 3ku^2 u')e^{i(\alpha x + \beta t)}, \tag{4.5}$$

Substituting Eqs. (4.3)-(4.5) into Eq. (4.1), we have

$$\left(\beta-\alpha^2\right)u-k^2u''+\alpha u^3+3ku^2u'=0. \tag{4.6}$$

When balancing u'' with u^2u' then gives

$$N+2=3N+1 \Rightarrow N=\frac{1}{2}. \tag{4.7}$$

To obtain an analytic solution, N should be an integer. This requires the use of the transformation

$$u(\xi)=(v(\xi))^{\frac{1}{2}}, \tag{4.8}$$

that transforms (4.6) to

$$4\left(\beta-\alpha^2\right)v^2+k^2(v')^2-2k^2vv''+4\alpha v^3+6kv^2v'=0. \tag{4.9}$$

Balancing vv'' with v^2v' in (4.9) gives

$$2N+2=3N+1, \tag{4.10}$$

so that $N=1$. Hence, we look for solutions to Eq. (4.9) in the form

$$v(\xi)=A_0+A_1\left(\frac{G'}{G}\right),\ A_1\neq 0. \tag{4.11}$$

Substituting Eqs. (3.11), (3.13) and Eq. (4.11) into Eq. (4.9), collecting the coefficients of $\left(\frac{G'}{G}\right)^l$, $(l=0,1,...,4)$ and set it to zero we obtain the system

$$-2k^2\lambda\mu A_0A_1+4\alpha A_0^3+4(\beta-\alpha^2)A_0^2+k^2\mu^2A_1^2-6k\mu A_0^2A_1=0,$$

$$-12k\mu A_0A_1^2-6k\lambda A_0^2A_1-2k^2(\lambda^2-\mu)A_0A_1-6k^2\mu A_0A_1+12\alpha A_0^2A_1+8(\beta-\alpha^2)A_0A_1=0,$$

$$-6k\mu A_1^3-12k\lambda A_0A_1^2-6kA_0^2A_1-2k^2(\lambda^2-\mu)A_1^2-6k^2\mu A_1^2-6k^2\lambda A_0A_1+2k^2\mu A_1^2+k^2\lambda^2A_1^2+12\alpha A_0A_1^2+4(\beta-\alpha^2)A_1^2=0,$$

$$-6k\lambda A_1^3-12kA_0A_1^2-4k^2\lambda A_1^2-4k^2A_0A_1+4\alpha A_1^3=0,$$

$$-6kA_1^3-3k^2A_1^2=0. \tag{4.12}$$

Solving this system by Maple gives

$$A_0=0,\ A_1=-\frac{k}{2},\ \mu=0,\ \alpha=-\frac{k\lambda}{2},\ \beta=\frac{k^2\lambda^2}{2},\ \mu=0, \tag{4.13}$$

where k and λ are arbitrary constants.
By using Eq. (4.13), expression (4.11) can be written as

$$v(\xi)=-\frac{k}{2}\left(\frac{G'}{G}\right), \tag{4.14}$$

From Eq. (2.9), Eq. (4.8) and Eq. (4.14), we obtain:

$$v(\xi) = \frac{k\lambda}{2}\left(\frac{C_2 e^{-\lambda\xi}}{C_1 + C_2 e^{-\lambda\xi}}\right), \tag{4.15}$$

and

$$u(\xi) = \sqrt{\frac{k\lambda}{2}}\left(\frac{C_2 e^{-\lambda\xi}}{C_1 + C_2 e^{-\lambda\xi}}\right)^{\frac{1}{2}}, \tag{4.16}$$

Thus, in (x,t)-variables we have the exact traveling wave solution of the Derivative nonlinear Schrödingers equation as follows:

$$q(x,t) = \sqrt{\frac{k\lambda}{2}}\left(\frac{C_2 e^{-ik\lambda(x-k\lambda t)}}{C_1 + C_2 e^{-ik\lambda(x-k\lambda t)}}\right)^{\frac{1}{2}} e^{-\frac{ik\lambda}{2}(x-k\lambda t)}. \tag{4.17}$$

If we choose $C_1 = C_2$ in Eq. (4.17), we obtain the exact solution of the Derivative nonlinear Schrödingers equation as follows:

$$q(x,t) = \sqrt{\frac{k\lambda}{2}}\left\{\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{ik\lambda}{2}(x-k\lambda t)\right)\right\}^{\frac{1}{2}} e^{-\frac{ik\lambda}{2}(x-k\lambda t)}. \tag{4.18}$$

5. Conclusion

In this paper, the $\left(\frac{G'}{G}\right)$-expansion is applied successfully for solving the Kundu-Eckhaus equation and derivative nonlinear Schrödingers equation. The results show that this method is efficient in finding the exact solutions of complex nonlinear partial differential equations.

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